

# A Disturbance Observer Based Nonlinear Controller for Friction Compensation in a Servomechanism

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*Abstract:* - In this paper, a disturbance observer based controller is proposed for the compensation of nonlinear friction in a servodrive system. The designed nonlinear controller uses the estimated system states (position, velocity) and disturbance to enhance tracking performance of the servodrive using high stiff gains for the compensation of the effects of nonlinear friction. The effectiveness of this disturbance observer is demonstrated under simulations.

*Key-Words:* - disturbance observer, disturbance rejection for friction compensation, stiffness control.

## 1 Introduction

The past few years witnessed a rapid growth in the use of nonlinear observers and disturbance observers for the compensation of nonlinearities in mechanical systems such as friction and backlash.

Compensation of disturbances such as nonlinear friction is desirable, because friction forces can not be ignored in motion control systems applications such as robotics where high performance and precision are required. However, direct compensation based on a complete model including the linear and nonlinear parts of the system is difficult because friction parameters are not available and vary with the temperature and mechanical wear.

Friction is still a complex phenomenon which has been subject to extensive research studies [1]. Various models have been formulated in the context of automatic control for compensation purposes [3] [6]. These methods depend on a number of other factors, such as the existence of more than one nonlinearity in the system, which makes these model based compensation techniques unsuitable for high performance applications. In this paper, a nonlinear disturbance observer is proposed for the compensation of nonlinear friction in a system without any knowledge of the friction parameters. The linear Luenberger observer may be considered as a special case of this structure. The servo drive system considered in this simulation study is represented by a one mass mechanical system with nonlinear dynamic friction as a disturbance. The dynamic model is used for its suitability in simulations and its good behaviour at low velocity regimes. The observer gains are initially tuned using a simple method for the linear case. The results are then applied to the overall nonlinear closed loop system. A comparative

simulation study of the nonlinear controller with a PD (Proportional + Derivative) controller is presented including many comparisons between the linear and nonlinear, with and without rejection cases to demonstrate the superiority of the proposed nonlinear observer/controller. The root mean square errors (RMS) of every result are reported in a table as a performance criteria.

## 2 System description

The mathematical model of the mechanical system in the presence of friction is assumed to be of the following form :

$$\begin{aligned} \frac{dx}{dt} &= v \\ \frac{dv}{dt} &= \frac{1}{m}u - \frac{1}{m}F \end{aligned} \quad (1)$$

$x$ ,  $v$  represent the position and velocity respectively,  $m$  denotes the mechanical inertia of the system and  $F$  is the friction force. The friction forces which are the main source of disturbance are represented by the dynamical model LuGre [2] :

$$\begin{aligned} \frac{dz}{dt} &= v - \frac{|v|}{g(v)}z \\ F &= \sigma_0 z + \sigma_1 \frac{dz}{dt} + f_v v \end{aligned} \quad (2)$$

Where the function characterizing the static map of friction is expressed by :

$$g(v) = \frac{1}{\sigma_0} \left( F_c + (F_s - F_c) \cdot e^{-\left(\frac{v}{v_s}\right)^2} \right) \quad (3)$$

Where,  $F_c$ ,  $F_s$ ,  $f_v$  representing coulomb friction, static friction and viscous friction respectively and  $v_s$  is the Stribeck velocity. The dynamic parameters  $\sigma_0$  and  $\sigma_1$  are the stiffness and damping. This model is used because it is one of the most representative in term of reproduction of friction related phenomenon and its suitability in simulations.

### 3 Nonlinear disturbance observer design

For the given nonlinear mechanical system, first, we try to design a linear observer to get information about the unknown part of the system including velocity and friction force. Assuming that we have access only to position measurement, the extended disturbance observer is of the form :

$$\begin{aligned} \frac{d\hat{x}}{dt} &= v - H_1(\hat{x} - x) \\ \frac{d\hat{v}}{dt} &= \delta - \frac{1}{m}u - H_2(\hat{x} - x) \\ \frac{d\hat{\delta}}{dt} &= -H_3(\hat{x} - x) \end{aligned} \quad (4)$$

The nonlinear function  $H_i$  ( $i=1, 2, 3$ ) is defined by

$$H_i(e) = \begin{cases} l_i |e|^a \operatorname{sgn}(e) & |e| > \varepsilon \\ \frac{l_i e}{\varepsilon^{1-\alpha}} & |e| < \varepsilon \end{cases} \quad (5)$$

where  $l_1$ ,  $l_2$  and  $l_3$  are the observer gains.

**Case 1 :** For  $a = 1$ , we get a linear Luenberger observer. A stable linear observer can be designed by ignoring the disturbance (nonlinear friction) and its estimation  $\delta$ , i.e the nonlinear part of the considered system. The eigenvalues of matrix  $A - LC$  are chosen to ensure fast convergence of estimated states (position and velocity) of the linear part of the system. Then, the third gain of the observer which is related to the disturbance ( friction in our case ) is modulated to improve the observation results and ensure convergence of the states. Please, leave two blank lines between successive sections as here.

**Case 2 :** For  $0 < a < 1$ , the observer is nonlinear and can give good dynamis results regarding to its high gains for smaller errors (Fig. 1)

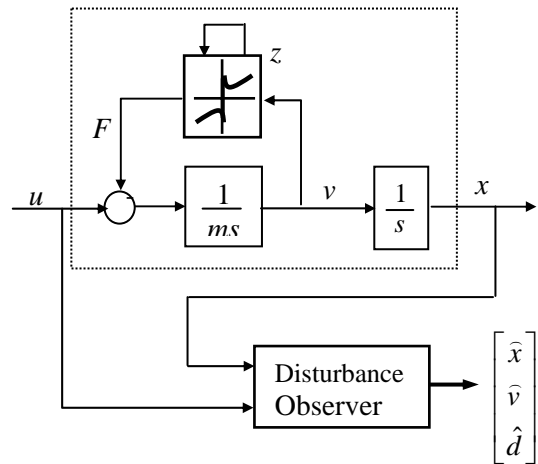


Fig. 1 The considered system with the disturbance observer

### 4 Nonlinear controller design

Increasing system stiffness is one of the well known methods for the compensation of the effects of friction [4]. A nonlinear controller is proposed here for improving tracking and eliminating errors caused by friction. The chosen strategy of control include the disturbance rejection which compensates for the nonlinear friction in the system and avoid the use a large stiff gains in the controller. The control law is characterized by the nonlinear function  $H$  introduced in (5), so that high gains are applied for small errors [5]. The control law is given by :

$$u = ms^2 + \Omega_1(x_r - \hat{x}) + K_d(v_r - \hat{v}) - \frac{\delta}{m} \quad (6)$$

where  $\Omega_1$  is a nonlinear function given by :

$$\Omega_1(x_r - \hat{x}) = \begin{cases} K_p |x_r - \hat{x}|^a \operatorname{sgn}(x_r - \hat{x}) & |x_r - \hat{x}| > \varepsilon \\ \frac{K_p (x_r - \hat{x})}{\varepsilon^{1-\alpha}} & |x_r - \hat{x}| < \varepsilon \end{cases} \quad (7)$$

with,  $x_r$ ,  $v_r$  reference trajectory and velocity respectively.  $K_p$ , proportional and derivative gain respectively.

Figure 2 shows the form of the nonlinear function used in the observer and also in the the controller. It is clear that high gains are equivalent to smaller errors. In the considered system and because the linear design of the observer and the controller are stable, but give poor performances, the use of the nonlinear gains in both the observer and the control is sure to be efficient and to give better results than the linear one.

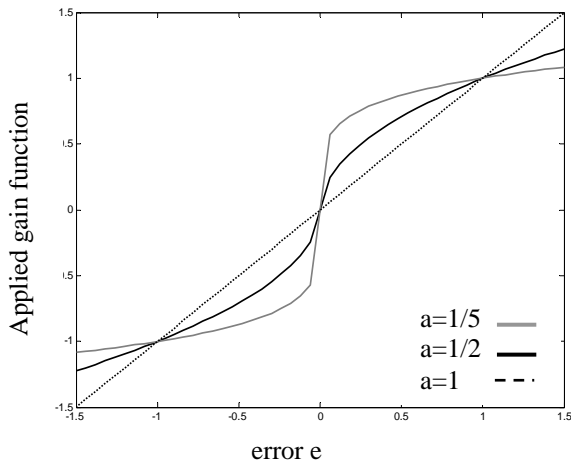


Fig. 2 Nonlinear function representing the gains of the observer and the controller

Fig. 3 depicts the overall control system including the nonlinear observer/controller and the disturbance rejection according to the control law introduced in (6).

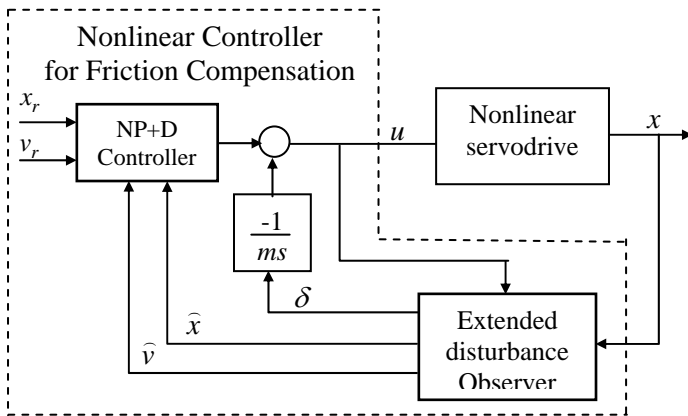


Fig. 3 Overall control system with friction compensation

### 5 Performance evaluation

The system parameters used in the simulations are listed below :

$\sigma_0$	280 Nm/rad	$\sigma_l$	1.0 Nms/rad
$F_c$	0.22 Nm	$F_s$	0.39 Nm
$f_v$	0.0 Nms/rad	$v_s$	0.1 rad/s
$m$	0.0025 Kg.m <sup>2</sup>		

The designed observer is now used to estimate the states of the servo drive system which are then supplied to the position tracking controller. The controller and observer parameters are obtained as  $K_p=2$ ,  $K_v=0.015$  and  $l_1=5$ ,  $l_2=500$ ,  $l_3=1000$  respectively. Letting  $a=1$  leads to a

linear observer whereas  $a=1/3$  corresponds to a nonlinear observer.

The RMS of the tracking error is taken as a performance measure of the designed disturbance observer

$$e_{rms} = \sqrt{\frac{1}{T} \int_0^T e^2(\tau) d\tau} \tag{8}$$

Initially the parameter  $a$  is set to 1 which leads to the following a linear control law

$$u = K_p(x_r - \hat{x}) + K_d(v_r - \hat{v}) - \frac{\delta}{m} \tag{9}$$

The open-loop responses related to the linear and nonlinear observers are shown in Fig. 4.

Fig. 5 shows the position tracking errors of the linear and nonlinear observers combined with a PD controller, both with the rejection of the estimated disturbance.

The RMS for the linear observer case is  $e_{rms}=0.0769$  rad has been reduced about 50% in the nonlinear observer case  $e_{rms}=0.0479$  rad.

These results have been improved by using the nonlinear observer to estimate the disturbance (friction forces which are the cause of the large error, especially at low velocities) and canceling them by rejection.

We notice that the estimation has been improved by using a high gain in the disturbance observation. We can also notice that estimation is more accurate for high velocities than for low velocities.

The linear disturbance observer gives good estimation but with noticeable errors. In order to improve the observer performance, we introduce a nonlinear function that modulates the observation error according to the idea that high gains are needed to improve results.

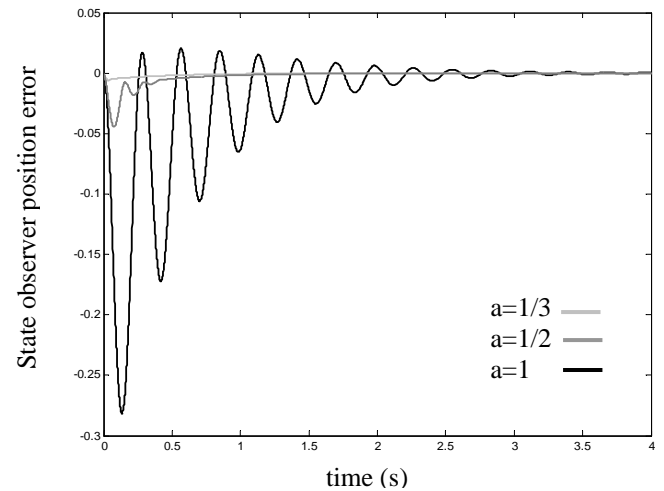


Fig. 4 Open loop responses of the linear and nonlinear observer

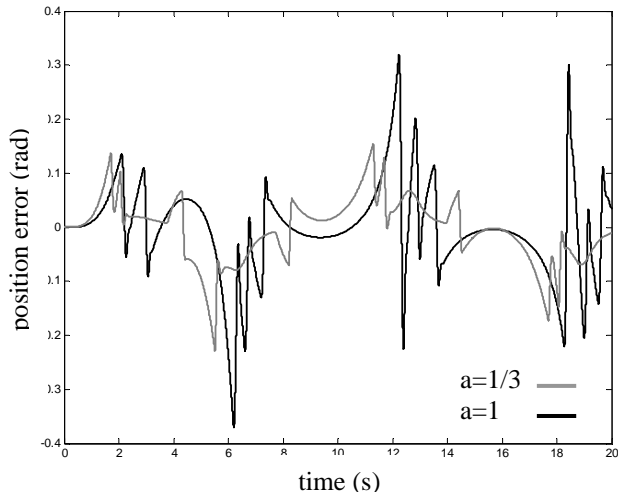


Fig. 5 : Tracking error under PD control with the disturbance rejection ,linear observer ( black ), non-linear observer ( gray).

The extended nonlinear disturbance observer is used here with the same gains as for the linear observer. This extended observer demonstrates an improved performance and robustness. We can also use high gains in our observer without causing any instability or divergence of the estimation because of the boundness of the nonlinear function  $H_i$  for relatively high input signals. The results show the improvement brought by both : the rejection of the disturbance and the use of nonlinear gains in the observer.

The tracking error is however important, so we must enhance the tracking performance by a nonlinear controller NPD with the disturbance rejection. This is demonstrated by Fig. 6 where a RMS error value of  $e_{rms}=0.0028$  has been achieved.

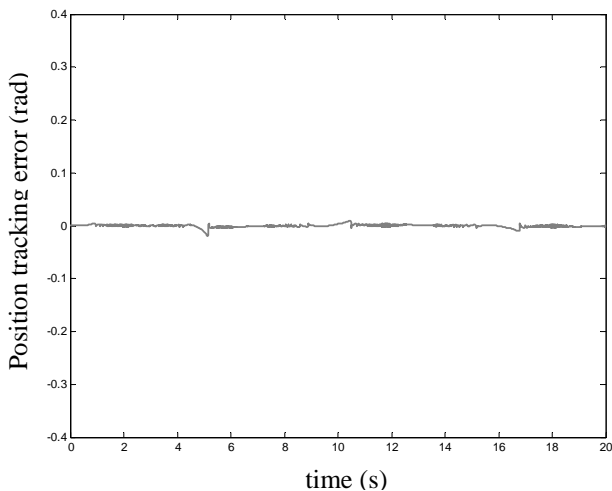


Fig. 6 Position tracking error , NPD + disturbance rejection.

Consider the reference trajectory drawn in Fig. 7 such that:

$$x_r = \tan^{-1} (4 \sin(0.5t)) (1 - \exp(-0.01t^3)), \text{ rad} \quad (10)$$

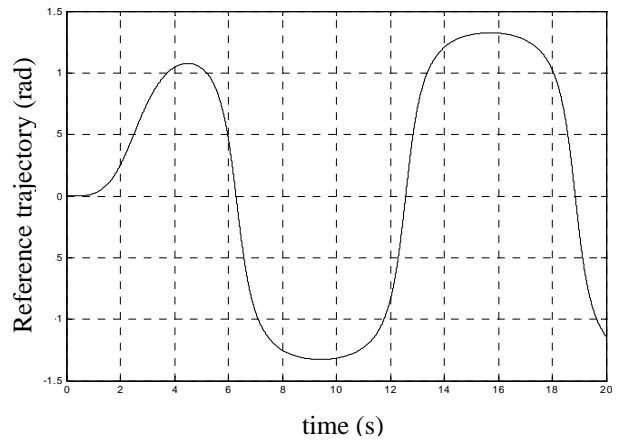


Fig. 7 Reference trajectory

Figure 8 shows improved tracking of the desired position achieved by the proposed control scheme. The undesirable effects caused by friction forces were cancelled using a combined nonlinear disturbance observer/controller.

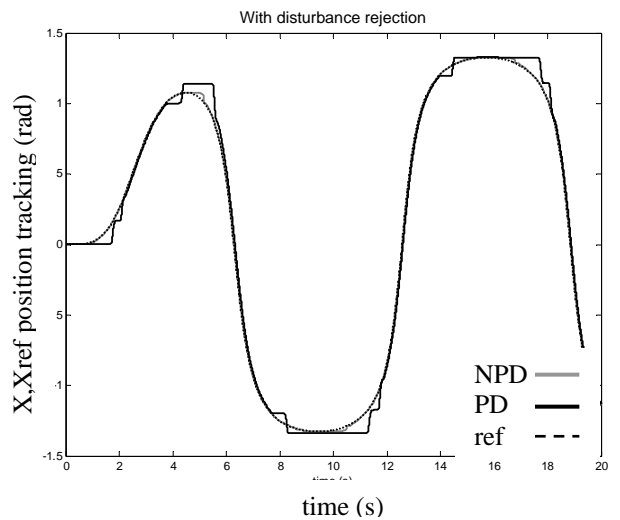


Fig. 8 Position tracking response

Figure 9 shows the real friction forces (which are the internal disturbance of the system) and the disturbance estimated and rejected by the observer.

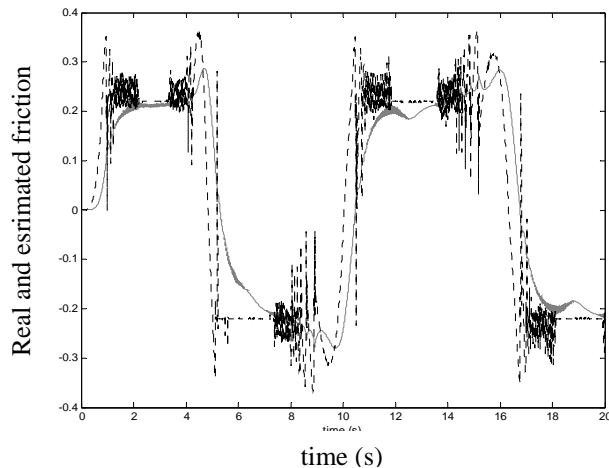


Fig. 9 Real (dark dotted) and estimated (gray ) friction forces with the non-linear observer

- [5] Zhiqiang Gao, Shaohua Hu, and Fangjun Jiang, *A Novel Motion Control Design Approach Based On Active Disturbance Rejection*. IEEE Conference on Decision and Control 2001.
- [6] P. Vedagarbha, D. M. Dawson & M. Feemster, *Tracking Control of Mechanical Systems in the Presence of Nonlinear Dynamic Friction Effects*. IEEE Trans. On Control Systems Technology, V.7, N.4, 1999

## 4 Conclusion

In this paper, we have verified by simulations the efficiency of a nonlinear disturbance observer/controller in canceling errors caused by friction in a one mass servomechanism, and this by means of high stiff gains of the controller. The proposed control scheme is simple to implement and the tuning of the observer/controller were done on the basis of linear methods which are a special case of a general nonlinear form. The dynamics of the observer was improved by using nonlinear gains for small estimation errors, and the controller gave very good performance in term of tracking error, this due to the high stiff gain used for small tracking errors. A part of friction could be compensated directly by the estimation and rejection, and a part has been cancelled by indirect stiff control.

### References:

- [1] Armstrong-Helouvry, B., Dupont, P. E., & Canudas de Wit, C. (1994). *A Survey of Analysis Tools and Compensation Methods for Control of Machines with Friction*. Automatica, 30(7), 1083-1138.
- [2] C. Canudas de Wit, H. Olsson, K. J. Aström & P. Lischinsky, *A new model for control of systems with friction*. IEEE T. on Automatic Control. 40(3): pp. 419-425, 1995.
- [3] K. J. Aström, Canudas de Wit, M. Gafvert & P. Lischinsky, *Friction Models and Friction Compensation*. European Journal of Control (1998) 4 : 176-195.
- [4] B. Armstrong, D. Neevel & T. Kusik, *New Results in NPD Control : Tracking, Integral Control, Friction Compensation and Experimental results*. Int. Conf. on Robotics and Automation, Detroit, 1999.