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Complex dynamic behavior of a CNN hardware system by an experimental and numerical analysis

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Abstract: - This paper presents an analysis of the complex dynamic behavior of a CNN hardware system by both an experimental and a numerical approach. Stable, unstable and chaotic behaviors have been observed by setting different CNN synaptic weights. For evaluating the CNN behavior the Lyapunov method has been adopted and a software called LeChaDe (Lyapunov exponent Chaos Detection) has been implemented. The experimental apparatus, as well as the validation of the implemented numerical algorithms are described.

Key-Words: - Cellular Neural Networks (CNNs), Chaotic dynamics, Lyapunov analysis

1 Introduction

The Cellular Neural Networks (CNNs) are analogical non-linear circuit suitable for real-time signal processing. A CNN can be described as a network made of an arbitrary number of fundamental sub-circuits, usually called "cells". The Chua-Yang CNN circuit model [1][2], which will be briefly recalled in a section of this paper, is the basic CNN unit. The CNN hardware presented in this work, uses VLSI chips and high velocity PCI based on a development of the Chua-Yang model [3][4]. By varying some Chua-Yang circuit parameters (template values) the CNN can show different dynamic behaviors. The CNN dynamic system studies has been analyzed by means of state equations and Lyapunov analysis [5] [6] [7]. The solutions of the state equations can be plotted on the state-plane. The locus of the solution waveforms on the state plane are called trajectories. On the base of the particular obtained trajectories, two main CNN behavioral classes has been established: 1) *stable class* - all the trajectories converge on an equilibrium point - 2) *unstable class* - a trajectory converges on an attractor which is not an equilibrium point - [5]. Examples of limit-of-stability are described in [6],[7]. Then, in this paper, after a brief recall of the Chua-Yang CNN model and the Lyapunov analysis, it will be shown as stable, unstable or chaotic responses can be obtained by a suitable modification of the CNN synaptic weights (template values).

2 The CNN system

2.1 CNN theory recall

In practical cases, each fundamental CNN cell is connected only with its bordering cells. If this connection is operated by a 2D architecture, the CNN can be regarded as a n-dimensional array (see Fig.1)

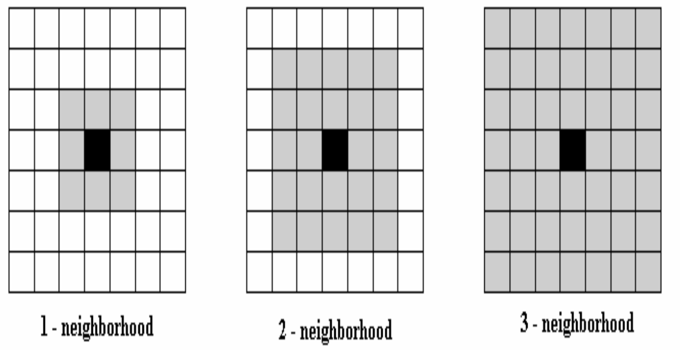

Fig. 1: Examples of CNN neighborhood

In a CNN, each cell has an internal state and an output state that are related by output functions. The internal state depends on the output states of nearest cell neighbors and changes according to an ordinary differential equation.
Thus, let us to consider an autonomous CNN system made of NxM cells on a regular grid. Each cell be indicated by a pair of indices (i,j). The CNN Chua-Yang model [1][2] can be briefly summarized as follows.
Let $C(i,j)$ be the cell relative to the i-th row and j-th column of the grid. The $C(i,j)$ r-neighborhood, $Nr(i,j)$, is given by:

$$Nr(i,j) = \{C(k,l) | \max\{|k-i|,|l-j|\} \leq r\} \quad (1)$$

where r is a positive integer and k and l two indices of row and column, respectively.

The r-neighborhood shows the following symmetric property:

$$\text{if } C(i,j) \in Nr(k,l) \text{ then } C(k,l) \in Nr(i,l)$$

The Chua-Yang circuit referred to a $C(i,j)$ cell is shown in the next Fig. 2.

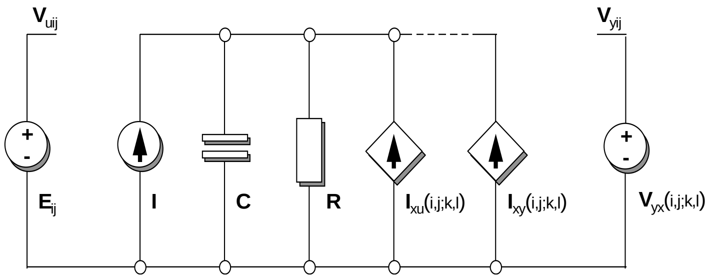

Fig. 2: - CNN Chua-Yang model

The symbols *u*, *x* and *y* indicate the cell input-node, state-node and output-node, respectively. Each cell is made of one independent voltage source *Eij*, one independent current source *I*, one ideal linear capacitor C, one ideal linear resistor *R* and *2m* voltage-controlled-current sources. The control voltages are indicated by V_{ukl} e V_{ykl}. They are measured on the *m* cells belonging to the neighborhood of the *C(i,j)* cell. In particular:

$$I_{xy}(i,j;k,l) = A(i,j;k,l)V_{ykl} \qquad (2)$$
$$I_{xu}(i,j;k,l) = B(i,j;k,l)V_{ukl}$$

where *A(i,j;k,l)* e *B(i,j;k,l)* are the so-called *cloning template matrices*. The dimensions of these matrices depend on the dimension of the neighborhood.
The non-linear components present in the Chua-Yang circuit are the voltage-controlled-current sources:

$V_{ukl} = f(V_{uij})$:

$$V_{ykl} = f(V_{yij}) \qquad (3)$$

where *f(.)* is a suitable non-linear function [1].
By applying Kirchhoff laws to the Chua-Yang circuit, the state equation can be written as:

$$C\frac{dV_{xij}(t)}{dt} = -\frac{1}{R_x}V_{xij}(t) + f_{ij} + g_{ij} \qquad (4)$$

$$f_{ij} = \sum_{C(k,l)\in Nr(i,j)} A(i,j;k,l)V_{ykl}(t) \qquad (5)$$

where

$$g_{ij} = \sum_{C(k,l)\in Nr(i,j)} B(i,j;k,l)V_{ukl} + I \qquad (6)$$

The output equation is:

$$V_{yij}(t) = \frac{1}{2}\left(\left|V_{xij}(t)+1\right| - \left|V_{xij}(t)-1\right|\right) \qquad (7)$$

and the boundary condition can be expressed as follows:

$$\left|V_{xij}(0)\right| \le 1 \qquad (8)$$

$$\left|V_{uij}(t)\right| \le 1$$

Moreover, are valid the next assumptions:

$$\begin{aligned} &A(i,j;k,l) = A(k,l;i,j) \\ &1 \le i \le M; 1 \le j \le N \\ &C > 0, R_x > 0 \end{aligned} \qquad (9)$$

It is important to note that *A(i,j;k,l)* takes into account the cell-output effects on the state voltages (feedback operator), whereas *B(i,j;k,l)* takes into account the cell-input effects on the circuit state (control operator).
These two matrices, together with the polarization current *I*, are the synaptic weights of the CNN (i.e. *A*, *B* and *I* are the *cloning template*). Thus, when a CNN input polarizes the cell nodes, an output is generated as the voltages on the output nodes.

2.2 CNN hardware and experimental setup

A global interconnection of each with each others introduces a net architecture too complicated for hardware design of VLSI. Thus, a simpler local interconnection has been adopted (i.e. each cell is connected only with its *r*-neighborhood).
As demonstrated in [5][8], a NxM CNN can be completely described by a space-invariant template as follows:

$$A = \begin{bmatrix} A_{-1,-1} & A_{-1,0} & A_{-1,1} \\ A_{0,-1} & A_{0,0} & A_{0,1} \\ A_{1,-1} & A_{1,0} & A_{1,1} \end{bmatrix} \qquad (10)$$

For 2D architectures, a NxM CNN with a Template expressed by (10) shows at least one equilibrium point if, and only if, the following set of inequalities are verified:

$$\begin{aligned} &A_{0,0} - 1 + A_{0,1}\cdot p_1 + A_{1,0}\cdot p_2 + A_{1,1}\cdot p_3 > 0 \\ &A_{0,0} - 1 + A_{0,-1}\cdot p_1 + A_{1,-1}\cdot p_1\cdot p_2 + A_{1,0}\cdot p_1\cdot p_3 > 0 \\ &A_{0,0} - 1 + A_{-1,0}\cdot p_2 + A_{-1,1}\cdot p_1\cdot p_3 + A_{0,1}\cdot p_1\cdot p_3 > 0 \\ &A_{0,0} - 1 + A_{-1,0}\cdot p_1\cdot p_3 + A_{-1,-1}\cdot p_3 + A_{0,-1}\cdot p_1\cdot p_3 > 0 \end{aligned} \qquad (11)$$

where, p_1, p_2, p_3: $(\exists p_1, p_2, p_3 \in \{1,-1\})$ are the CNN synaptic weights.

The designed CNN hardware system is a 6x24 DPCNNBoard. In particular, the SVCNNBoard (6x24DPCNNboard) and the functional model of one of the four dedicated chips 6x6DPCNN applied for the CNN integration (6x6 cells, Digitally Programmable Cellular Neural Network) are shown in Fig. 3.

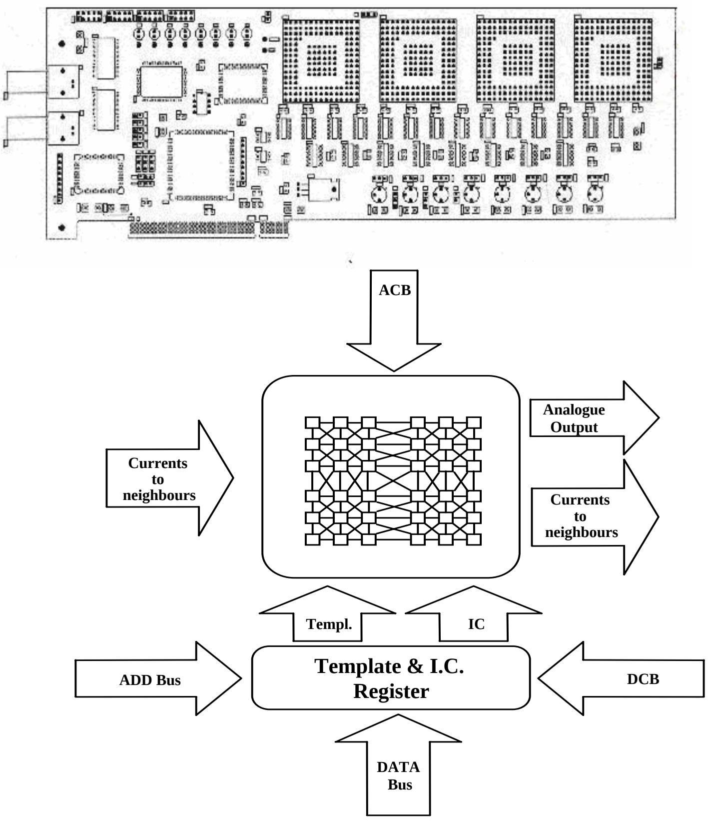

Fig. 3: SVCNNBoard and 6x6 DPCNN

2.3 Experimental tests

A set of tests have been performed with different synaptic weights. The aim is to observe the CNN dynamic behavior from stability to chaos. For the updating of the synaptic weights, the following four discrete configurations have been chosen:

1), equal synaptic weight value for each parameter $p_1 = p_2 = p_3$

2) The inequality set (11) has been solved by obtaining the following template:

$$A = \begin{bmatrix} a & a & a \\ -a & b & a \\ -a & a & -a \end{bmatrix} \tag{12}$$

In this case the stability is achieved, if b and a verify the inequality $b-1-a>0$.

3) "*b*" has been set to the value 1.5 to avoid the saturation of electronic components of the CNN

4) "*a*" has been step-by-step modified into the range [0.5-1]

For each implemented template, measurements have been made. The 144 (6x24) cell voltages are the state variables. Each acquired and sampled waveform has been analyzed by the software LEChaDe (Lyapunov Exponent Chaos Detection).

In all analyses, five behavior classes have been considered:
The influence cell-by-cell has been investigated by plotting the trajectories of the state variables generated by the state variables measured on different cells.

In all analyses, five behavior classes have been considered:
a) asymptotic global stability
b) stable oscillating
c) global oscillating
d) oscillating – chaotic
e) global chaotic

2.4 Software for stability analysis

2.2.1 LeChaDe

Maximum Lyapunov exponents ELP has been estimated using the programs BASGEN (dataBASe GENerator) and FET (Fixed Evolution Time) written by Alan Wolf and described in papers [9][10]. A code has been developed and called LeChaDe (Lyapunov exponent Chaos Detection) to manage the executable programs BASGEN and FET. BASGEN and FET allow to evaluate the Lyapunov Principal Exponent (ELP) (i.e. the highest value of the Lyapunov Exponent). The knowledge of the ELP allow to estimate the dynamic behavior of a CNN. The Fig. 4 shows the software LEChaDe flow-chart. In particular, LEChaDe acquires measured data from the hardware CNN. The imported sampled waveforms are sent as file data to the executable BasGen and FET programs. Finally, by the analysis of the output file returned by the program FET (Fet.out in Fig. 4), LeChaDe indicates the behavioral class that the CNN input produces. Exhaustive details on the theoretical aspects of the modules BASGEN and FET are available on papers of Wolf et al. [9] [10].

3 Measurements and numerical analysis

Voltage waveforms have been acquired from the CNN board output-node by varying the "*a*" parameter of equation (12). Five different classes of behavior have been identified:

a) asymptotic global stability: at each state-node (cell) it is measured a continuous stable signal for "*a*" range is 0.5 – 0.6
b) stable oscillating behavior has been observed for some state-cells, whereas for other cells (mainly the on the bound) a stable behavior for "*a*" range 0.6 – 0.65;
c) global oscillating behavior: at each stae-node is present a periodic signal for "*a*" equal to 0.65 – 0.7 (see Fig.5)
d) oscillating – chaotic behavior for "*a*" range equal to 0.7 – e 0.8);

e) global chaotic behavior for "*a*" equal to 0.8 -1 (see Fig. 6)

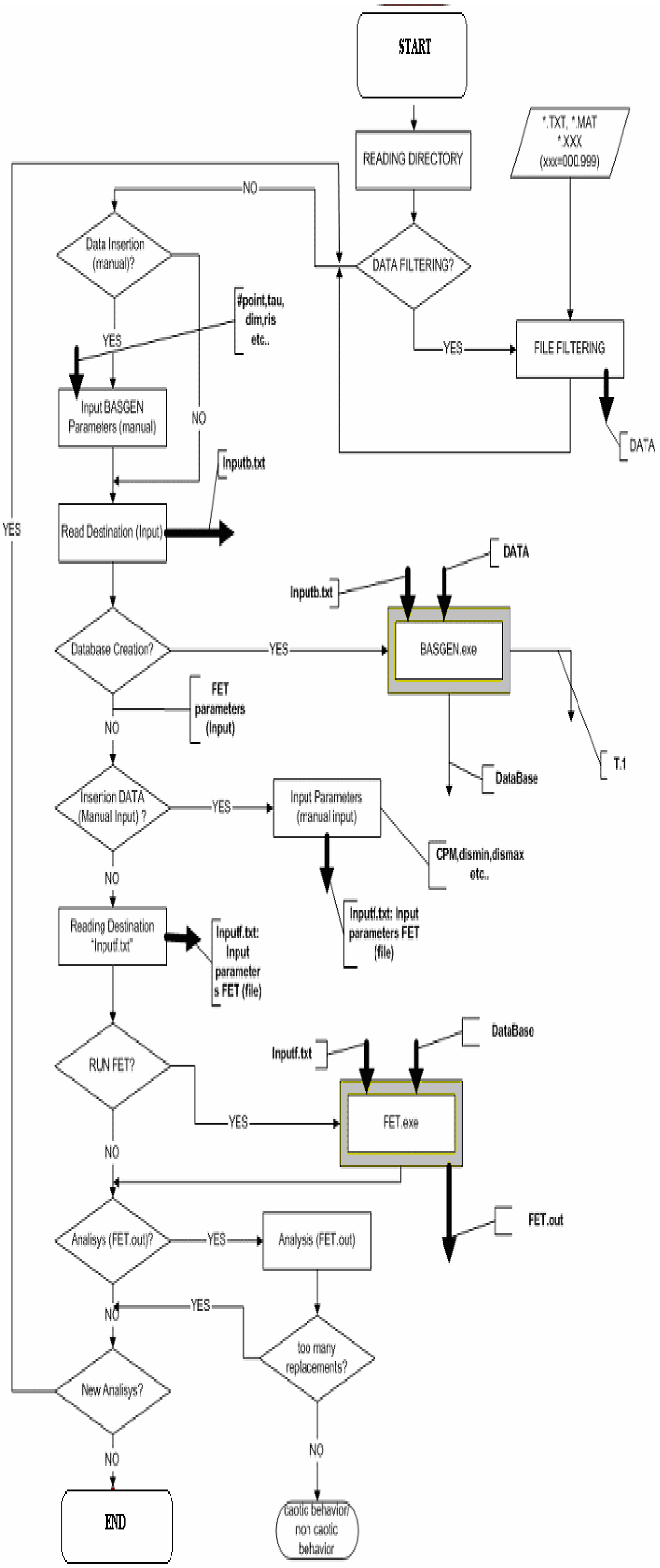

Fig. 4 : LeChaDe Flow-Chart.

Obviously, for waveforms which belong to the classes d) and e) the LEP is positive (as evaluated by analyses performed by LEChaDe). By referring to Fig. 7, it has been observed that the cells on the border (cell number 4 versus cell number 1) are not notably influenced by the adjacent cells. However they show a chaotic behavior (see Fig. 8 class d) on the same chip. On the other hand, on different chips (cell 37 vs. cell 1) the neighborhood produces a strong influence (see Fig. 9 class e) for the border cells. Similarly from Figs. 10 to 13 other examples are shown for class e.

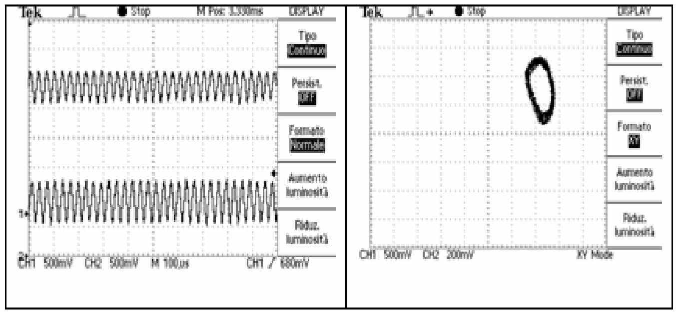

Fig. 5: - Global oscillating behavior: at each state-node is present a periodic signal for "*a*" equal to 0.65 – 0.7

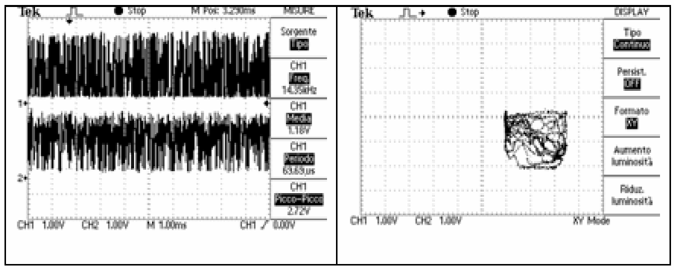

Fig. 6: - Global chaotic behavior for "*a*" equal to 0.8 -1

1	2	3	4	5	6	37	38	39	40	41	42	73	74	75	76	77	78	109	110	111	112	113	114
7	8	9	10	11	12	43	44	45	46	47	48	79	80	81	82	83	84	115	116	117	118	119	120
13	14	15	16	17	18	49	50	51	52	53	54	85	86	87	88	89	90	121	122	123	124	125	126
19	20	21	22	23	24	55	56	57	58	59	60	91	92	93	94	95	96	127	128	129	130	131	132
25	26	27	28	29	30	61	62	63	64	65	66	97	98	99	100	101	102	133	134	135	136	137	138
31	32	33	34	35	36	67	68	69	70	71	72	103	104	105	106	107	108	139	140	141	142	143	144

Fig. 7: - Scheme of cell disposition for the 6x24 board

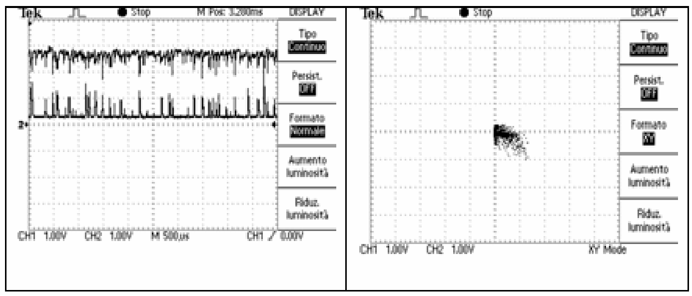

Fig. 8: Cell 4 vs. cell 1 for *a* belonging to the class d.

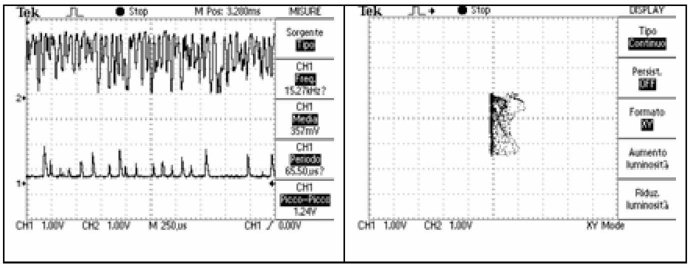

Fig. 9: Cell 37 vs.cell 1 for *a* belonging to the class d.

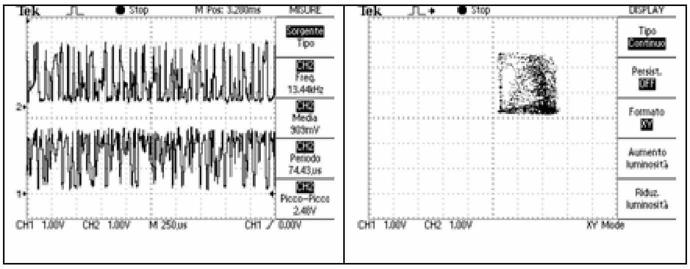

Fig. 10: Cell 22 vs.cell 15 for *a* belonging to the class e. .

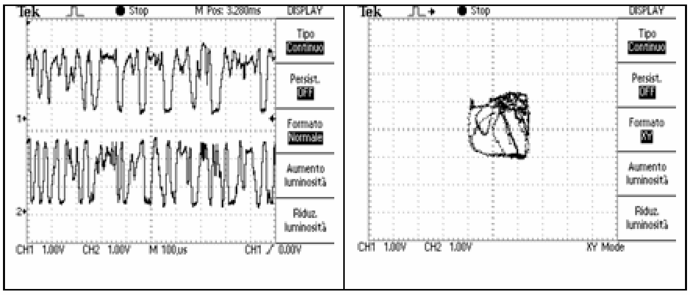

Fig. 11: Cell 22 vs.cell 58 for *a* belonging to the class e.

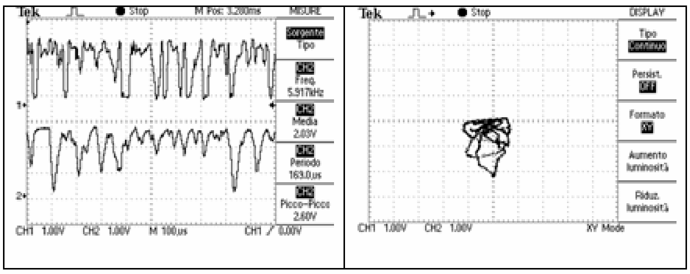

Fig. 12: Cell 22 vs.cell 114 for *a* belonging to the class e.

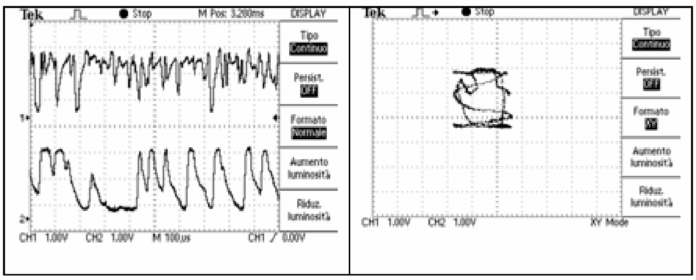

Fig. 13: Cell 22 vs.cell 144 for *a* belonging to the class e.

4 Conclusion

In this paper the experimental analysis of the complex dynamic behavior of a CNN hardware system (non-periodic, chaotic behavior, attractors) have been shown. The experimental data have been analyzed and classified by means of a dedicated software (LeChaDe (Lyapunov exponent Chaos Detection)) based on BASGEN and FET analyses [9],[10] for the evaluation of the Lyapunov coefficients. Both experimental apparatus and numerical analysis have been presented. Finally, several ranges of synaptic weights able to produce different CNN behaviors (from stable state to chaotic state) have been detected.

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