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A HYBRID WAY TO CONSTRUCT STATIC VOLTAGE STABILITY REGION BASED ON COMBINING LOOP CURRENT WITH NODE VOLTAGE

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Abstract: - A fast method to construct static voltage stability region using bus power injection and network parameter is presented which is based on the analysis of π-shaped basic loop model and a hybrid way combining loop current and bus voltage. In the hybrid method the impedance branch and ground branch are treated as the link and tree branch respectively. A new voltage stability margin index (VSMI) in state-space is presented. In VSMI the difference between the square of current injected magnitude and node reactive power is used to denote the voltage stability margin. The method is illustrated by IEEE-30 system.

Key-Words: - Power system; Static voltage stability Region; Loop current; Nodal voltage; Margin index

1 Introduction

The power system was forced to operate closed to its stability limit because of the increase in power demand. The potential threat of a heavily loaded line or system is voltage instability or collapse. This phenomenon has become an issue which requires the static voltage stability analysis to be conducted at the planning stage.

Static voltage stability analysis is processed with the assumption that the system is operating in the steady state[1]. It provides the ability of the transmission network to support a specified load demand. The results of such studies can be used to determine how far the system's present operating point is from voltage collapse.

The P–V or Q–V curves are used as a tool to assess the static voltage stability limit of a power system. These curves are generated from the results of repetitive power flow simulations for various load conditions and thus require a significant amount of computational time[2-4].

Several other methods also have been presented such as modal analysis[5], artificial neural networks[6], neuro-fuzzy networks[7], energy function methods[8], sensitivity analysis[9] and bifurcation theory methods[10].

It is an established fact that the voltage collapse occurs when the system load (P and/or Q) increases beyond a certain limit. If the limiting values of P and Q are known, the voltage stability margin for a given operating point can directly be determined. Therefore many indices are put forward to determine the static voltage stability margin [11-13]. However all of these methods cost much time and storage. Therefore, it is disadvantageous for the on-line data processing for the bulk power system.

In this paper, the π-shaped basic loop model is analyzed by using a hybrid analysis method combining loop current with nodal voltage. The impedance branch and ground branch are treated as the link and tree branch respectively in the π loop model. A method to construct voltage stability region in state space by use of nodal injection power and network parameters is given. A new voltage stability margin index (VSMI) is proposed, in which the voltage stability margin index is characterized by the difference between the square of the current injected at load node and the reactive power at the same node. The proposed method has been applied to IEEE 30-bus system to illustrate its effectiveness.

2 Foundational circuit model

The π-model circuit of a transmission line is composed of one impedance branch and two ground ones as shown in Figure 1, which denotes a foundational loop while all impedance and ground branches are considered as the links and tree branches respectively where the amount of links and lines is equal to that of impedance branches.

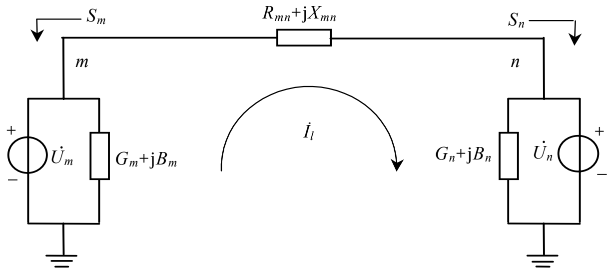

Fig.1 The basic loop model

$S_m = p_m + \mathrm{j}q_m$ and $S_n = p_n + \mathrm{j}q_n$ denote the injected power at node m and n respectively, p and q denote the active power and reactive power respectively; $\dot{U}_m = e_m + \mathrm{j}f_m$ and $\dot{U}_n = e_n + \mathrm{j}f_n$ denote the voltage vectors of nodal m and n respectively; G and B denote the conductance and susceptance to ground respectively; subscript $m, n = 1,2,\cdots,N$ denotes the nodal number; the loop current is $\dot{I}_l = i_{l,a} + \mathrm{j}i_{l,r}$; subscript $l = 1,2,\cdots,L$ denotes the loop number. Therefore, loop current equation can be represented:

$$\dot{I}_l(R_{mn} + \mathrm{j}X_{mn}) = \dot{U}_m - \dot{U}_n \tag{1}$$

where R_{mn} and X_{mn} are the resistance and reactance of transmission line between nodal m and n.

It is obtained by expanding formula (1):

$$\begin{cases} i_{l,a}R_{mn} - i_{l,r}X_{mn} - e_m + e_n = 0 \\ i_{l,a}X_{mn} + i_{l,r}R_{mn} - f_m + f_n = 0 \end{cases} \tag{2}$$

The ground branch circuit is shown in figure 1 where the load is considered as voltage source, so the loop current passes two sub-branches: the capacitive branch and the load branch.

The equivalent voltage source of the load branch is:

$$\overset{*}{U}_m = \frac{p_m - \mathrm{j}q_m}{\sum\limits_{l\in m}\dot{I}_{lm} - \dot{U}_m\sum\limits_{l\in m}(G_l + \mathrm{j}B_l)} \tag{3}$$

It follows from (3) that:

$$\overset{*}{U}_m\sum_{l\in m}\dot{I}_{lm} - \overset{*}{U}_m\dot{U}_m\sum_{l\in m}(G_l + \mathrm{j}B_l) = p_m - \mathrm{j}q_m \tag{4}$$

Above equation is represented as the plural form:

$$(e_m - \mathrm{j}f_m)(\sum_{l\in m}i_{lm,a} + \mathrm{j}\sum_{l\in m}i_{lm,r}) - (e_m^2 + f_m^2)\sum_{l\in m}(G_l + \mathrm{j}B_l) = p_m - \mathrm{j}q_m \tag{5}$$

Furthermore, it is obtained:

$$\begin{cases} e_m\sum\limits_{l\in m}i_{lm,a} + f_m\sum\limits_{l\in m}i_{lm,r} - (e_m^2 + f_m^2)\sum\limits_{l\in m}G_l = p_m \\ e_m\sum\limits_{l\in m}i_{lm,r} - f_m\sum\limits_{l\in m}i_{lm,a} - (e_m^2 + f_m^2)\sum\limits_{l\in m}B_l = -q_m \end{cases} \tag{6}$$

As for node n, there is the same form of nodal voltage function with node m. The equations (2) and (6) are the basic models of the power system.

3 The nodal voltage and loop current analysis

It is obtained from equation (2):

$$\begin{cases} i_{l,a} = \dfrac{X_{mn}(f_m - f_n) + R_{mn}(e_m - e_n)}{R_{mn}^2 + X_{mn}^2} \\ i_{l,r} = \dfrac{R_{mn}(f_m - f_n) - X_{mn}(e_m - e_n)}{R_{mn}^2 + X_{mn}^2} \end{cases} \tag{7}$$

The classical nodal voltage equations represented in the form of rectangular coordinates can be obtained by substituting formula (7) into equation (6). For the same reason, when the conductance of the ground branch is ignored as $\sum\limits_{l\in m}G_l = 0$ and $x_m = \sum\limits_{l\in m}i_{lm,a}$, $y_m = \sum\limits_{l\in m}i_{lm,r}$, $B_m = \sum\limits_{l\in m}B_l$, the formulas can be obtained from equation (6) as follows:

$$e_m = (p_m - f_m y_m)/x_m = h_m(I) \tag{8}$$

$$f_m = \frac{[2B_m p_m y_m + x_m(x_m^2 + y_m^2)] \pm \varphi_m}{2B_m(x_m^2 + y_m^2)} = g_m(\dot{I}) \tag{9}$$

where,

$\varphi_m = x_m\sqrt{(x_m^2 + y_m^2)^2 - 4B_m q_m(x_m^2 + y_m^2) - 4B_m^2 p_m^2}$,

and $\dot{I}$ is the loop current vector. Accordingly, loop current equations are:

$$\begin{cases} i_{l,a}R_{mn} - i_{l,r}X_{mn} - h_m(\dot{I}) + h_n(\dot{I}) = 0 \\ i_{l,a}X_{mn} + i_{l,r}R_{mn} - g_m(\dot{I}) + g_n(\dot{I}) = 0 \end{cases} \tag{10}$$

Thus it can be observed that loop current equations are linear after the corresponding nodal voltages are determined. Therefore it is very easy to ascertain the value of $i_{lm,a}$ and $i_{lm,r}$.

4 Voltage stability Region

4.1 Expression

From formula (9), it is obvious that power flow solutions are existent when the following formulas are satisfied:

$$(x_m^2 + y_m^2)^2 - 4B_m q_m(x_m^2 + y_m^2) - 4B_m^2 p_m^2 \geq 0 \tag{11}$$

Namely:

$$(x_m^2 + y_m^2 - 2B_m q_m)^2 \geq 4B_m^2 q_m^2 + 4B_m^2 p_m^2 \tag{12}$$

It means that solutions are existent only when the square of nodal injected current amplitude locates out of the circle, which is centered at $2B_m q_m$ and the radius is $2B_m\sqrt{q_m^2+p_m^2}$.

From formula (6), it is obvious that only one solution of the node voltage is actual when the equal sign of formula (11) is available, otherwise two solutions which include one higher voltage solution and one lower one are existent when the sign of inequality from (11) is available. The amount of power flow solutions is determined by the solution amount of the of node voltage equations which equal to 2^{N-T}. T denotes the amount of non-degenerated nodes or loops.

The transmission capacity reaches the upper limit in the electric power network when the equal sign of formula (11) is available. In other words the saddle bifurcation point is reached on the PV curve, which means thet operation point of the power system is very close to the edge of voltage stability. Therefore, the formula (12) also provides the region of static voltage stability in the space of state variables.

Therefore the upper limit of the voltage stability region is:

$$I_{\mathrm{VSM,max}} = 2B_m\sqrt{q_m^2+p_m^2} \tag{13}$$

The index for voltage stability margin is shown as below:

$$I_{\mathrm{VSM}} = \left|x_m^2+y_m^2-2B_m q_m\right| - 2B_m\sqrt{q_m^2+p_m^2} \tag{14}$$

It is obvious that the calculation of VSMI is very simple.

4.2 Arithmetic procedure

1) Calculate the upper limit of voltage stability margin $2B_m\sqrt{q_m^2+p_m^2}$ and $2B_m q_m$ according to formula (13).
2) Draw the circular voltage stability boundary that is centred at $2B_m q_m$ and the radius is $2B_m\sqrt{q_m^2+p_m^2}$ in state-space. The inner area of the circle is voltage instability region.
3) Substitute the power flow result into (20) and work out $i_{lm,a}$ and $i_{lm,r}$, then calculate x_m and y_m.
4) Substitute x_m and y_m into (14), if $I_{VSM} > 0$ that means the node m is in a voltage-stability state; if $I_{VSM} = 0$ means node m is in a critical state; if $I_{VSM} < 0$ means the voltage stability is lost.

5 Case study

The method presented is applied to IEEE 30-bus system and the result is shown in Table1. As shown in Table 1, the stability margin of node 20 and 29 is very little at the current load level. There is a serious chance of voltage collapse if the load demand increases ulteriorly. The stability of node 15 and 18 is weakly. The stability margin of node 3, 7 and 12 is great. At the current load level, node 20 and 29 are close to the boundary of the circular instability region that means voltage collapse will occur if the load demand increases continuously.

In order to validate the effectiveness of this method, we take node 17 as an example and sturdy the voltage stability in detail. We increase the load on node 17 with invariable power-factor and execute power flow calculation and voltage stability analysis. The result is shown in Table 2. The visualizing graph of the voltage stability region based on the calculation result is shown as figure 2.

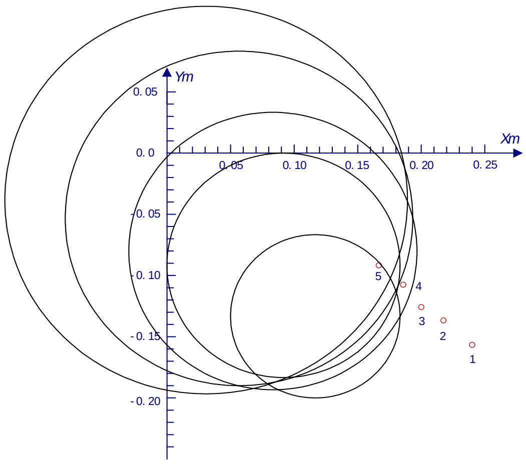

Fig 2 The regions on different load condition

The visualizing graphs of stability region on load condition No.1 and No.5 are compared as shown in Figure 3 in order to illuminate the variation of the stability region clearly.

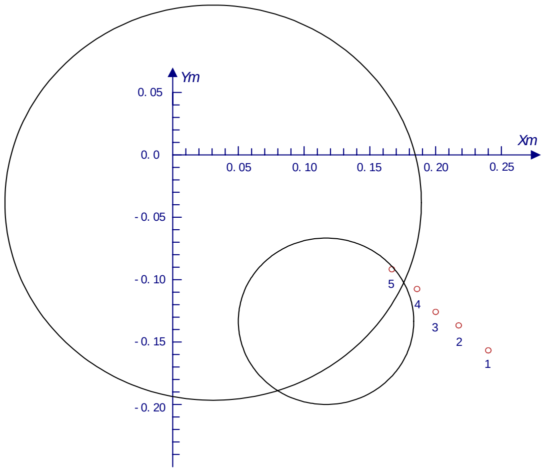

Fig 3 The regions on load condition No.1 and No.5

As shown in Table2, the value of VSMI decreases gradually as well as the nodal voltage when the load demand increases continuously. At the same time, the radius of instability region($2B_m\sqrt{q_m^2+p_m^2}$) becomes greater which means the circular instability region extend gradually as shown in Figure 2 and 3. The VSMI is negative on load condition 5 and as shown in Figure3 the node is located in the circle that means the node has entered the instability region and voltage collapse will happen.

Tab.1 The voltage stability analysis results of IEEE 30-bus system

Node	3	7	12	15	17	18	20	24	29
I_{VSM}	0.2715	0.3114	0.8174	0.0116	0.0532	0.0130	0.0036	0.0525	0.0042
$2B_m\sqrt{q_m^2+p_m^2}$	0.0084	0.1391	0.0676	0.0516	0.0645	0.0200	0.0139	0.0661	0.0169
x_m	0.5246	–0.5996	–0.6372	–0.1848	0.2310	0.0387	–0.1156	0.0438	0.0765
y_m	–0.0322	0.1760	0.6888	0.1188	–0.1716	–0.0100	0.0040	–0.2765	–0.0295

Tab.2 The voltage stability margin of bus 17 in IEEE 30-bus system

Load level	P/pu	Q/pu	V/pu	θ/(°)	x_m	y_m	I_{VSM}	$2B_m\sqrt{q_m^2+p_m^2}$
1	0.0900	0.0580	1.04282	–10.3012	0.2431	–0.1575	0.0544	0.0645
2	0.1100	0.0709	1.04089	–7.32361	0.2173	–0.1363	0.0296	0.0788
3	0.1200	0.0773	1.03903	–7.43736	0.2044	–0.1258	0.0181	0.0869
4	0.1350	0.0870	1.03725	–7.62030	0.1851	–0.1096	0.0019	0.0967
5	0.1500	0.0966	1.03505	–7.79935	0.1658	–0.0933	–0.0131	0.1075

6 Conclusion

A fast method to construct static voltage stability region is presented which is based on the analysis of π-shaped basic loop model and a hybrid way combining loop current and bus voltage. A new fast voltage stability margin index (VSMI) in state-space is presented.

In VSMI the difference between the square of current injected magnitude and node reactive power is used to denote the voltage stability margin. A circular voltage stability boundary is established in state-space which can supply the demand of power system operator. The graph of the stability region can be drawn in the two-dimensional surface clearly and visualized. The calculation of VSMI is simple and fast so that it can be applied on-line.

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