# **Dynamical Wavelet Network Model of Iranian Male Labor Supply**

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*Abstract:* - High rate of unemployment is one of the most important economical problems causing various considerable social problems. In order to decrease the unemployment rate, economists should possess mathematical models of both parts of labor market, labor supply and demand. The labor supply is based on human behavior and economical variables, but the labor demand is based on economical factors. Thus, analytical modeling of labor supply is very difficult. The relationship between labor supply inputs is dynamic and very complex. This paper develops wavelet network to model the male labor supply. In fact, it assumes labor supply as a dynamic system presented in a black box model. By using wavelet network, a dynamic model of male labor supply is obtained dealing with system complexity.

*Key-Words*: Male labor supply, Dynamic wavelet network modeling, RBP algorithm.

# **1** Introduction

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The labor supply depends on two kinds of variables, first is economical variables such real wage and real nonlabor income and second is population variables such as growth of population, education, marriage, and etc. Thus, labor supply is based on human behavior resulting in various uncertainties.

The labor demand depends on economical factors such as production, productivity, real wage, and etc.

There are various techniques in analytically modeling the labor supply; however this paper models the labor supply in a black box manner that is based on wavelet networks. The most important result of using the black box modeling technique for labor supply is to overcome the system complexity.

Section 2 explains economical theories and economical empirical results of labor supply. Data is required to identification of models, introduces in section3. Iranian labor supply models are exposed in section4 and conclusion of paper is expressed in section5.

# **2** Backgrounds

Theoretical investigation is the first step in labor supply modeling. In this way, we acquire some knowledge about effective causes on the male labor supply, which clarifies the causality diagram. Subsection 1 briefly explains economical theories of labor supply. Historical backgrounds are reviewed in subsection 2.

# 2.1 Theory

Generally speaking, there are two models expressing the behavior of labor force to supply, static and life cycle model. The static model explains effective causes on male labor supply in every age of life time while ignoring their future's effects. On the other hand, the life cycle model considers their present and future. It should be regarded that the life cycle model theoretically is based on the static model.

Despite this fact, in both of them, behavior of labor forces to supply is developed based on a solution of optimization problem. The problem is the way of allocating a life time to working and leisure such that the utilization function of life is maximized subject to a budget and time limitations. Thus, economical theories emphasize that the behavior of male labor forces and the female one are totally different. In fact, male labor force is to divide life time in two fractions, work and leisure. While female labor force involves allocation of their life time in three fractions, working in a market, working in house and leisure.

In this paper, the results of the optimization problem are the matter of interest, not the problem or its solution. If a behavior of male labor supply is to be considered regardless of the age of the life time, the static model is the right choice, while the life cycle model is used when the behavior of male labor supply is to be considered during the whole life time.

# 2.1.1 Static Model

In the static model, according to reference [1] a utility function is assumed as follow:

$$LF^{s} = f(w, y) \tag{1}$$

where, LF<sup>S</sup> is active population that functions of a real wage rate and nonlabor income. Economic references [1]-

5th WSEAS Int. Conf. on WAVELET ANALYSIS and MULTIRATE SYSTEMS, Sofia, Bulgaria, October 27-29, 2005 (pp55-61) [3], state relationship among LF<sup>S</sup> and w or y and prove is categorizing labor forces to some groups. For e

those relationships. Those references emphasize that the real wage rate have two effects on male labor supply. One is a substitution effect and another is an income effect.

If real wage rate and male labor supply are both increased while nonlabor income is constant, then we call real wage rate has a substitution effect. In this situation, leisure time is expensive. Thus, people prefer to assign more time to working instead of leisure.

If real wage rate is increased while male labor supply is decreased (nonlabor income is still constant), then we call real wage rate has an income effect. In this situation, leisure time is cheap. Thus, people prefer to assign more time to leisure instead of working.

Also, economical references state the relationship among nonlabour income and male labor supply and it is dependent on kind of nonlabor income.

If the nonlabor income is a normal good, an increase in nonlabor income causes to decrease male labor supply. In this case, we call the nonlabor income has an income effect and if the nonlabor income is a inferior good, an increase in nonlabour income causes to increase male labor supply. In this case, we call nonlabor income has a substitution effect. However we emphasize that for usual people, the nonlabour income has an income effect.

Up to now, the static model of male labor supply is explained briefly and two important causes are determined that influence on male labor supply. Next subsection, the life cycle model is expressed.

# 2.1.2 Life Cycle Model

The static model describes how persons divide their time among the labor market, and the nonmarket section. The framework helped us understand how some economical variables, such as real income and wage rate, influence the decisions of whether to work. Despite its usefulness, the static models of labor supply dose not provide a completely satisfying story of how we allocate our time. We make labor supply decisions continuously over the life cycle, and our current decisions influence economical opportunities in the future and are obviously influenced by decisions we made in the past. It is evident that we allocate our time in different ways at different stages of our life cycle.

In life cycle model, the decisions of consumption and working at every stage of life time are based on a price of goods and wage rates in total stages of life. Thus every one's utilization is function of consumption and working time of total life time. Also in life cycle model like the static model, the model of male labor supply is different from female one.

The most important result of life cycle model is that models of labor supply for every stages of life are different. Thus one technique in modeling labor supply

is categorizing labor forces to some groups. For example we categorize them to three groups:

- 1. Group of people that their ages are between 10-19 years. In this group, some people usually have a primary education.
- 2. Group of people that their ages are between 20-24 years. In this group, some people usually have an undergraduate education.
- 3. Group of people that their ages equal or more than 25 years. In this group, some people usually have a graduate education or higher than it.

Finally, we understand from life cycle theory that we can't model the male labor supply with one model totally, and in order to describe the male labor supply effectively, we should categorize the male labor supply population and present one model for every group. So we use three introduced education groups to obtain Iranian male labor supply.

In this subsection, we explain economical theories of male labor supply briefly. If every one needs to more details can return to references [1]-[6].

Up this now, we explain effective variables on the male labor supply. These variables are the real wage rate and real nonlabor income. These variables are called economical variables. But there are some variables have an influence on the male labor supply that are called population variables like the unemployment rate, a ratio of educated population per total population, ratio of students per total population, ratio of married population per total population. These variables are found in historical researches that explained in next subsection.

# **2.2 Historical researches**

In this subsection, we review historical researches associated to the male labor supply. These researches contain Iranian researches and foreign ones. First, foreign researches are investigated and then Iranian ones are reviewed.

# 2.2.1 Foreign researches

Generally speaking, a research on modeling of male and female labor supply in a various age groups is very valuable and is based on life cycle model. Briscoe and Wilson (1992) studied Britain participation rates of male and female labor in various age groups separately. In this study, they divided the labor supply in nine groups and these models were estimated for a short and long run. The labor supply generally was function of real wage rate, domestic production, and unemployment rate. In this study, a coefficient of GDP (domestic production) for females labor supply had a negative sign but in males labor supply didn't have an equal sign, so it means that it has a substitution and income effect on male labor supply models. A coefficient of real wage rate had a strong positive sign in female labor supply models, so it means that it has the substitution effect and the real wage rate had a strong negative sign in high and low age groups, so 5th WSEAS Int. Conf. on WAVELET ANALYSIS and MULTIRATE SYSTEMS, Sofia, Bulgaria, October 27-29, 2005 (pp55-61) it means that it has the income effect on labor supply After 1989, three development programs have run in Iran.

it means that it has the income effect on labor supply models.

Clark and Anker (1990) studied participation rates of old men and women internationally. The participation rate was assumed generally function of the personal domestic production, ratio of 65 ages and older population per active population and ratio of old women per old men. Clark and Anker stated high personal income causes to decrease a participation rate because of income effect. Also they emphasized that increasing of welfare budget causes to decrease a participation rate.

Leoni (1994) studied Italian labor supply. In his paper, the labor supply was modeled based on Keynesian and Neoclassic views and participation rates were assumed as the labor supply variable. In Neoclassic models, participation rates were assumed as functions of unemployment rate, natural immigration rate, fertility, real hour wage rate, real nonlabor income, and virtual variables. In some models, real wage rate had a positive effect on labor supply. But in Keynesian models, the labor supply was a function of nominal wage rate, nominal nonlabor income, unemployment rate, consumer price index and virtual variables. In some models, a coefficient of consumer price index had a negative sign. Also Leoni studied dynamic models for the participation rates.

Elhorest (1996) researched men and women participation rate in twelve European countries and he assumed generally models of participation rates were function of unemployment, ratio of population younger than 25 years per total population, average of hour wage rate, education of older active population. In his study, a coefficient of unemployment had a negative sign and a coefficient of education and wage had a positive sign. Finally, Elhorest stated that his study was according to neoclassic model.

Hans G. Bloemen (2000) modeled a labor supply with limitation on job order. The job order defined on wage rate and hours of work and the job order limited and its distribution was assumed Poisson. In this paper, models of job orders, hours of work per week and wage rate were based on probability functions. He presented that the wage rate is a function of individual properties like level education, working experiments, etc.

Daniel Houser (2003) presented a dynamic and stochastic model of labor supply and investment was based on life cycle theory. He could use Bayesian econometrics for unbiased parameters estimation. The most important idea in his paper is that he was capable to model the labor supply in a stochastic model to decrease a need of expected variables for modeling. In this study, a wage rate function assumed generally function of working experiments, number of work hours in last age, level education, and age.

# 2.2.2 Iranian researches

Labor supply models of third development program:

In third one, programmers modeled Iranian male and female labor supply separately in 10 age groups because of high unemployment rate in 1997 and 1998. Models equations were econometrics relations with logarithm variables. In some models, we found dynamics. Generally speaking, Iranian labor supply models were function of real wage rate, real personal income, unemployment rate, a ratio of student per total population, ratio of high educated population per total population, consumer price index, inflation rate and welfare budget so that real wage rate had the positive and negative effect that means it has a substitution and income effect. Also a real personal income had a positive effect on the labor supply models means it have a substitution effect. that The unemployment rate in all models had a negative effect that means it causes to depress the labor force from finding jobs. Also the ratio of student per total population in all models had a negative effect that means people tend to continue their education. The ratio of high educated population per total population had positive effect that means the high educated people tend to supply their labor more than others. The consumer price index and inflation rate showed costs of life and they had positive effect on labor supply. The welfare budget had negative effect in older groups. It showed an earning income after retirement then more welfare budget causes more earning income after retirement and then decreases the labor supply of older people.

Labor supply models of Iranian industrial development strategy: This strategy is design in 2001 by the suggestion of industrials ministry of Iran. In this plan, designers of strategy modeled Iranian male and female labor supply separately in three age groups consist of 10-19, 20-24, and 25 and more than 25 years because of high unemployment of young people have exited for some years ago. In these models, various factors influence on the labor supply such that the personal income or domestic production (GDP), real wage rate, population, unemployment rate, ratio of student per total population and ratio of high educated population per total population. In this plan, the real wage rate and the personal income or GDP had positive effects on the labor supply models that mean both variables have a substitution effect and the real wage rate didn't appear in female labor supply that means this variable doesn't have a signification effect on the labor supply. Also in this plan, the population and the ratio of high educated population per total population had positive effects and the ratio of student per total population and the unemployment rate had negative effects.

Farjadi and Falihi (1999) estimates models of male and female participation rate separately in separate age groups. In these models, we find some factors influence on the participation rates like this: the real wage rate, personal income, and ratio of student per total population so that the real wage rate and personal income had positive effects on participation rate models and the ratio 5th WSEAS Int. Conf. on WAVELET ANALYSIS and MULTIRATE SYSTEMS, Sofia, Bulgaria, October 27-29, 2005 (pp55-61) student per total population had negative effect on RD: Real Deposit of money in back, participation rate models.

Up to now, we explain theories of labor market and review researches on labor supply. Thus how effects of factors on labor supply are found. Now we are able to start black box modeling of Iranian male labor supply, however we prefer to introduce dataset that we use to identification in next section.

# 3 Data

We use required data to model male labor supply from Statistic Central of Iran that include some variables such that the real wage index of building workers, real gross domestic products, real bank deposit, real welfare budget, ratio of student per population, ratio of high educated population per population, and ratio of married population per population. Then these types of data are available in the programs of humans and homes census in some years consist of 1956, 1966, 1976, 1986, 1996 and we have them in sampling of employment plan in 2003. As we see, we don't have measured data between years of census programs, thus we use the spline interpolation to generate extra data. For the detail of spline interpolation, should be return to [19]-[23]. Totally, there exist 48 samples of data for modeling. Because of complexity of labor supply system and leakage of required data, we can't use 25 percent of samples for validation; thus we should use 45 samples for training and 3 samples for validation.

As we see above, unfortunately we don't have important economical variables i.e. the real wage rate and real personal income in every age, so that we aren't able to model life cycle of male labor supply. Therefore we only use the most important result of life cycle theory that states different models of labor supply exist for every age group. Consequently we categorize male labor supply to three age groups consist of: 10-19, 20-24, and 25 and more ages and we use the static theory of labor supply to model male labor supply. Next section explains the process of dynamic wavelet network modeling of male labor supply. Finally, we emphasize that due to improper ranges of data, we must use scale factor to proper ranges, so that data is in range of zero to one.

# 4 Model

All models of Iranian male labor supply follow from an equal structure that is shown in Fig1. This figure shows generally the influence of economical and population variables.

In this paper, we assume the economical active population as the labor supply. Economical variables consist of real wage rate of building workers, real domestic gross production, real deposit, and real welfare budget. We define abbreviation letters for above economic variables as follow:

RWB: Real Wage index rate of Building worker, RGDP: Real GDP,



model of male labor supply

#### RW: Real Welfare budget.

Population variables consist of ratio of student per population, ratio of high educated people per population, and ratio of married people per population, value of population. We define abbreviation letters for above population variables in follow:

SR: Student per population Rate.

HR: High educated people per population Rate.

MR: Married people per population Rate.

P: value of Population.

We show the delayed male labor supply one time as  $L_{t-1}$  and the male labor supply as  $L_t$ .

As we see in Fig1, we design a dynamic wavelet network model for male labor supply. Also we see the inner of wavelet network in form of Fig2.



The output of wavelet network is written as follow:

$$\overline{\mathbf{Y}} = \sum_{i=1}^{N} \mathbf{W}_{i} \Psi(\mathbf{D}_{i} (\mathbf{x} - \mathbf{t}_{i})) + \overline{\mathbf{g}}$$
(2)

where, X is horizontal vector, N is degree of inner network,  $W_i$  is scalar scale coefficient,  $D_i$  is resolution coefficient and it is a vertical vector that its size equals to input vector,  $t_i$  is transfer coefficient and it is a horizontal vector that its size equals to input vector,  $\overline{g}$  is scalar that

estimates mean of output, and  $\overline{Y}$  is scalar output of wavelet network model.

And the wavelet function is written as follow:

$$\psi(\mathbf{x}) = (1 - \mathbf{x}^2)e^{-\frac{\mathbf{x}^2}{2}}$$
 (3)

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Now we are able to design the wavelet network model of male labor supply. In this research, we use a resilient back propagation that introduced in [25] to training of models parameters. The objective function in the identification is a sum of square errors and we use some validity experiments for validation of models that are consist of in bellow:

- 1- Simulation result,
- 2- Prediction result,
- 3- R<sup>2</sup>: that defined in below[13]:

$$R^{2} = 1 - \frac{\sigma_{\varepsilon}^{2}}{\sigma_{y}^{2}} \approx 1 - \left(\frac{N-1}{N-d}\right) \frac{\sum \varepsilon_{t}^{2}}{\sum y_{t}^{2}} \qquad (4)$$

where,  $\sum \epsilon_t^2$  is sum of errors square,  $\sum y_t^2$  is sum of measured outputs square, N is number of sampled data, and d is number of parameters.

4- PP: that is like  $R^2$  and defined in below [18]:

$$PP := 1 - \left(\frac{N + d - N_n}{N - d + N_n}\right) \frac{\sum_{t=N+1}^{N_n} \varepsilon_t^2}{\sum_{t=N+1}^{N_n} y_t^2}$$
(5)

where,  $N_n$  is number of validity data.

#### 4.1 Iranian male labor supply

In this section, we show the result of three models of Iranian male labor supply respectively:

- Male Labor Supply at 10-19 ages
- Male Labor Supply at 20-24 ages
- Male Labor Supply at 25 and more ages.

We describe results of models in every group. In every part, we show the simulation and prediction results. Up to now, we do two validity experiments, i.e. simulation and prediction for model validation. However, we see in table1  $R^2$  and PP validity experiments of all models.

#### 4.1.1 Male labor supply at 10-19 ages

In this model, the number of inputs is 7 and inputs consist of RBW, RD, RGDP, P, SR, U, and  $L_{t-1}$  respectively. Also we assume the inner degree of network is 4 for wavelet network model of male labor supply at 10-19 ages. We show simulation and prediction results in Fig3.



Simulated Male Labor Supply Real Male Labor Supply at 10-19 1.4 0 Predicted Male Labor Supply 1.7 1.7 Milion Persons 1.65 1.0 1.55 1.5 1996 1997 2000 200 2002 2003 Years

*Fig3-b.* Prediction We see results are very well.

#### 4.1.2 Male labor supply at 20-24 ages

In this model, the number of inputs is 9 and inputs consist of RBW, RD, RGDP, P, SR, HR, MR, U, and  $L_{t-1}$  respectively. Also we assume the inner degree of network is 4 for wavelet network model of male labor supply at 20-24 ages. We show simulation and prediction results in Fig4.





We see results are very well.

5th WSEAS Int. Conf. on WAVELET ANALYSIS and MULTIRATE SYSTEMS, Sofia, Bulgaria, October 27-29, 2005 (pp55-61) **4.1.3 Male labor supply at 25 and more ages** modeling that is based on wavelet network. The results of

In this model, the number of inputs is 10 and inputs consist of RBW, RD, RGDP, RW, P, SR, HR, MR, U, and  $L_{t-1}$  respectively. Also we assume the inner degree of network is 2 for wavelet network model of male labor supply at 25 and more ages. We show simulation and prediction results in Fig5.

We present table1 that shows the  $R^2$  and PP validity experiment for all models.

|                | Male     | Male     | Male           |
|----------------|----------|----------|----------------|
|                | labor    | labor    | labor          |
|                | at 10-19 | at 20-24 | at 25 and      |
|                |          |          |                |
|                |          |          | more           |
| $\mathbb{R}^2$ | 0.9998   | 0.9990   | more<br>0.9999 |

Table1. The R<sup>2</sup> and PP validity experiment



# **5** Conclusion

The analytical modeling of labor supply is very difficault and very complex. To dfeat model complexity, ths paper develops a kind of black box

modeling that is based on wavelet network. The results of designed models are very well. It means that models performances are acceptable.

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