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Simulation and multirate control of a mini-helicopter robot

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Abstract: - This paper presents the application and testing via simulation of a multirate control to the stabilization of a mini-helicopter robot. A multirate modeling method is described, which is the base for a specialized software developed for the modeling and simulation of multirate control systems – the Multirate Control Toolbox (MCT). A model in Matlab/Simulink of a mini-helicopter is used and briefly described.

Key-Words: - Multirate systems, mini-helicopter, CACSD, modeling and simulation, control, Matlab, UAV

1 Introduction

Since 1957 diverse techniques have been proposed and used for the modeling and design of multirate control systems ([2],[4]). A common characteristic in these approaches is the representation of the multirate system by means of an equivalent single-rate linear time-invariant equation with a T_o sampling period (frame period), which is the least common multiple of all sampling periods in the system, and which contains more inputs and outputs than the original system (this is the so-called lifting representation of the multirate system [2],[6],[7]). Several authors ([1],[7]) have shown that this representation preserves many of the properties of the original multirate system (reachability, observability, stability).

In this paper a method for multirate systems modeling is presented, which considers more general and arbitrary schemes of input and output sampling. So, it is possible to model multi-input-multi-output (MIMO) systems with input and/or output offset and irregular sampling rates. The input offset (see Fig. 2) is presented, for example, in the case of control signals that are applied sequentially through a shared communication channel and the communication between controller and plant is not always available (serial systems).

A regular sampling scheme is one in which each signal could be sampled at an arbitrary period, but it is constant over the frame period. In an irregular sampling scheme each signal maybe arbitrarily sampled at irregular intervals, however the sequence is repeated over a frame period. The irregular sampling has some interesting characteristics and it is a more realistic way of modeling sampling processes in some environments.

The presented modeling method was implemented with Matlab/Simulink [10] as a toolbox called the Multirate Control Toolbox (MCT), designed in our laboratory and now available for free to contribute to the scientific community [11]. The software allows the modeling and simulation of any type of multirate sampled-data system from its lifting representation and it can be carried out in usual single-rate manner. The introduced tool is applied to the control and testing via simulation of a mini-helicopter robot.

The mini-helicopter, which is part of the Colibri project [12], is a multidisciplinary effort to create an unmanned helicopter with low cost-off-the-shell tools and to test different mathematical approaches for control and identification. Applications for this research vary from open field monitoring to search and rescue. At the moment, the research group is working in control methods, identification, avionics, real-time software, rapid prototyping, and a Controller Area Network (CAN).

The paper is organized as follows. Section 2 introduces the multirate modeling method. Section 3 presents the modeling and simulation of a mini-helicopter using Matlab/Simulink. The simulation of a multirate control applied to a mini-helicopter is developed in section 4, where some illustrative graphics are presented.

2 Irregular multirate model – IMM

An irregular multirate system of order n with m inputs and p outputs is considered. The system and sampling scheme are shown in the Fig. 1 and Fig. 2, respectively. An input and output offset is allowed (this is a relevant characteristic of the proposed method).

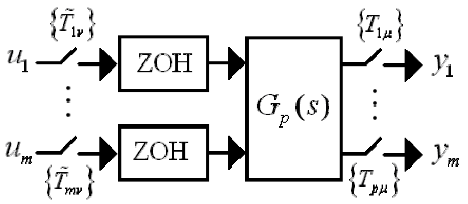


Fig.1. Irregular multirate system

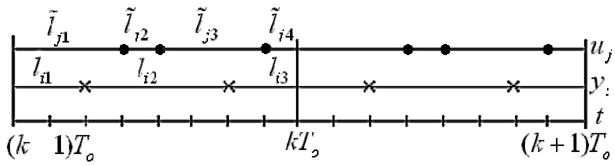


Fig.2. Multirate sampling pattern

T is the base sampling period (the greatest common divisor of all sampling periods in the system) and T_o is the frame sampling period (the enclosing period).

$$i = 1,2,3,...,p\,,\; j = 1,2,3,...,m \qquad (1)$$

$\tilde{N}_j$ is the number of samples of input u_j over the frame period T_o (input multiplicities).
N_i is the number of samples of output y_i over the frame period T_o (output multiplicities).
$\tilde{T}_j = \left[\tilde{T}_{j\nu}\right]$ is the set of the sampling instants $\tilde{T}_{j\nu}$ of input u_j.
$T_i = \left[T_{i\mu}\right]$ is the set of the sampling instants $T_{i\mu}$ of output y_i.

$$\mu = 1, 2 ,.. .,N_i\,,\; \nu = 1, 2, ..., \tilde{N}_j \qquad (2)$$

$$\overline{\overline{N}} = \tilde{N}_1 + ... + \tilde{N}_m\,,\; \overline{N} = N_1 + ... + N_p \qquad (3)$$

$\tilde{l}_j = \left[\tilde{l}_{j\beta}\right]$ is the set of sampling interval lengths $\tilde{l}_{j\beta}$ of input u_j, with respect to base sampling period T.
$l_i = \left[l_{i\alpha}\right]$ is the set of sampling interval lengths $l_{i\alpha}$ of output y_i, with respect to base sampling period T.

$$\tilde{l}_{j\beta} = \frac{\tilde{T}_{j\beta} - \tilde{T}_{j,\beta-1}}{T}\,,\; \sum_{\beta=1}^{\tilde{N}_j+1} \tilde{l}_{j\beta} = N \qquad (4)$$

$$\beta = 1, 2, ..., \tilde{N}_j + 1\,,\; \tilde{T}_{jo} = 0\,,\; \tilde{T}_{j,\tilde{N}_j+1} = T_o \qquad (5)$$

$$l_{i\alpha} = \frac{T_{i\alpha} - T_{i,\alpha-1}}{T}\,,\; \sum_{\alpha=1}^{N_i+1} l_{i\alpha} = N \qquad (6)$$

$$\alpha = 1, 2, ..., N_i + 1\,,\; T_{io} = 0\,,\; T_{i,N_i+1} = T_o \qquad (7)$$

A continuous-time linear invariant plant of order n is considered:

$$\begin{cases} \dot{\mathbf{x}} = \mathbf{A}_p\mathbf{x} + \mathbf{B}_p\mathbf{u} \\ \mathbf{y} = \mathbf{C}_p\mathbf{x} + \mathbf{D}_p\mathbf{u} \end{cases} \qquad (8)$$

The continuous-time plant must be controllable and observable. The discrete-time plant model at base sampling period T is:

$$\begin{cases} \mathbf{x}((k+1)T) = \mathbf{A}\mathbf{x}(kT) + \mathbf{B}\mathbf{u}(kT) \\ \mathbf{y}(kT) = \mathbf{C}\mathbf{x}(kT) + \mathbf{D}\mathbf{u}(kT) \end{cases}$$
$$\mathbf{A} = \mathbf{A}(T) = e^{\mathbf{A}_pT}\;,\; \mathbf{B} = \mathbf{B}(T) = \int_0^T e^{\mathbf{A}_p\tau}\mathbf{B}_p d\tau \qquad (9)$$

The matrices **(A, B, C, D)** have the proper sizes.

Based on progressive substitutions from (9) (see [1],[9] for details), the irregular multirate model (IMM) is obtained, which allows the above mentioned offset.

$$\begin{cases} \mathbf{x}((k+1)T_o) = \tilde{\mathbf{A}}\mathbf{x}(kT_o) + \tilde{\mathbf{B}}\mathbf{u}^D(kT_o) \\ \mathbf{y}^D(kT_o) = \tilde{\mathbf{C}}\mathbf{x}(kT_o) + \tilde{\mathbf{D}}\mathbf{u}^D(kT_o) \end{cases} \qquad (10)$$

The matrices of multirate model are given in detail in [9] and used and documented in [11].

The multirate state vector is:

$$\mathbf{x}^D(kT_0) = \begin{bmatrix} x_1(kT_o) \\ \vdots \\ x_n(kT_o) \\ u_1((k-1)T_0 + \tilde{T}_{1,\tilde{N}_1}) \\ \vdots \\ u_m((k-1)T_0 + \tilde{T}_{m,\tilde{N}_m}) \end{bmatrix} \qquad (11)$$

The multirate input and output vectors are:

$$\mathbf{u}^D(kT_0) = \begin{bmatrix} \vdots \\ u_j(kT_0 + \tilde{T}_{j1}) \\ \vdots \\ u_j(kT_0 + \tilde{T}_{j,\tilde{N}_j}) \\ \vdots \end{bmatrix}$$
$$\mathbf{y}^D(kT_0) = \begin{bmatrix} \vdots \\ y_i(kT_0 + T_{i1}) \\ \vdots \\ y_i(kT_0 + T_{iN_i}) \\ \vdots \end{bmatrix} \qquad (12)$$

The vectors and matrices sizes are:

$$\begin{aligned}
&\mathbf{x} \in \mathbb{R}^{(n+m)\times 1},\ \mathbf{u}^D \in \mathbb{R}^{\bar{\bar{N}}\times 1},\ \mathbf{y}^D \in \mathbb{R}^{\bar{N}\times 1} \\
&\tilde{\mathbf{A}} \in \mathbb{R}^{(n+m)\times(n+m)},\tilde{\mathbf{B}} \in \mathbb{R}^{(n+m)\times\bar{\bar{N}}} \\
&\tilde{\mathbf{C}} \in \mathbb{R}^{\bar{N}\times(n+m)},\tilde{\mathbf{D}} \in \mathbb{R}^{\bar{N}\times\bar{\bar{N}}}
\end{aligned} \tag{13}$$

The main characteristics of the obtained irregular multirate model method IMM are:

- The modeling of multirate sampled-data systems with regular and irregular sampling patterns is possible
- The synchronization of inputs and outputs at each frame period T_0 is not necessary, i.e. an offset is allowed (see Fig. 2)
- The obtained model is causal and linear time-invariant, but with increased minimal dimension ("lifting model")
- An arbitrary number of inputs and outputs can be considered
- Particular models are easily derived (MRIC, MROC, serial, etc)
- The real-time implementation is straightforward using software of rapid prototyping like Matlab

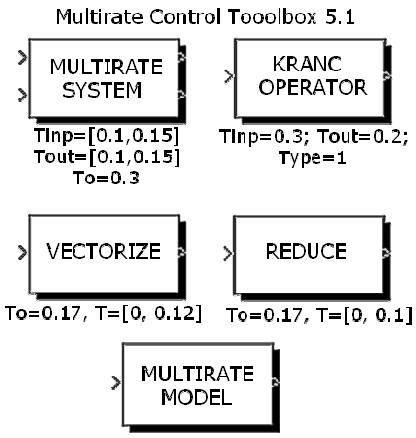


Fig.3. Multirate Control Toolbox

3 Modeling of a mini-helicopter

The mini-helicopter used for the present approach is an X-Cell Gas Graphite hobby helicopter (Fig.4). This helicopter is used in the Colibri project [12], which is being instrumented to perform autonomous flight in real time, using rapid prototyping tools like Matlab/Simulink. At present a simulation model, a Kalman Filter and some controllers have been developed based on [3] and [5]. The model has 15 basic states.

Characteristics of the X-Cell miniature helicopter modeling:

- It is assumed that the fuselage center of pressure coincides with the center of gravity, therefore the moments created by the fuselage aerodynamic forces are neglected
- A quaternion attitude representation is used to allow extreme attitudes
- The rotor head is relatively more rigid, allowing for large rotor control moments
- The blades of the main rotor have no twist
- Some effects are "overpowered" (e.g., interaction between the rotor wake and the fuselage or tail)
- There is a stabilizer bar which is useful to control attitude dynamics
- It is assumed that the inflow is steady and uniform
- The rotor forces and moments largely dominate the dynamic response, and this significantly simplifies the modeling task
- The coupled rotor and stabilizer bar equations can be lumped into one first-order rotor equation of motion (for both lateral and longitudinal tip-path-plane flapping)
- There is an electronic governor for maintaining a near constant rotor speed

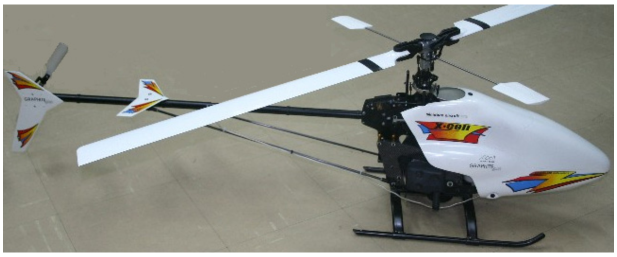


Fig.4. X-Cell 0.60 Gas Graphite

Fig.5. Simulation model of a mini-helicopter in Matlab/Simulink

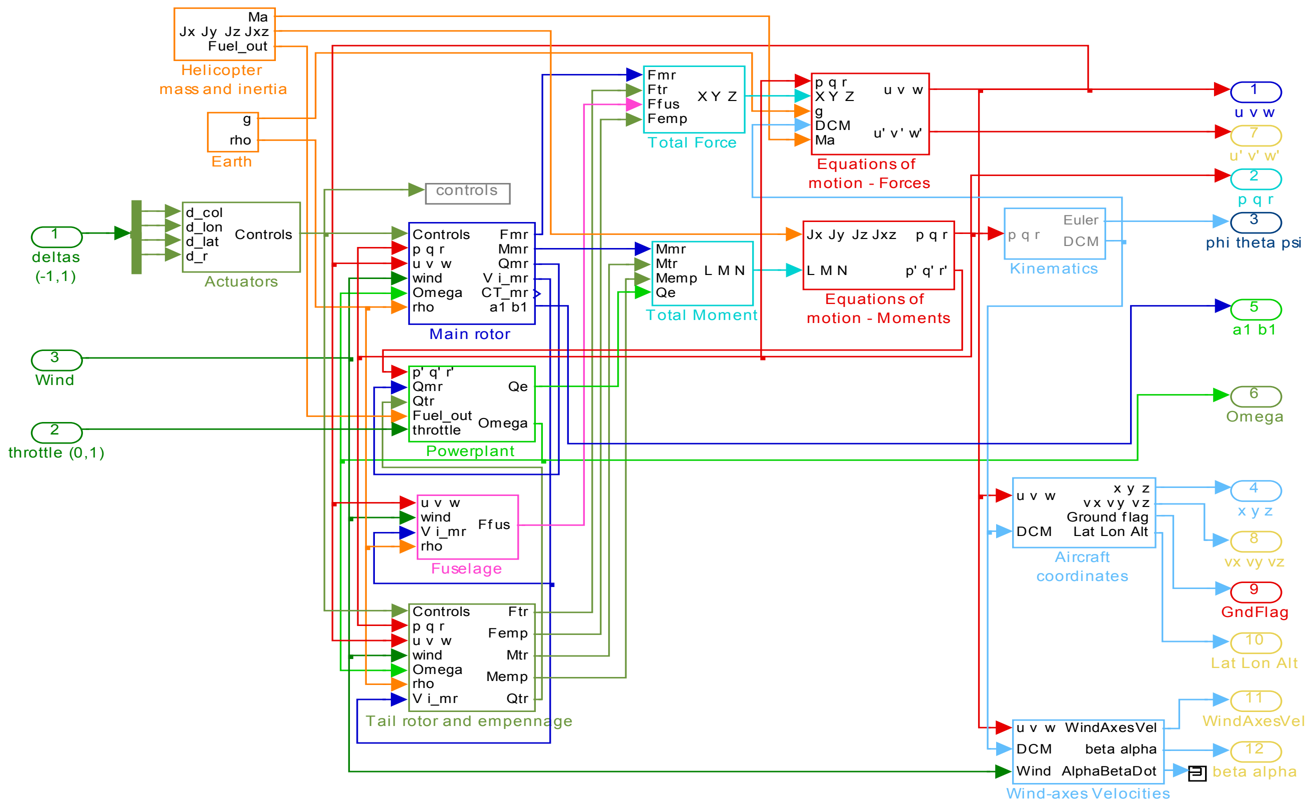
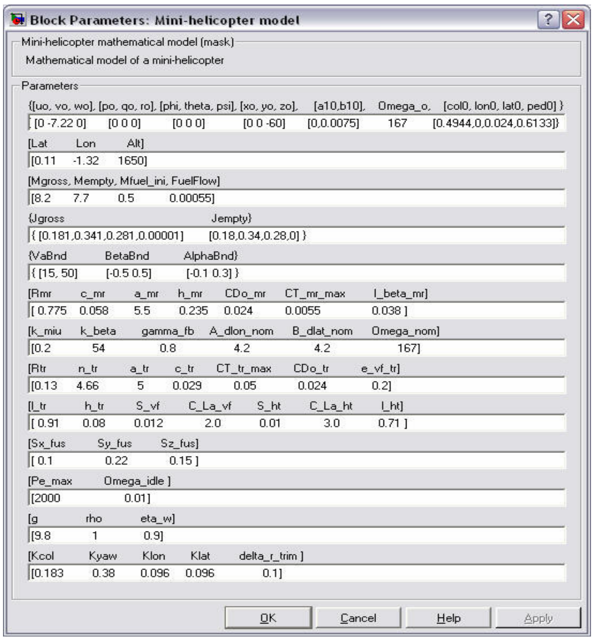


Fig.6. Inside the simulation model of a mini-helicopter in Matlab/Simulink


Block Parameters: Mini-helicopter model

Mini-helicopter mathematical model (mask)

Mathematical model of a mini-helicopter

Parameters

{[uo, vo, wo], [po, qo, ro], [phi, theta, psi], [xo, yo, zo], [a10,b10], Omega_o, [col0, lon0, lat0, ped0] }
[[0 -7.22 0] [0 0 0] [0 0 0] [0 0 -60] [0,0.0075] 167 [0.4944,0,0.024,0.6133]}

[Lat Lon Alt]
[0.11 -1.32 1650]

[Mgross, Mempty, Mfuel_ini, FuelFlow]
[8.2 7.7 0.5 0.00055]

{Jgross Jempty}
{ [0.181,0.341,0.281,0.00001] [0.18,0.34,0.28,0] }

{VaBnd BetaBnd AlphaBnd}
{ [15, 50] [-0.5 0.5] [-0.1 0.3] }

[Rmr c_mr a_mr h_mr CDo_mr CT_mr_max I_beta_mr]
[0.775 0.058 5.5 0.235 0.024 0.0055 0.038]

[k_miu k_beta gamma_fb A_dlon_nom B_dlat_nom Omega_nom]
[0.2 54 0.8 4.2 4.2 167]

[Rtr n_tr a_tr c_tr CT_tr_max CDo_tr e_vf_tr]
[0.13 4.66 5 0.029 0.05 0.024 0.2]

[l_tr h_tr S_vf C_La_vf S_ht C_La_ht l_ht]
[0.91 0.08 0.012 2.0 0.01 3.0 0.71]

[Sx_fus Sy_fus Sz_fus]
[0.1 0.22 0.15]

[Pe_max Omega_idle]
[2000 0.01]

[g rho eta_w]
[9.8 1 0.9]

[Kcol Kyaw Klon Klat delta_r_trim]
[0.183 0.38 0.096 0.096 0.1]

OK Cancel Help Apply

Fig.7. Parameters of simulation Model in Matlab/Simulink

The inputs and states of simulation model are the following (Fig. 5):

$$\mathbf{u} = \begin{bmatrix} \delta_{col} & \delta_{lon} & \delta_{lat} & \delta_{r} & \delta_{t} \end{bmatrix}^T \qquad (14)$$

δ_{col} – collective control input
δ_{lon} – longitudinal cyclic control input
δ_{lat} – lateral cyclic control input
δr – tail rotor yaw control input
δ_t – throttle

$$\mathbf{x} = \left[u, v, w, p, q, r, \varphi, \theta, \psi, x, y, z, a_1, b_1, \Omega\right]^T \quad (15)$$

(u, v, w): Linear velocities: the 3×1 vector of aircraft velocities in body axes
(p, q, r): Angular rates, the 3×1 vector of body angular rates
(φ, θ, ψ): (roll, pitch, yaw). The 3×1 vector of Euler angles
(x, y, z): Cartesian coordinates of aircraft relative to fixed earth
a_1, b_1: The longitudinal and lateral flapping angles
Ω: Main rotor angular speed

The mini-helicopter is an eight-degree-of-freedom system: three lineal displacements (u, v, w), three angular movements (p, q, r) and two main rotor flapping angles (a_1, b_1). The rigid-body dynamics of such vehicles are described by the Newton-Euler equations of motion. There are two reference frames: body-fixed and earth-fixed.

The differential equations describing the mini-helicopter translational and rotational motion in the body-fixed reference are given in equations (16), where $[X\ Y\ Z]^T$ is the vector of external forces acting on the vehicle center of gravity (c.g.), $[L\ M\ N]^T$ is the vector of external moments, M_a is the helicopter mass and I is the inertial tensor.

$$\begin{aligned}
\dot{u} &= (vr - wq) - g\sin\theta + X/M_a \\
\dot{v} &= (pw - ru) + g\cos\theta\sin\varphi + Y/M_a \\
\dot{w} &= (qu - pv) + g\cos\theta\cos\varphi + Z/M_a \\
\dot{p} &= qr(I_y - I_z)/I_x + L/I_x \\
\dot{q} &= pr(I_z - I_x)/I_y + M/I_y \\
\dot{r} &= pq(I_x - I_y)/I_z + N/I_z
\end{aligned} \tag{16}$$

Higher-order effects are taken into account to improve the rigid-body model accuracy. These extensions are rotor dynamics, engine-drive train and actuators dynamics. The coupled rotor and stabilizer bar equations are lumped into one first-order equation of motion. This procedure is done for both lateral and longitudinal tip-path-plane flapping. The equations are expressed as follows:

$$\begin{aligned}
\dot{b}_1 &= -\frac{b_1}{\tau_e} - p + \frac{1}{\tau_e} 2k_\mu \left(\frac{4\delta_{col}}{3} - \lambda_o\right)\frac{v - v_w}{\Omega_{mr}R_{mr}} + \frac{B_{\delta_{lat}}}{\tau_e}\delta_{lat} \\
\dot{a}_1 &= -\frac{a_1}{\tau_e} - q + \frac{1}{\tau_e}\left[\begin{array}{l} 2k_\mu \left(\dfrac{4\delta_{col}}{3} - \lambda_o\right)\dfrac{u - u_w}{\Omega_{mr}R_{mr}} \\ +k_\mu \dfrac{16\mu_{mr}^2}{8|\mu_{mr}| + a_{mr}\sigma_{mr}} \operatorname{sign}\mu_{mr} \dfrac{w - w_w}{\Omega_{mr}R_{mr}} \end{array}\right] \\
&\quad + \frac{A_{\delta_{lon}}}{\tau_e}\delta_{lon}
\end{aligned} \tag{17}$$

$B_{\delta_{lat}}$ and $A_{\delta_{lon}}$ are effective steady-state lateral and longitudinal gains from the cyclic inputs to the main rotor flap angles; (u_W, v_W, w_W) are the wind components, τ_e is the effective rotor time constant for a rotor with the stabilizer bar.

4 Multirate control of a mini-helicopter

A multirate control is now applied to a mini-helicopter robot using the Multirate Control Toolbox (MCT) and the mini-helicopter simulation model described before (see Fig. 10).

It is assumed that all variables are measured and the sample rate of each variable is chosen arbitrarily but near real values. A continuous-time state feedback control **u** = **-Kx** is obtained (see Table 1) for a linearized model around a hover operating point and then is translated into a multirate control using the multirate modeling method exposed above (see [11] for code details). The linearization is performed using tools available with Matlab. Several regular multirate patterns were tested and one is selected to show the possibilities of multirate control.

The MCT and the mini-helicopter model are developed in a way that Matlab with the Real-Time Workshop toolbox [10] allows the automatic generation of code for several operating systems like QNX, xPC, Windows, VxWorks. The analysis, design and code generation are available and easily understanding by a wide group of researchers and students.

Table 1. Matlab control design code

```
% Linearization
x_op= [0,0,0,0,0,0,0.0795,0,0,0,0,-60,0,0.00769,167]
u_op = [0.6123,0,0.0191,0.4273,0.5283 ]
[A,B,C,D]=linmod('colibri', x_op, u_op)

% Continuous state feedback control design
Ko = place (A, B,[-1 -1 -2 -2 -2.5 -2.5 -3 -3 -3.5 -3.5 -
1.6 -1.6 -1.7 -1.7 -1.8])
```

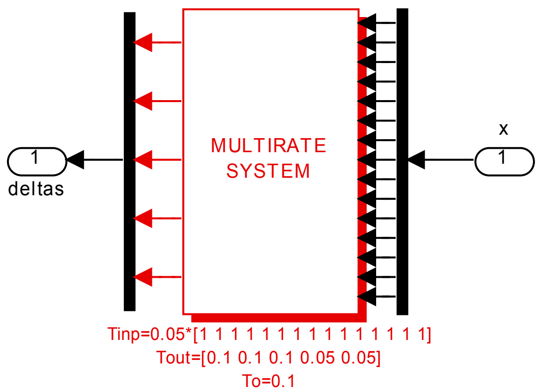


Fig.8. Multirate controller

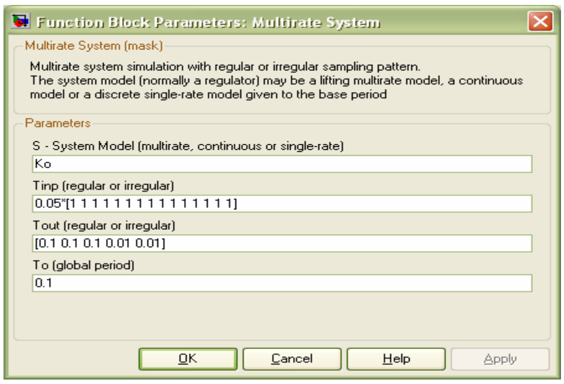


Fig.9. Multirate controller parameters

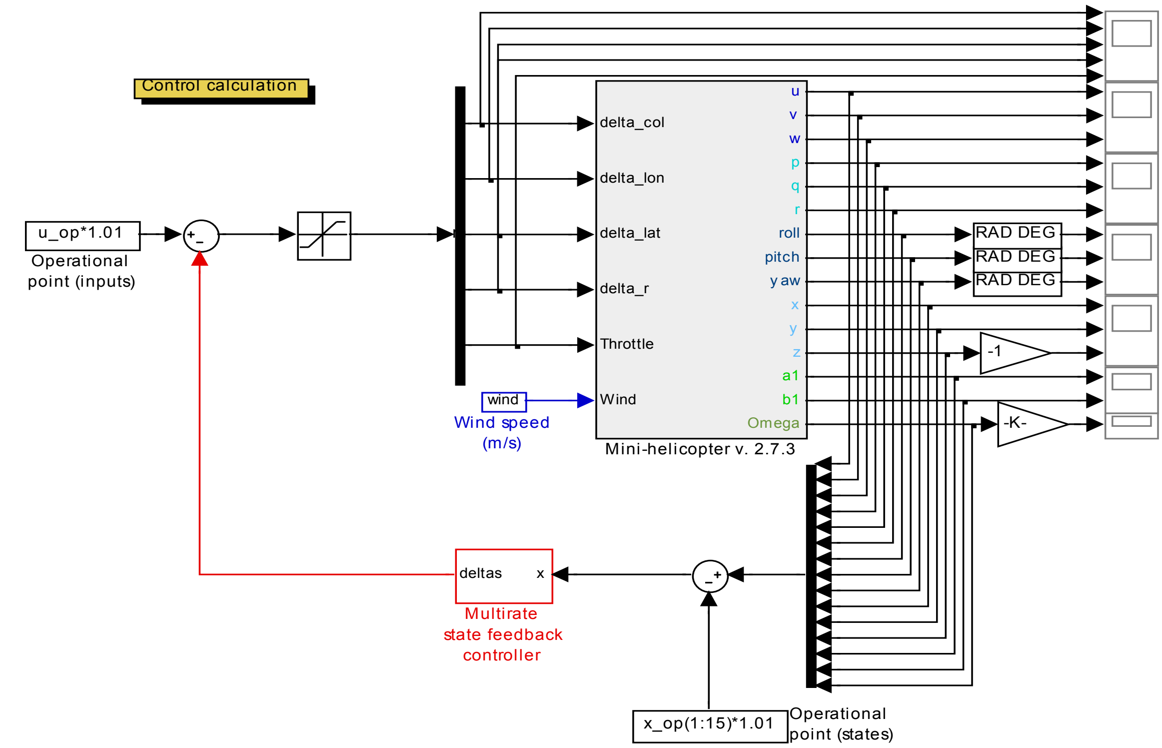


Fig.10. Multirate Control in Matlab/Simulink

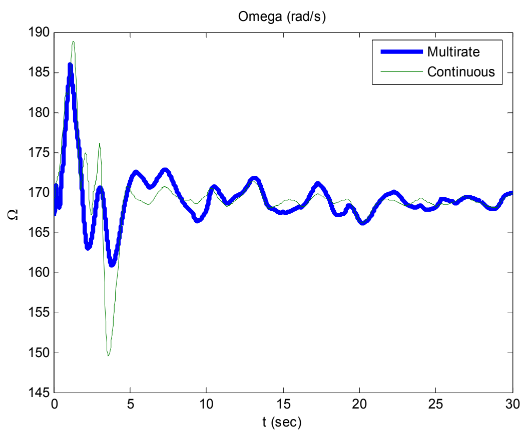


Fig.11. Main rotor angular speed control

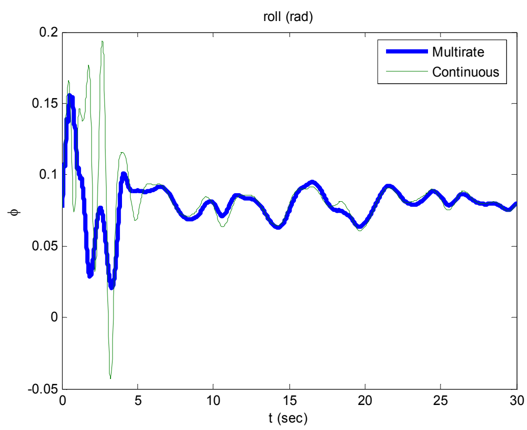


Fig.13. Roll control

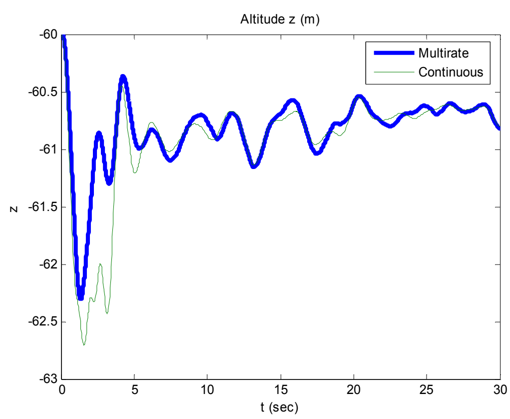


Fig.12. Mini-helicopter altitude control

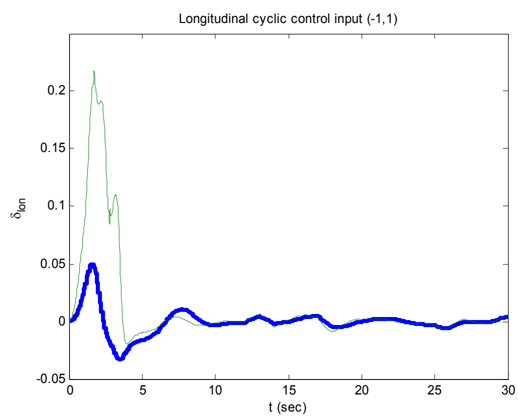


Fig.14. Longitudinal cyclic control input

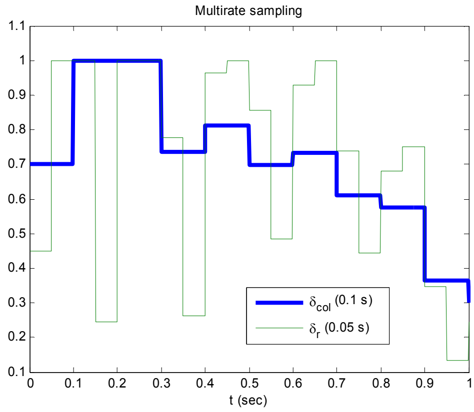


Fig.15. Multirate sampling

The Fig. 11-15 show that the performance of a control system can be improved selecting an appropriate multirate sampling pattern. Another advantage is the possibility to sample each signal at different frequencies, releasing computer time.

5 Conclusions

This paper presents basic ideas about the multirate control of a mini-helicopter robot using techniques of rapid prototyping with CACSD tools like Matlab. The control method is tested via simulation.

The modeling and simulation of multirate control systems were performed based on an irregular multirate model (IMM) which enables the simulation, in an intuitive and usual manner, of a wide variety of multirate control systems. This characteristic is valuable for studying, analyzing and teaching of theory and practice of multirate sampled-data systems.

The MCT is based on the IMM. The more important properties of this toolbox are:

- Computation and simulation of sampled systems with irregular o regular multirate sampling patterns, with or without input and output offset
- It has compact simulation blocks that are similar to single-rate conventional blocks
- A hybrid simulation can be performed, which allows the intersample analysis of continuous-time signals
- Direct extension to periodic systems
- Automatic code generation for several real-time operating systems like QNX, xPC, VxWorks

The results given in last figures show that the Multirate Control Toolbox works well and can be used in an easy manner. The next step of the research is obtaining a better mathematical model via identification and test over the real system the multirate controllers using the capability of Matlab to generate code using the Real-Time Workshop and other toolboxes.

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