7th WSEAS Int. Conf. on MATHEMATICAL METHODS and COMPUTATIONAL TECHNIQUES IN ELECTRICAL ENGINEERING, Sofia, 27-29/10/05 (pp305-309)

Proving the Controllability for Arbitrary Operating Point of STATCOM

ZHANG LIUCHUN, XIAO DENGMING, ZHANG DONG
School of Electronics, Information and Electrical Engineering
Shanghai Jiaotong University
No. 1954, Huanshan road, Shanghai
CHINA

Abstract：-The controllability for arbitrary operating point of STATCOM (Static Synchronous Compensator) is proved in this paper. First the nonlinear mathematical model of STATCOM is derived based on switching functions. Then, using the method of Jacobian, the mathematical model of the STATCOM is linearized around a given operating point. Last, based on the linearized mathematical model, this paper proves that the STATCOM can be controlled at arbitrary operation points. Proved result extends the viewpoint that the STATCOM can only be controlled at a given operating point, and means the design of STATCOM controller is reasonable and feasible.

Key Words：-high voltage power grid; STATCOM; controller; controllability; arbitrary operation point

1 Introduction

The STATCOM is one of the new generation flexible AC transmission systems (FACTS) devices with a promising future application, which is recognized to be one of the key advanced technologies of future power system. The technology of STATCOM improving system transmission capability has been successfully applied in power systems in developed countries, such as Japan, USA and Britain. In china, electrical resources mainly centralize in the west region, while electric power consumers centralize in southern foreland where developed. The inconsistence between the geography distribution of power resources and that of consumers demands long distant and high capable power transmission. The 500KV and even higher rank voltage power grid has been developed due to the requirements for long distant and high capable power transmission. The installation of STATCOM is suggested to support voltage at the middle of distant power transmission lines by the North China Power System (NCPS)[1].

The successful application of STATCOM home and abroad means the STATCOM is mature in technique. At present the researches on STATCOM mainly concentrate on modeling and controller design to STATCOM. Paper [2] points out that system control performance can be determined by controllability, observability, and etc. Plenty of work has been done on the controller design while little work has been done on the analysis of the controllability and observability of STATCOM [3~5]. Paper [6] only gave a qualitative analysis of the relationship between controllability and the controller design. Although paper [7] proved the controllability of STATCOM controller, it was limited to a given operation point. Based on paper [7], this paper proves the controllability for arbitrary operation point of STATCOM.

The paper is organized as follows: first derives the mathematical model of STATCOM based on switching functions. Then the mathematical model is linearized using the method of Jacobian and the controllability for arbitrary operating point of STATCOM is proved. Finally, the major contribution of this paper is summarized.

2 Mathematical model derivation and Linearization

For the convenience of the research on STATCOM modeling, this section first states its principle. The diagram of STATCOM principle is showed in Fig.1. Where, direct current side is capacitor storing energy and supporting direct voltage for STATCOM. GTO or IGBT is composed of several shunt or series inverters. Its main function is to transform direct voltage to alternating voltage. By driving GTO pulses, the magnitude, frequency and phase of the alternating voltage can be controlled. Connected transformer can transform the out-put voltage to the same voltage level of the system voltage. Thus the STATCOM can be connected to the power system. STATCOM can be regarded as a voltage source converter connected to an energy storage unit, usually a DC capacitor, which controls STATCOM absorbing or sending out reactive power [8].

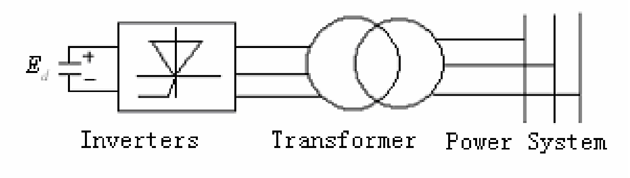

Fig.1 Diagram of STATCOM Principle

There are several methods can be used to model STATCOM. At present the methods often used are topology modeling, output modeling and switching functions modeling, etc [9]. In this paper switching functions method is adopted to model STATCOM because of its advantages over the former two modeling methods.

2.1 Mathematical model derivation

The voltage source converter based STATCOM is the dominant topology in practice. Fig. 2 is the circuit diagram of a typical STATCOM.

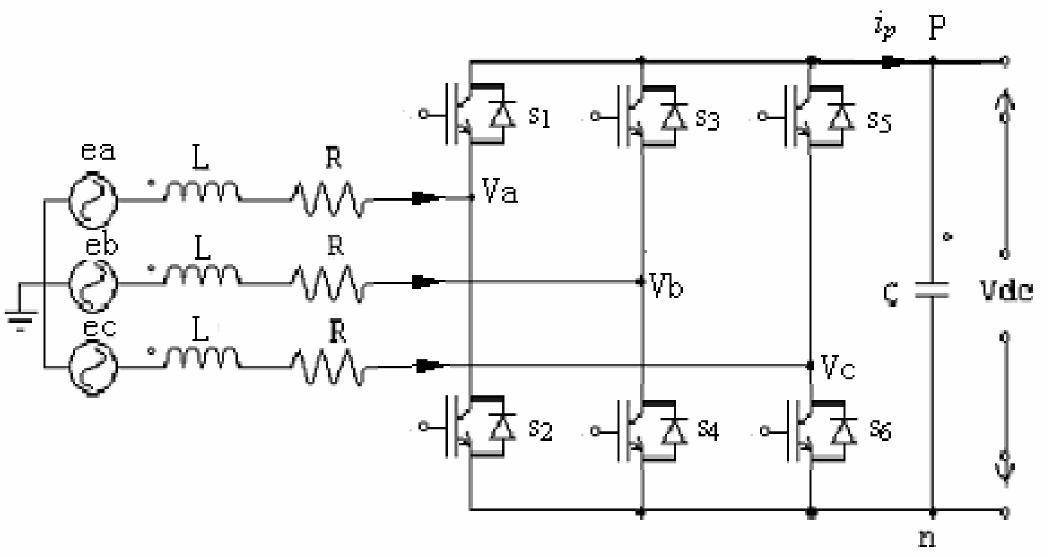

Fig.2 Equivalent Circuit of STATCOM

Where, i_a ,i_b and,i_c represent line current; V_a, V_b *and* V_c represents converter phase voltage; e_a, e_b and e_c are AC source phase voltage; $Vdc=Vpn$ is DC side voltage; i_p expresses DC side current; L is inductance of the line reactor; R is resistance of the line reactor; C is DC side capacitor.

Based on the equivalent circuit of STATCOM shown in Fig.2 we can derive the mathematic model of STATCOM as following procedure.

From power electronics principles and circuit principles we get

$$\frac{d}{dt}\begin{bmatrix} i_{ab} \\ i_{bc} \\ i_{ca} \end{bmatrix} = \frac{1}{3L}\begin{bmatrix} e_a - e_b \\ e_b - e_c \\ e_c - e_a \end{bmatrix} - \frac{1}{3L}\begin{bmatrix} D_{ap} - D_{bp} \\ D_{bp} - D_{cp} \\ D_{cp} - D_{ap} \end{bmatrix} V_{pn} - \frac{R}{L}\begin{bmatrix} i_{ab} \\ i_{bc} \\ i_{ca} \end{bmatrix} \quad (1)$$

And

$$C\frac{dV_{pn}}{dt} = i_p = \begin{bmatrix} D_{ap} - D_{bp} \\ D_{bp} - D_{cp} \\ D_{cp} - D_{ap} \end{bmatrix}^T \begin{bmatrix} i_{ab} \\ i_{bc} \\ i_{ca} \end{bmatrix} \quad (2)$$

Where, D_{kp} are switching functions and k=a, b, c

$$i_{ab} = \frac{1}{3}(i_a - i_b),\, i_{bc} = \frac{1}{3}(i_b - i_c),\, i_{ca} = \frac{1}{3}(i_c - i_a)$$

And

$$\begin{bmatrix} V_a - V_b \\ V_b - V_c \\ V_c - V_a \end{bmatrix} = \begin{bmatrix} D_{ap} - D_{bp} \\ D_{bp} - D_{cp} \\ D_{cp} - D_{ap} \end{bmatrix} V_{pn}$$

It is common practical in power system application to transform 3 phases AC dynamics into orthogonal components in a rotating reference frame. From the power system theory we get the real and reactive currents relative to a rotating reference frame with angular frequency ω as:

$$\begin{bmatrix} i_d \\ i_q \end{bmatrix} = P\begin{bmatrix} i_{ab} \\ i_{bc} \\ i_{ca} \end{bmatrix} \quad (3)$$

And

$$P=\frac{2}{3}\begin{bmatrix}\cos(\omega t) & \cos(\omega t-\frac{2}{3}\pi) & \cos(\omega t+\frac{2}{3}\pi)\\ -\sin(\omega t) & -\sin(\omega t-\frac{2}{3}\pi) & -\sin(\omega t+\frac{2}{3}\pi)\\ \frac{1}{2} & \frac{1}{2} & \frac{1}{2}\end{bmatrix} \quad (4)$$

Where, i_d represents active current component and i_q represents reactive current component.

By applying equation (3) to equation (4), we have:

$$\begin{bmatrix}i_{ab}\\ i_{bc}\\ i_{ca}\end{bmatrix}=\frac{1}{3}\begin{bmatrix}i_a-i_b\\ i_b-i_c\\ i_c-i_a\end{bmatrix}=\overline{T}^{-1}\begin{bmatrix}i_d\\ i_q\\ 0\end{bmatrix} \quad (5)$$

Where

$$\overline{T}^{-1}=\frac{1}{\sqrt{3}}\begin{bmatrix}-\sin(\omega t-\frac{1}{3}\pi) & \cos(\omega t-\frac{1}{3}\pi) & 1\\ \sin(\omega t) & -\cos(\omega t) & 1\\ -\sin(\omega t+\frac{1}{3}\pi) & \cos(\omega t+\frac{1}{3}\pi) & 1\end{bmatrix} \quad (6)$$

If we set T as the first two 2×3 subspace of matrix $\overline{T}$, we can get

$$\begin{bmatrix}i_d\\ i_q\end{bmatrix}=T\begin{bmatrix}i_{ab}\\ i_{bc}\\ i_{ca}\end{bmatrix} \quad (7)$$

Similarly, we can get

$$\begin{bmatrix}D_d\\ D_q\end{bmatrix}=T\begin{bmatrix}D_{ab}\\ D_{bc}\\ D_{ca}\end{bmatrix} \quad (8)$$

$$\begin{bmatrix}e_d\\ e_q\end{bmatrix}=T\begin{bmatrix}e_{ab}\\ e_{bc}\\ e_{ca}\end{bmatrix} \quad (9)$$

From power system principle we can get $e_d=V_m$ and $e_q=0$.

By above transforming (7), (8) and (9) the STATCOM mathematical model (1) and (2) can be expressed in standard state space form as(10):

$$\frac{d}{dt}\begin{bmatrix}i_d\\ i_q\\ V_{dc}\end{bmatrix}=\begin{bmatrix}-\frac{R}{L} & \omega & -\frac{D_d}{3L}\\ -\omega & -\frac{R}{L} & -\frac{D_q}{3L}\\ \frac{3}{2C}D_d & \frac{3}{2C}D_q & 0\end{bmatrix}\begin{bmatrix}i_d\\ i_q\\ V_{dc}\end{bmatrix}+\begin{bmatrix}\frac{1}{3L}\\ 0\\ 0\end{bmatrix}V_m \ . \quad (10)$$

2.2 Model linearization

From the STATCOM mathematic model (10), it can be easily seen that STATCOM is a nonlinear system. To analyze the performance of STATCOM easily, equation (10) must be linearized to linear equations around a given operating point using the method of Jacobian. And then prove the controllability based on the linearized mathematical model.

For a nonlinear system, the dynamics may be expressed in the general vector function form as $\dot{X}=f(x,u)$, and the nonlinear vector function can be linearized around the operation point as follow:

$\dot{x}=Ax+Bu$.

Where,

$$A=\frac{\partial f}{\partial x}\mid x^0,u^0,\ \ B=\frac{\partial f}{\partial x}\mid x^0,u^0$$

For the STATCOM model, this procedure can be used to get the linearized model. First operating point is set as: $x^0=\begin{pmatrix}i_{d0}\\ i_{q0}\\ v_{dc0}\end{pmatrix}$,$v^0=\begin{pmatrix}D_{d0}\\ D_{q0}\end{pmatrix}$

Then compute the Jacobians:

$$A=\frac{\partial f}{\partial x}\mid x^0,u^0=\begin{pmatrix}-\frac{R}{L} & \omega & -\frac{D_{d0}}{3L}\\ -\omega & -\frac{R}{L} & -\frac{D_{q0}}{3L}\\ \frac{3D_{d0}}{2C} & \frac{3D_{q0}}{2C} & 0\end{pmatrix} \quad (11)$$

$$B=\frac{\partial f}{\partial x}\mid x^0,u^0=\begin{pmatrix}-\frac{V_{dc0}}{3L} & 0\\ 0 & -\frac{V_{dc0}}{3L}\\ \frac{3i_{d0}}{2C} & \frac{3i_{d0}}{2C}\end{pmatrix} \quad (12)$$

So, by the method of the Jacobian, linearized mathematical model of STATCOM is in the form as:

$$\dot{x}=Ax+Bu \quad （13）$$

3 CONTROLLABILITY PROVING FOR ARBITRARY OPERATING POINT OF STATCOM

Having the linearized system state space model as equation (13), the characteristics of STATCOM model can be analyzed by linear system method. In paper [7], the stability, observability and its open loop response characteristic has been analyzed. Though paper [7] has proved the controllability of STATCOM at a given operating point, proved results did not obtain a general knowledge of operating point controllability. This section will prove the controllability for arbitrary operating point extending the proved results in paper [7].

For the convenience of proving and without influencing the proving results, we suppose the following component parameters be expressed in per-unit. That is:

R＝1，L＝1，C＝1，V_m＝1，ω＝1

Besides, the arbitrary operating point can be expressed as $V_{dc} = l$, $i_d = 0$, $i_q = m$, $D_{d0} = 0$, $D_{q0} = n$ (Where l, m, and n are non-zero).

Thereby, from (11) and (12) we will get,

$$A=\begin{pmatrix} -1 & 1 & 0 \\ -1 & -1 & -\frac{n}{3} \\ 0 & \frac{3n}{2C} & 0 \end{pmatrix} \tag{13}$$

$$B=\begin{pmatrix} -\frac{l}{3} & 0 \\ 0 & -\frac{l}{3} \\ 0 & \frac{3m}{2} \end{pmatrix} \tag{14}$$

The system controllability matrix is:

$$M=\begin{pmatrix} B & AB & A^2B \end{pmatrix}=$$

$$\begin{pmatrix} -\frac{l}{3} & 0 & \frac{l}{3} & -\frac{l}{3}-\frac{m}{2} & 0 & \frac{l}{3} \\ 0 & -\frac{l}{3} & \frac{l}{3} & -\frac{mn}{2} & 0 & \frac{l}{3}+mn+\frac{n^2l}{6} \\ 0 & \frac{3m}{2} & 0 & -\frac{nl}{2} & \frac{nl}{2} & \frac{3nm}{4} \end{pmatrix}$$

$\neq 0$

M$\neq$0 means that the M matrix has full rank, that is STATCOM is controllable at arbitrary operating point instead of only limited to a given point. Proved result obtains a general knowledge of operating point controllability. Besides, it means the design of STATCOM is reasonable and feasible. What is more, the research on STATCOM is perfected.

4 Conclusion

Much research shows that the installation of STATCOM, usually in the middle of distant AC lines, can supply reactive power to support system voltage rapidly and can damp the oscillation by accommodating its reactive power output/absorb, which can primarily improve transmitting capability of high voltage AC lines.

At present, scholars home and abroad mainly concentrate on the research of STATCOM controller design. Little work has been done on the research of STATCOM controller performance analysis such as controllability observability etc.

In this paper, the controllability for arbitrary operating point is proved. Proved result extends the viewpoint that the controllability of STATCOM is limited to a given operating point, and means the design of STATCOM controller is reasonable and feasible.

References:

[1] Xia Zuhua, Cui Wenjin, etc. Research of STATCOM Application in Improving the Transmitting Capabilities of 500KV AC Lines. Power System and Communications Infrastructures for the Future.Beijing, September 2002.

[2] Larsen EV, Sanchez-Gasca J J, Chow J H. Concept for Design of FACTS Controller to Damp Power Swings IEEE Trans on PWRS, 1995, 10(2) :948~956.

[3] Yonezawa, H., Shimato, etc. study of a STATCOM application for voltage stability evaluated by dynamic PV curves and time simulations. Power Engineering Society Winter Meeting. 2000, IEEE Volume 2, 23-27 Jan. 2000 Page(s):1471 - 1476 vol.2.

[4] Sekoguchi, M., Konishi, H. etc. Nonlinear optimal control applied to STATCOM for power system stabilization. Transmission and Distribution Conference and Exhibition 2002: Asia Pacific. IEEE/PES Volume 1, 6-10 Oct. 2002 Page(s):342 - 347 vol.1.

[5] Yonezawa, H., Shimato, etc. study of a STATCOM application for voltage stability evaluated by dynamic PV curves and time simulations. Power Engineering Society Winter Meeting. 2000, IEEE Volume 2, 23-27 Jan. 2000 Page(s):1471 - 1476 vol.2.

[6] Li Chun, Jiang Qirong, Wang Zhonghong, Lichengye. Input Signal Selection For the

Damping Controller of ±20 Mvar STATCOM. Power System Automation (China). 2000,12（10）：50～53

[7] Li Qunxing. A Comparison of Pole Assignment & LQR Design Methods for Multivariable Control for STATCOM. Master Dissertation, the Florida State University College of Engineering, 2003.

[8] Geng Juncheng, Liu Wenhua, Yu Xufeng, Yuan Zhichang Modeling of Cascade STATCOM. Proceedings of the CSEE.2003,23(6):66~70

[9] Liu Baohong. Research on STATCOM Topology and Simulation of the Damp Influence, Master Dissertation. Beijing, Huabei Power University，2003.