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Application of Iterative Version of Fiction Sources Method for Calculation of Leakage Field in 3D Transformer Model

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Abstract: - The method and results of calculations of electromagnetic leakage field in the 3D transformer model is presented in the paper. In the model, the geometry of windings, core (with 3 or 5 limbs) and tank very close to the actual one, was taken into account. The possibility of the tank's screening was also taken into account. The mathematic problem comes down to search, in a dielectric area, for a complex magnetic potential that fulfils the Laplace's equation and boundary conditions of an impedance type on conducting surfaces. Not commonly known and rarely used numerical method of boundary type was applied to solve the problem in question. The Fiction Sources Method (FSM) relies on an iterative approximation of searched potential function using the linear combination of fundamental solutions. The results of calculations of field components and stray loss density distribution on the surface of transformer tank is presented for several example versions of model.

Key-Words: - electrodynamics, transformer, leakage field, impedance condition, boundary method, approximation, iteration, fiction sources

1 Introduction

The purpose of this paper was to create a numerical program to calculate the quasi-stationary distribution of electromagnetic leakage field in the 3D model of the high-power transformer. This model takes into account the most essential transformer's elements of shapes very close to the reality (Fig.1). The model under consideration consists of system of cylindrical windings, 3 or 5 limb core and an oval tank with a flat bottom and cover. It enables also regarding the tank surface screening. It is assumed that:

- the conductivity and permeability of magnetic metal parts are constant (media uniformity, isotropy, and linearity),
- the metal thickness is significantly larger than the depth of electromagnetic field penetration (this assumption is adequately met for commonly used materials and frequencies in power engineering industry),
- the time variations of the field are sinusoidal (quasi-stationary state),
- the displacement currents may be neglected (the field frequency is not too high).

The problem of leakage field calculation comes down to the solution of the Laplace's equation for the complex magnetic potential with an impedance boundary conditions on the surfaces of core and tank [1, 2, 3, 5].

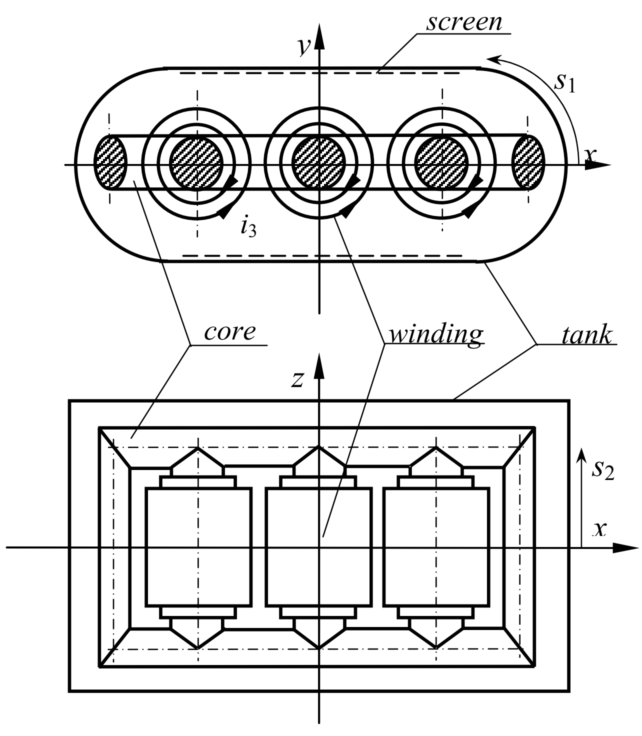

Fig.1 The analysed model of transformer (with five limb core version)

The solution of such a problem in question was obtained via application of certain not commonly known and rarely used numerical method, of which main idea relies on approximation of searched potential function using the linear combination of fundamental solutions of the Laplace's equation [4]. The singular points of those solutions ("fiction sources") are assumed outside of the considered area of a dielectric interior of transformer (i.e. inside of the core and tank).

Before, this model was considered using much more complex boundary elements method (BEM) [1, 2, 3]. In this paper, the comparison of results obtained by using both methods, and measurements performed on the real model of transformer are presented.

2 Description of transformer model

The geometry of the analysed model, and the coordinate system are presented in Fig. 1.

The transformer windings are presented by the infinitely thin cylinders with currents flowing circumferentially and having uniform density along each winding height. It is assumed that for each limb the currents flowing through N_w windings (only 2 windings per limb are presented in Fig. 1). It means that their phase replacement in relation to the phase of the middle limb current are equal to $\pm 2/3\pi$.

The transformer core consists of three limbs of circular cross-section, two external limbs of elliptic cross-section and two yokes of circular cross-section. The limbs and yokes are of ideal ferromagnetic ($\mu = \infty$).

The tank and the covers are formed by regular shells of planar and cylindrical shape, and are made of an isotropic metallic conductor of constant conductivity γ and magnetic permeability μ. The possibility of tank's screening is taken into account by assuming different parameters γ and μ for some part of the tank surface.

3 Mathematical expression of problem

The mathematical formulation of the problem is described in details in [3] (see also [1, 2]).

In the dielectric region, the complex scalar magnetic potential is defined as:

$$\boldsymbol{B} = grad\varphi \tag{1}$$

which satisfies the Laplace's equation:

$$\Delta\varphi = 0 \tag{2}$$

with the following boundary condition:

- for the core surfaces Ω_1 ($\mu = \infty$)

$$\varphi = 0 \tag{3}$$

- for the tank and cover surfaces Ω_2 (impedance type condition):

$$\tilde{\Delta}\varphi = (1+j)\beta\frac{\partial\varphi}{\partial n} \tag{4}$$

where: $j = \sqrt{-1}$, $\beta = \sqrt{\dfrac{\omega\gamma\mu_0}{2\mu_r}}$, and $\tilde{\Delta}$ is the Laplace's operator with respect to the tangential surface coordinates s_1, s_2 (see Fig.1):

$$\tilde{\Delta} = \frac{1}{h_1 h_2}\left[\frac{\partial}{\partial s_1}\left(\frac{h_2}{h_1}\frac{\partial}{\partial s_1}\right) + \frac{\partial}{\partial s_2}\left(\frac{h_1}{h_2}\frac{\partial}{\partial s_2}\right)\right] \tag{5}$$

and

$$h_i = \sqrt{\sum_{j=1}^{3}\left(\frac{\partial x_j}{\partial s_i}\right)^2} \quad i = 1, 2 \tag{6}$$

are Lame parameters.

Let us introduce the convenient operator:

$$L_\beta = \begin{cases} 1 & \text{for} \quad \beta = 0 \\ \tilde{\Delta} - (1+j)\beta\dfrac{\partial}{\partial n} & \text{for} \quad \beta \neq 0 \end{cases} \tag{7}$$

The equations (3), (4) can be rewritten together in more simple form:

$$L_\beta[\varphi(\boldsymbol{r})] = 0 \quad \text{for} \quad \boldsymbol{r} \in \Omega = \Omega_1 \cup \Omega_2 \tag{8}$$

The determination of the potential φ allows to determine the magnetic field in the dielectric areas from (1), as well as tangential components of the electric field on the tank surface:

$$E_1 = (1+j)\frac{\omega}{2\beta}B_2, \quad E_2 = -(1+j)\frac{\omega}{2\beta}B_1 \tag{9}$$

and then consequently stray loss density S in a conducting media by the normal component of the Poynting's vector:

$$\Pi_3 = (1+j)\frac{\omega}{4\beta\mu_0}\left(B_1^2 + B_2^2\right), \quad S = \mathrm{Re}(\Pi_3) \tag{10}$$

4 Solution method

Due to three planes of symmetry of the model ($x = 0$, $y = 0$, $z = 0$) it was possible to limit the solution to only one of eight symmetric parts, e.g. $x \geq 0$, $y \geq 0$, $z \geq 0$. The borders of the region, representing the surfaces of the tank, the cover and the core have been divided into seven planar and cylindrical areas (three limbs, yoke, cylindrical and flat parts of tank and cover). The local curvilinear orthogonal coordinate systems s_1, s_2, s_3 are introduced for each area. The coordinates s_1, s_2 are current coordinates tangential to the surface area (circuital and axial, respectively), and s_3 is an outward normal to the surface.

The searched potential φ is presented as a sum of two components, which individually satisfy the Laplace's equation:

$$\varphi(\boldsymbol{r}) = \varphi_0(\boldsymbol{r}) + \varphi_1(\boldsymbol{r}) \tag{11}$$

where $\varphi_0(P)$ is an exciting (applied) field potential, due to currents through the windings, $\varphi_1(\boldsymbol{r})$ is an induced field potential, due to the presence of the core and tank in the system.

The exciting field potential for one winding with radius R_k and height h_k (k- number of winding) can be determined in local cylindrical coordinates r, θ, z by means of integration, from formulae [3]:

$$\varphi_{0,k}(r,z) = = \frac{-\mu_0 i_k}{2\pi h_k} \int_0^{\pi} \left(\rho_k - \frac{r\cos\theta}{2} \ln \frac{\rho_k + r\cos\theta - R_k}{\rho_k - r\cos\theta + R_k} \right) \Bigg|_{w_1}^{w_2} d\theta \tag{12}$$

where $\rho_k = \sqrt{r^2 + w^2 - 2rR_k\cos\theta + R_k{}^2}$,

$w_1 = z + \frac{1}{2}h_k$, $w_2 = z - \frac{1}{2}h_k$.

The integral (12) is solved numerically. Full exciting field potential is presented by the sum:

$$\varphi_0(\boldsymbol{r}) = \sum_k \varphi_{0,k}(\boldsymbol{r}) \tag{13}$$

The potential $\varphi_1(\boldsymbol{r})$ is approximated by the following formulae:

$$\varphi_1(\boldsymbol{r}) \approx \tilde{\varphi}_1(\boldsymbol{r}) = \sum_{n=1}^{N} \lambda_n F_n(\boldsymbol{r}) \tag{14}$$

where

$$F_n(\boldsymbol{r}) = \frac{1}{|\boldsymbol{r} - \boldsymbol{r}_n|} \tag{15}$$

are fundamental solutions of the Laplace's equation. Their singular points $\boldsymbol{r}_n$ ("fiction sources") lay outside the dielectric area, where the solution is searched. For considered model, they are fixed in the nodes of the regular network on the plane parallel to the boundary of dielectric area surfaces, inside the core (n = 1,.., N_c) and outside the tank and cover ($n = N_c+1$, .., N).

The complex coefficients λ_n, are found iteratively based on the boundary conditions. Let us define the boundary error of the solution at the i-th step of iteration as the functional:

$$\delta_i = \iint_{\Omega} |g(\boldsymbol{r}) - \tilde{g}_i(\boldsymbol{r})|^2 \, d\Omega \tag{16}$$

where $g(\boldsymbol{r}) = L_\beta[\varphi(\boldsymbol{r})]$, $\tilde{g}_i(\boldsymbol{r}) = L_\beta[\varphi_0(\boldsymbol{r}) + \tilde{\varphi}_{1,i}(\boldsymbol{r})]$.

At each step of iteration, set K_i (i - step number) of parameters λ_n is determined in such manner that boundary error is minimized. Let us assume that after i-1 steps, M parameters λ_n have been found. Based on the best approximation (least squares) method, at the i-th step we have obtained the linear set of equations:

$$\sum_{n=M+1}^{M+K_i} S_{m,n}\lambda_n = T_m \quad m = 1,...,K_i \tag{17}$$

where

$$S_{m,n} = \iint_{\Omega} G_m^*(\boldsymbol{r}) G_n(\boldsymbol{r}) d\Omega \tag{18}$$

$$T_m = \iint_{\Omega} G_m^*(\boldsymbol{r}) \left(g(\boldsymbol{r}) - \tilde{g}_i(\boldsymbol{r}) \right) d\Omega \tag{19}$$

and $G_n(\boldsymbol{r}) \equiv L_\beta[F_n(\boldsymbol{r})]$.

The integrals (18), (19) and system of equations (17) are solved numerically (using the Gauss method). It has been proven [3] that consecutive values of boundary errors (16) represent a non-increasing sequence, hence it is convergent.

After determining the factors λ_n and substituting them to (14) and (11), we obtain the searched solution. Then, based on (1) we can find the components of the magnetic field, and from (9) and (10), the electric field and stray loss density on the tank surface.

5 Results of calculations (examples)

The algorithm as described above was the basis for a computing programs written in Fortran 77 and adopted to widely available personal computers. Before, this model was considered using much more complex boundary elements method (BEM) [3]. To verify both methods as well as to estimate its accuracy, the calculated results were compared with those obtained form the measurements, carried out on a small size model of a the power transformer[1] (with 3 limb core). The comparison of the of the calculated and measured results is given in Fig. 2 - 6. The complete distribution of flux density components on the transformer tank surface of the model under consideration, calculated by the fiction sources method, are presented in Fig 7 – 13.

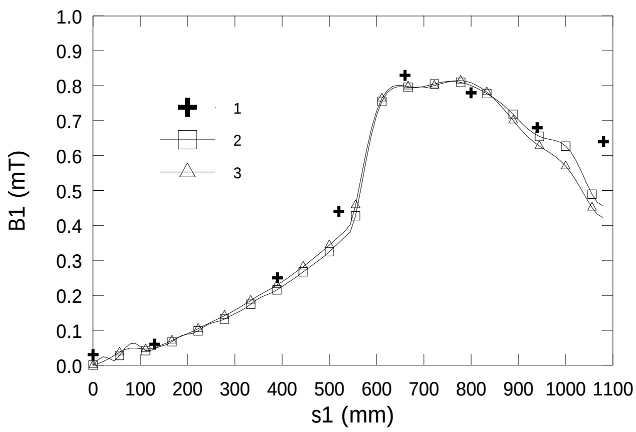

Fig. 2 Distribution of the circuital component of the flux density along the transformer tank circumference at the height corresponding to the winding. 1 – measurements, 2 – results of BEM; 3 – results of FSM.

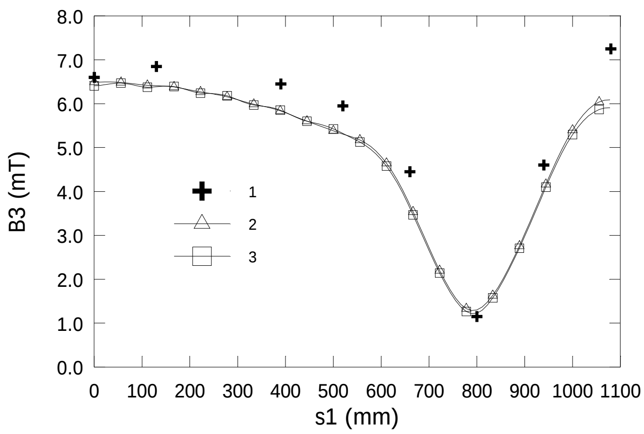

Fig. 3 Distribution of the normal component of the flux density along the transformer tank circumference at the height corresponding to the winding. 1 – measurements, 2 – results of BEM; 3 – results of FSM.

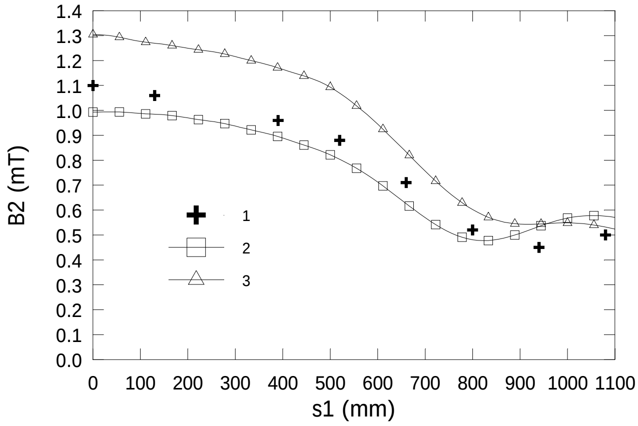

Fig. 4 Distribution of the axial component of the flux density along the transformer tank circumference at the half of the tank height. 1 – measurements, 2 – results of BEM; 3 – results of FSM.

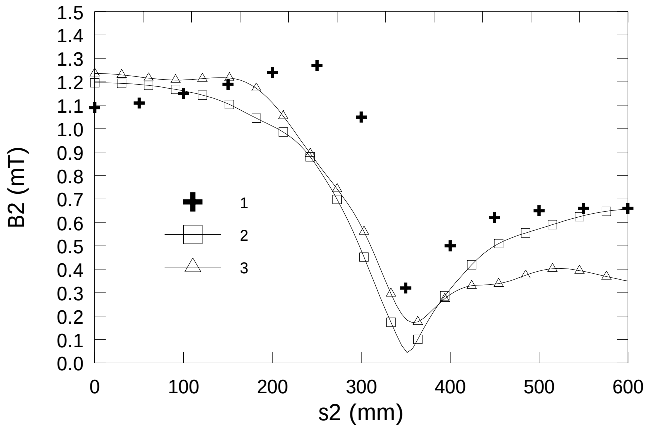

Fig. 5 Distribution of the axial component of the flux density along the transformer tank height for $y = 0$. 1 – measurements, 2 – results of BEM; 3 – results of FSM.

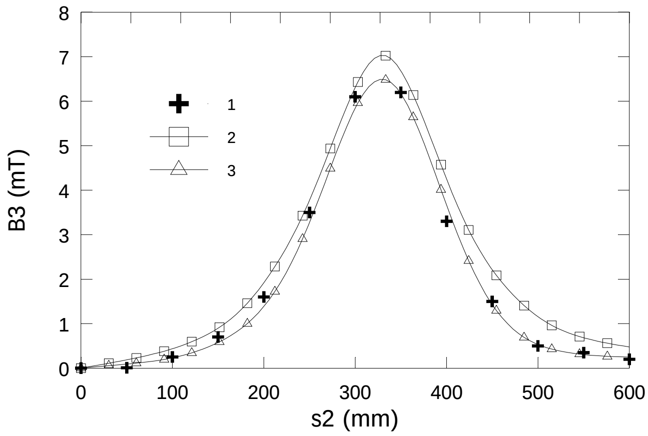

Fig. 6 Distribution of the normal component of flux density along the transformer tank height for $y = 0$. 1 – measurements, 2 – results of BEM; 3 – results of FSM.

[1] Measurements on a model were carried out by Jacek Lasociński D. Sc. (E.E.) from Transformer Division of Institute of Power Engineering.

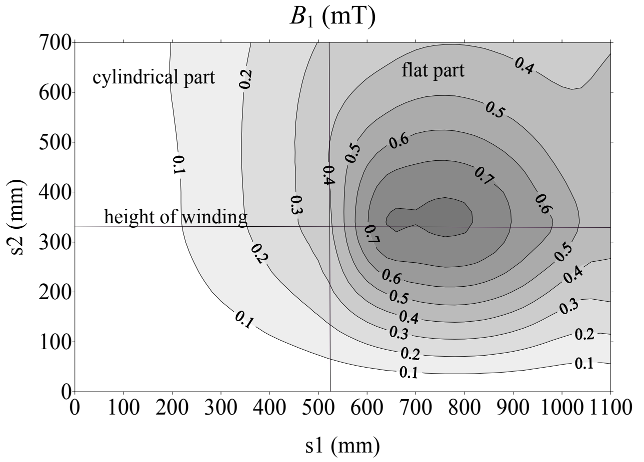

Fig.7 Distribution of the circuital component of the flux density on the transformer tank surface (3 limb core).

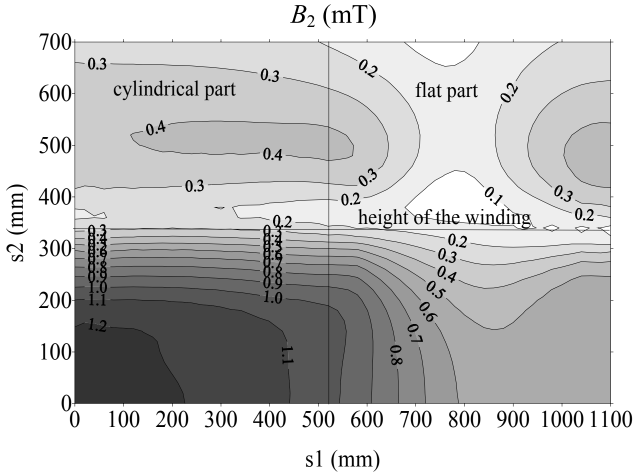

Fig.8 Distribution of the axial component of the flux density on the transformer tank surface (3 limb core).

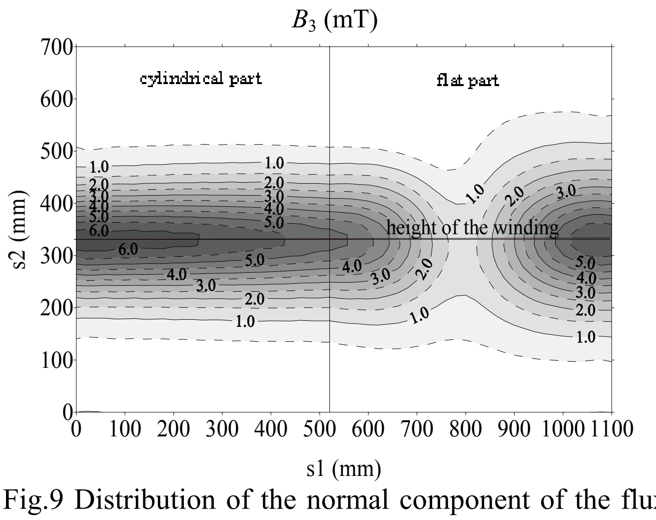

Fig.9 Distribution of the normal component of the flux density on the transformer tank surface (3 limb core).

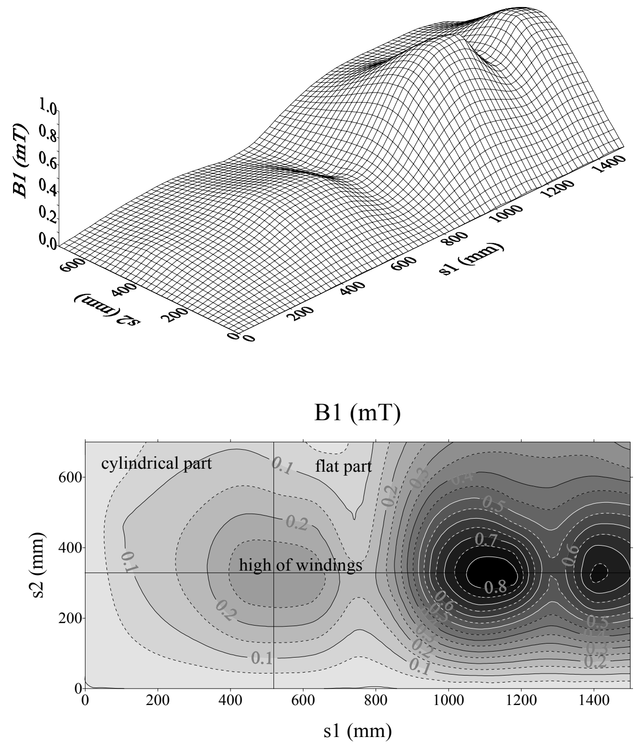

Fig. 10 Distribution of the circuital component of the flux density on the transformer tank surface (5 limb core).

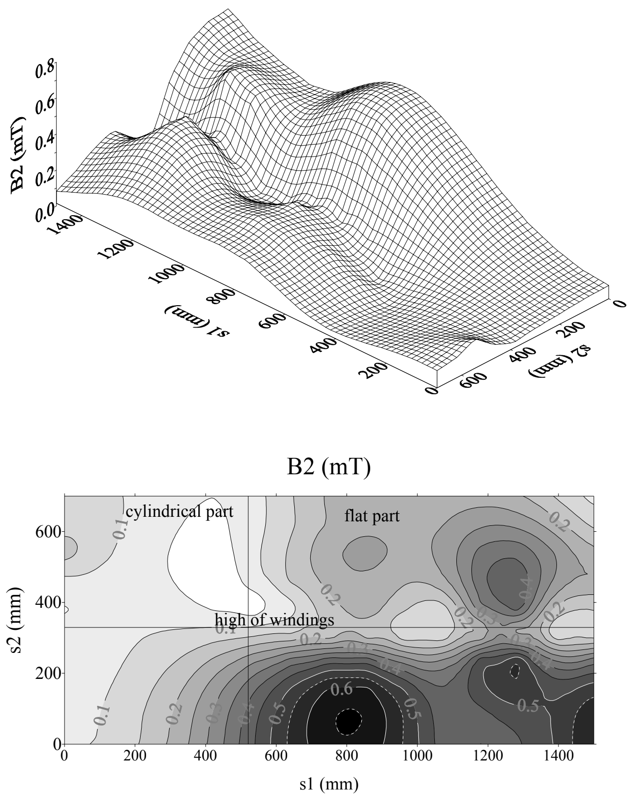

Fig.11 Distribution of the axial component of the flux density on the transformer tank surface (5 limb core).

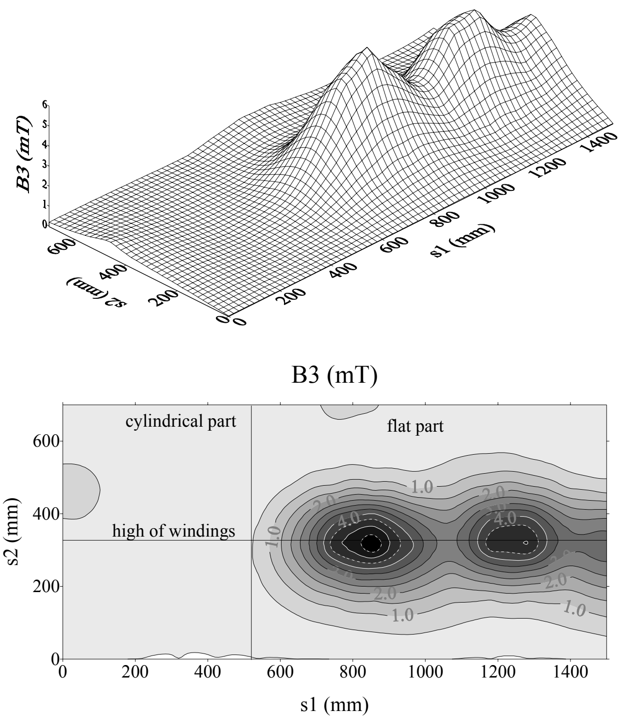

Fig. 12 Distribution of the normal component of the flux density on the transformer tank surface (5 limb core).

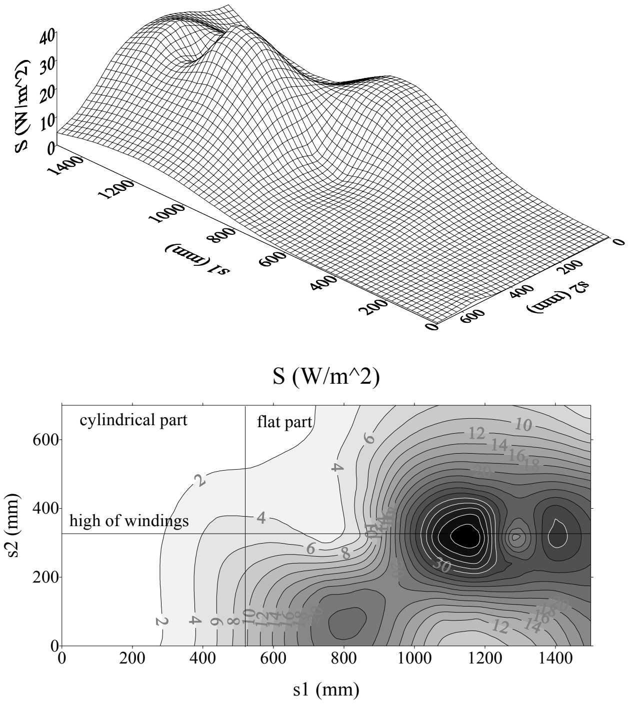

Fig.13 Distribution of the stray loss density on the transformer tank surface (5 limb core).

5 Conclusions

The presented method enables, with use of commonly accessible PC equipment, determining the electromagnetic field distribution in complex three dimensional systems, and achieved accuracy is fully satisfactory for technical purposes. By the application of the boundary type method and iteration fulfilment of boundary conditions, a relatively small numerical model was achieved despite the fact that complex model of transformer was considered. The accuracy of a method (error of boundary conditions' fulfilment) relies mainly on the digitisation size of the boundary surfaces, a number of fundamental solutions used for approximation of searched function of magnetic potential, way of arrangement of their singular points, and a number of iteration steps. It was stated that best results are achieved when those points are located in the relation to the boundary of considered area (metal surfaces of transformer) with distance about 1.5 - 3 times greater than linear size of digitising elements on boundary to calculation integrals (14), (15). It was stated that the iteration version of method enables, in the principle, unlimited extension of model without deterioration of accuracy of boundary conditions' fulfilment (and therefore calculations' accuracy) and the only cost of that is a longer time of program execution (larger number of iteration steps).

The comparison results of calculations achieved sing the proposed method with results achieved using much more complex boundary elements method shown that there are not significance differences between them. The computing program, suitable for personal computers may be already practically used for studies when designing new transformers.

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