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Robust Feedback Linearization

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Abstract: - This research deals to introduce a new method for generalizing the well-famed Quantitative Feedback Theory (QFT) in granting robustness of Feedback linearization on nonlinear systems. In this new scheme, first applying feedback linearization, with selecting operating points in domain of system parameter's variations, nonlinear systems replaced with linear systems, then robust controller is designed for this linearzed system to control real nonlinear system. Weather, after feedback linearization, robust controller easily can be designed for this system to cope with uncertainties on practical nonlinear system. Finally this new technique is compared with Linear Quadratic Regulator (LQR).

Key-Words: - Quantitative feedback theory- Nonlinear control-Feedback linearization-Robust control-Linear Quadratic Regulator

1 Introduction

Feedback linearization is a technique on nonlinear systems control, which transforms all or part of dynamical systems to linear system(s) described by algebraic equations. In other words, linearization hypothesis transforms main model of the system to a simple form of equivalent models. Hence, it can be used in developing robust or adaptive controllers for MIMO nonlinear systems. The main idea behind using feedback from all linear systems is based on some known efficient design techniques which are used to finalize the controller design. One of the disadvantages of the feedback linearization is lack of enough robustness in presence of system parameter's uncertainties, in such a way with varying the parameters in system, the resulted responses won't be desired and may be unstable. So, in nonlinear systems with parameter uncertainties, feedback linearization can linearize the nonlinear system around some of its operation points of the parameter variation's domains. On the other hand, the nonlinear system will not become linear perfectly or even may remain nonlinear. That is why the designed controller couldn't present an appropriate response except in one point or in some points of parameter's variations domain which the system has been linearized in its neighborhood. In this article, a new method will be presented defining "Robust Feedback Linearization”. This method is based on QFT in the design of robust controllers for nonlinear systems. It is based on the of Schouder's fixed point theory, which has been presented by Horowitz. Through combining this theory with the feedback linearization, an algorithm will be designed to control nonlinear systems. The advantage of this procedure is that after linearization with feedback we will be faced with a linear system. Firstly, it seems possible either to design the controller by using some other methods such as Linear Quadratic Regulator (LQR), and then make it robust enough by this method, or to design a robust controller for this system after direct linearization with feedback.

2 Linearization Through Feedback

Two types of linearization through feedback are as follows.

2.1 Input State linearization

Suppose that the control input u, is posed to the following single input nonlinear system $\dot{X} = f(x,u)$. This problem can be solved during two steps using linearization techniques. Firstly, finding a state mapping $T : z \to x$, the system equation should be transformed into $\dot{Z} = f(z) + g(z)u$. Then the transformation of $U = u(z,v)$ is regarded to transform system dynamics in to linear and time invariant dynamics alike $\dot{z} = Az + bu$. Now, the custom linear design techniques can be used to design u. It ought to noted that T, T^{-1} should be continues and derivative. This type of mapping is called Diffeamorphism.[1,2]

Definition: The nonlinear system $g: D_x \to R^n$, $f: D_x \to R^n$, $\dot{x} = f(x) + g(x)u$ is called input-state linearizable, if a diffeamorphism mapping $T: D_x \to R^n$ exists such that $D_z = T(D_x)$ includes the origin and the variable transformation $z = T(x)$ transforms the above system as : $\dot{z} = Az + B\beta^{-1}(x)[x - \alpha(x)]$

Which A *and* B are controllable and $\beta(x)$ is non-singular in domain of all rate of $x \in D_x$. The above expression indicates that the main condition to make the nonlinear system input-state feedback linearizable is that the rates A, B, α, β, T satisfy the partial differential equations which are as follows:

$$\begin{aligned} &\frac{\partial T}{\partial x} f(x) = AT(x) - B\beta^{-1}(x)\alpha(x) \\ &\frac{\partial T}{\partial x} g(x) = B\beta^{-1}(x) \end{aligned} \qquad (1) \quad (1)$$

Without reducing the generality, regarding the canonical form of A_c, B_c for matrices A, B we will have:

$$T = \begin{bmatrix} T_1(x) \\ T_2(x) \\ . \\ . \\ T_{n-1}(x) \\ T_n(x) \end{bmatrix} \qquad (2)$$

$$A_c T(x) - B_c\beta^{-1}(x)\alpha(x) = \begin{bmatrix} T_1(x) \\ T_2(x) \\ . \\ . \\ T_{n-1}(x) \\ T_n(x) \end{bmatrix}, B_c\beta^{-1}(x) = \begin{bmatrix} 0 \\ 0 \\ . \\ . \\ \frac{1}{\beta(x)} \end{bmatrix} \qquad (3)$$

By replacing the statements of (3) in (1), we will have:

$$\begin{bmatrix} \frac{\partial T_1}{\partial x} f(x) \\ \frac{\partial T_2}{\partial x} f(x) \\ : \\ \frac{\partial T_{n-1}}{\partial x} f(x) \\ \frac{\partial Tn}{\partial x} f(x) \end{bmatrix} = \begin{bmatrix} T_1(x) \\ T_2(x) \\ : \\ T_n f(x) \\ \alpha(x)/\beta(x) \end{bmatrix}, \begin{bmatrix} \frac{\partial T_1}{\partial x} g(x) \\ \frac{\partial T_2}{\partial x} g(x) \\ : \\ \frac{\partial T_{n-1}}{\partial x} g(x) \\ \frac{\partial Tn}{\partial x} g(x) \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ : \\ 0 \\ 1/\beta(x) \end{bmatrix} \qquad (4)$$

the above equation indicates that the relation $T_2 - T_n$ is an obvious function of the first element T_1. Therefore, it seems necessary to find the function $T_1(x)$ in such a way, it satisfies the following statements:

$$\begin{cases} \frac{\partial T_i}{\partial x} g(x) = 0; i = 1,2,..,n-1, \frac{\partial T_n}{\partial x} g(x) \neq 0 \\ T_{i+1}(x) \frac{\partial T_i}{\partial x} f(x), i = 1,2,...n-1 \end{cases} \qquad (5)$$

Afterwards, if the function $T_1(x)$ can be calculated so that it makes the (5) balance, then α, β will be determined as follows:

$$\beta(x) = \frac{1}{(\partial T_n(x) / \partial x) g(x)} \quad , \alpha(x) = \frac{-(\partial T_n / \partial x) f(x)}{(\partial T_n / \partial x) g(x)} \qquad (6)$$

2.2 Input-Output Linearization

Since special certain output variables are considered such as tracking problem, the state space model would be indicated in the state form and output equations. In this situation, the mentioned method in the linearization of state equations doesn't necessarily result to output linearization. Consider the following system as follows:

$$\begin{cases} \dot{x} = f(x) + g(x)u \\ y = h(x) \end{cases} \qquad (2) \qquad (7)$$

Where f, g, h in the field of $D \subseteq \Re^n$ are smooth sufficiently. Supposing $\psi(x) = h(x)$, we will have:

$$\dot{y} = \frac{\partial \psi_1}{\partial x}[f(x) + g(x)u] \qquad (8)$$

If $[\frac{\partial \psi_1}{\partial x}]g(x) = 0$ is satisfied, then we will have:

$$\dot{y} = \frac{\partial \psi_1}{\partial x} f(x) = \psi_2(x) \qquad (9)$$

And after that the second derivate of y can be obtained as follows:

$$y^{(2)} = \frac{\partial \psi_2}{\partial x}[f(x) + g(x)u] \qquad (10)$$

Next, if $[\frac{\partial \psi_2}{\partial x}]g(x) = 0$ be satisfied, we will have:

$$y^{(2)} = \frac{\partial \psi_2}{\partial x} f(x) = \psi_3(x) \qquad (11)$$

By repeating this progress, it is resulted that if $h(x) = \psi_1(x)$ satisfies (5), then u will not appear in the equations $y, \dot{y}, ...y^{(n-1)}$ except in equation

$y^{(n)}$ as follows:

$$y^{(n)} = \frac{\partial \psi_n}{\partial x}[f(x) + g(x)u] \qquad (12)$$

In such a situation, the input-output linearization system conforms to the following control rule:

$$u = \frac{1}{\frac{\partial \psi_n}{\partial x} g(x)}[-\frac{\partial \psi_n}{\partial x} f(x) + v] \Rightarrow y^{(n)} = v \qquad (13)$$

If $h(x) = \psi_1(x)$ satisfies the following equations in interval of the rates $1 \le r \le n$ we have:

$$\frac{\partial \psi_i}{\partial x} g(x) = 0; i = 1,2,...,r-1, \frac{\partial \psi_r}{\partial x} g(x) \neq 0 \qquad (14)$$

Then $y^{(r)}$ will be equal to:

$$y^{(r)} = \frac{\partial \psi_r}{\partial x}[f(x) + g(x)u] \qquad (15)$$

And the control rule, which can be formulized as follows:

$$u = \frac{1}{\frac{\partial \psi_r}{\partial x} g(x)}[-\frac{\partial \psi r_r}{\partial x} f(x) + v] \Rightarrow y^{(r)} = v \qquad (16)$$

linearizes the input-output mapping as a chain form of linear $y^{(r)} = u$ integrators. In this case, the constant r is called degree of relation in the system where $r \in \{..-2,-1,0,1,2,...\}$. If the degree of relation is n, then the system will be both input-state linearizable and input–output linearizable.[1,3]

3 Design of Controller through the Quantitative Feedback Theory

Because of some uncertainties existence in system modeling, input disturbances in practical systems, and incoming input disturbances and uncertainties into process of system designing, controllers are required that in interval of all uncertainties at the model of the system and unknown disorders meet the characteristics of desired closed loop system, input tracking, removing disturbances, noise and etc. One of the efficient and reliable methods which can satisfy the above conditions is to use QFT in designing robust controller, which has been presented by Horowitz for the linear and time-invariant SISO system. In this paper, the robust controller has been totalized for nonlinear plants.

3.1 Quantitative Feedback Theory

QFT is an equipped theory that indicates how to use the feedback to obtain the desired system reaction in spite of uncertainties in the system model and disturbances. These two factors are modeled in the QFT as follows:

1) Sets $\tau_R = \{T_R\}$ from desired input-output tracking statement and sets $\tau_D = \{T_D\}$ from desired statements for removing the disturbances and disorders.

2) Sets $P = \{p\}$ from the change of system operating points.

Generally, QFT uses the two-degrees-of-freedom structure regarding Fig.1. The main aim is designing the controller G and the prefilter F so that in the domain of all system variations, the control proportions T_R and T_D, lie in the set of τ_R and τ_D

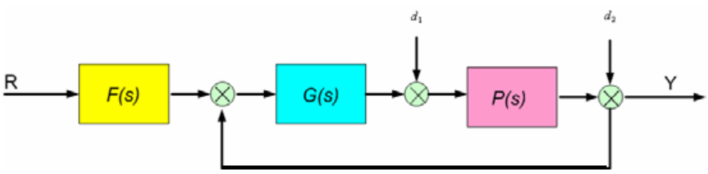

Fig. 1. The fundamental QFT design

3.2 Design of the Controllers through QFT Method for Nonlinear Systems

In design of the controller procedures for nonlinear system, the use of linearization in some cases is inevitable. Through the local linearization method, utilizing some robust controller designing method such as QFT technique, the uncertainty of a nonlinear system may be replaced by a collection of uncertain linear and time-invariant systems.

The main idea in design of the robust controller through the QFT method for a nonlinear system, is transforming of the nonlinear system into an equivalent linear system using feedback Linearization Theory in addition of disturbances on output with consider to the desired output properties. In fact, the deference between nonlinear systems and the obtained equivalent linear system is regarded as disturbances in the equivalent linear system output. To better understanding an example is planned as follows.

Example.1: Consider the indicated system in Fig. 2, where N is a nonlinear (SISO) system of a known member in $\{N\}$, and $\{y\}$ is a collection of permissible output responses. Also the primary conditions on y have been indicated. Our purpose is to design G and F_r so that the system output for all $N \in \{N\}$ lies in the collection of $\{y\}$.

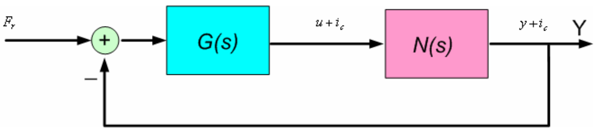

Fig. 2. Typical block diagram of a SISO Nonlinear system (*ic*: initial conditions)

Lemma 1*:* Regarding the following mapping on $\{y\}$:

$$\phi(y) = \frac{P_{N,y} G F r + y^{i}_{N,y} + d_{N,y}}{1 + P_{N,y} G} \qquad (17)$$

$$y = Nu = P_{N,y} u + y^{i}_{N,y} + d_{N,y} \qquad (18)$$

Where for any y through equation $y=N.u$, an u is determined. The (18) is used to select a linear time-invariant system ($P_{N,y}$) and a disturbance signal ($d_{N,y}$). Next, the mapping, $\phi(y)$, can be obtained from (17). If the obtained mapping has a constant point for any $N \in \{N\}$ in $\{y\}$, the G and F_r will be a type of solution for the intended problem. According to the above lemma, a designing process for the proposed problem is as follows:

1. For any $N \in \{N\}$ and $Y \in \{y\}$, $P_{N,y}$ and $d_{N,y}$ must be selected such that the (24) would be satisfied.

2. G and F_r should be designed in such a way the mapping, $\phi(y)$ would consist a constant point in $\{y\}$.

In this process, two main questions are posed:

a. How to select the pair $P_{N,y}$ and $d_{N,y}$?

b. How to guarantee that the obtained mapping has a constant point in $\{y\}$?

To answer these questions, the use of Schouder's fixed point theory which is based on the following lemma seems usefull. [7, 5, 2]

Lemma 2*:* Suppose that $\{y\}$ is a complete and closed set in the Banach space, and ϕ is a mapping from $\{y\}$ to itself. In such a situation, ϕ has a fixed point in $\{y\}$.

Based on the above lemma, if $\{y\}$ is complete and closed set in Banach space, then the G and F_r are such that the mapping $\phi(y)$, is a smooth mapping in interval of all $N \in \{N\}$ from $\{y\}$ to itself. In such a condition, the G and F_r will be a solution of Example 1.

According to the mentioned conditions If the pair $P_{N,y}$ and $d_{N,y}$ are selected correctly, Example.1 will be transformed to the following problem.

Example 2*:* Consider the indicated system in Fig.3, in which P, d from the set of $\{P, d\}$ are linear time-invariant system (LTI) and disturbance respectively. $y_0(s)$ is a nominal output and $e(w)$ is an index function. Design the controllers G and F_r such that for all the pairs of $\{P, d\}$, the system be stable and the system output be limited as follows:

$$|y(j\omega) - y_0(j\omega)| < e(\omega) \qquad (19)$$

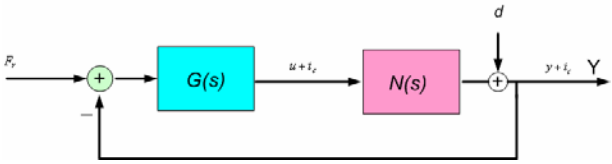

Fig.3. The schematic of LTI feedback in place of main system

So, the main step in the solution of the mentioned problem is calculation of the bounds $G(j\varpi)$ through (19), which is a solution of the following inequality. (F_r is supposed to be obvious).

$$\left|\frac{PGF_r + d}{1 + PG} - y_0(j\omega)\right| \le e(j\omega); \forall P, d \in \{P, d\} \qquad (20)$$

A suggested method for solution of the above inequality is to select a medium state regarding P_0, d_0, and to achieve F_r so that the left side in (20) deals to zero.

Clearly, we have:

$$F_r = y_0 + \frac{y_0 - d_0}{P_0 G} \qquad (21)$$

In this situation, (20) is transformed as follows:

$$\left|\frac{(y_0 - d_0)P / P_0 + d - y_0}{1 + PG}\right| < e(\omega); \forall P, d \in \{P, d\} \qquad (22)$$

Finally, the design steps would be summarized as follows:

a. Use (22) to determine the bounds on $G(j\omega)$.

b. Design the controller $G(s)$ such that it satisfies obtained bounds and the calculated F_r from (21).

4 Combining QFT and Feedback Linearization

As indicated throughout the feedback linearization, a nonlinear system can be transformed to a linear system and complete the design step throughout the designing methods for the linear system. But, the important subjects in the feedback linearization are the uncertainty parameters in the nonlinear system, which cause the linearization error. In some cases when the feedback linearization is designed in certain operating point of system parameters, the nonlinear system is linearized only in this point of parameter's variations domain, and in other points the system doesn't linearized perfectly or it may never become linear. Hence, in spite of existence of some uncertain parameters in the nonlinear system and the linearization error, the designing of a robust controller throughout feedback for this type of the systems after linearization is required. Due to this purpose, we can use QFT to design the robust controller according to the mentioned method.

In this idea, the nonlinear system is firstly linearized by the feedback linearization in some operating points of the parameters range. Then, the linearization error in other points of the system parameters throughout QFT is regarded as the collection of disturbances in the output of the linearized system. Finally, the robust controller is designed for the proposed system in spite of these disturbances.
Based on the theory and Lemmas, which are indicated in section 3, it is guaranteed that the obtained response meets the desired output in the range of all system parameter's changes.

Example 3*:* Consider to the Vanderpol nonlinear system which contains the parameter uncertainties and regarding the nominal system parameters as follows**:**

$$\ddot{y} + A\dot{y}(By^2 - 1) + Ey = Ku \qquad A \in [1,3], B \in [1,4], E \in [-2,1], K \in [1,4] \tag{23}$$

$$A_0 = 1, B = 1, E = -2, K = 4 \tag{24}$$

Using the feedback linearization, if the input is considered as (25), then the above system will be linearized perfectly in the nominal point and its equation will be as (26):

$$u = \frac{1}{K_0}(v + A_0 B_0 \dot{y} y^2 - A_0 \dot{y} + E_0 y) \tag{25}$$

$$\ddot{y} = v \tag{26}$$

For this system, the design steps are accomplished throughout two methods:
a. Designing the state feedback throughout LQR.
b.Designing the controller throughout robust feedback linearization according to the introduced method.

4.1 Designing of the Optimal State Feedback

The transfer function of the linearized system after linearization with the feedback is as (26), where the above state space system's equations are as follows:

$$\dot{X} = AX + Bv \qquad y = CX + Dv \tag{27}$$

where:

$$A = \begin{bmatrix} 0 & 0 \\ 1 & 0 \end{bmatrix}, B = \begin{bmatrix} 1 \\ 0 \end{bmatrix}, C = [0 \;\; 1] \;, D = [0] \tag{28}$$

If $R = [0.01]$ and $Q = \begin{bmatrix} 0 & 0 \\ 0 & 0.25 \end{bmatrix}$ is satisfied, the values of coefficient matrix *K*, in the control rule of optimized feedback $u = -KX$, will be obtained as:

$K = [5.9407 \quad 0.8417]$

The results of simulation throughout exerting of the above control rule is indicated in Fig. 4, which shows the output of closed loop system in domain of all system parameters changes.
As indicated in Fig. 4, it is observed that properties of the system output is ideal in the nominal point and in the other points doesn’t meet ideal expected properties. In other word, the feedback linearization isn’t robust in presence of the system parameters variations. The results of simulation throughout exerting the above control rule is indicated in Fig.4, which shows the output of closed loop system in domain of all system parameters changes. In the other word the feedback linearization isn’t robust enough in presence of system parameters changes.

4.2 Designing the Controller throughout QFT

The transfer function of equivalent linear system is obtained from (26) and for all parameter's changes the regarded disturbance which is the same linearization error determined by (25), (23).
Know, concerning these disturbances in the output of linear system in (26) and throughout the quantitative feedback theory, the robust controller is designed. Fig.(5) shows the bounds and the type of gain function forming for nominal loop in Nicholes diagram. Fig.(6) shows step response for all parameter variations. Finally, the controller *G* and prefilter *F* are obtained as follows:

$$G = \frac{6.76 \times 10^3 (s + 35.94)(s + 2.76)}{s^2 + 162.1s + 388.8} \tag{29}$$

$$F = \frac{0.0829s + 5.628}{s^2 + 4.11s + 5.63}$$

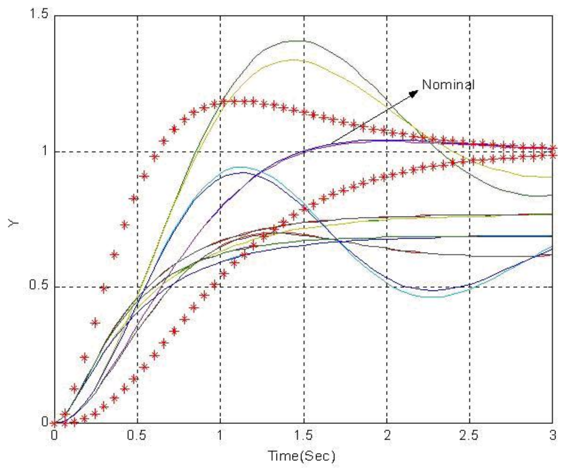

Fig. 4. The output of the closed loop system in presence of the parameters' variations and uncertainties using LQR controller and the typical bounds and nominal loop function.

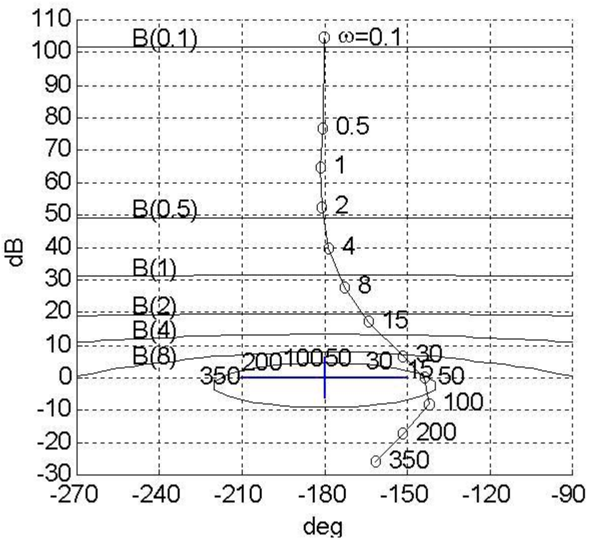

Fig. 5. The bounds and nominal loop gain.

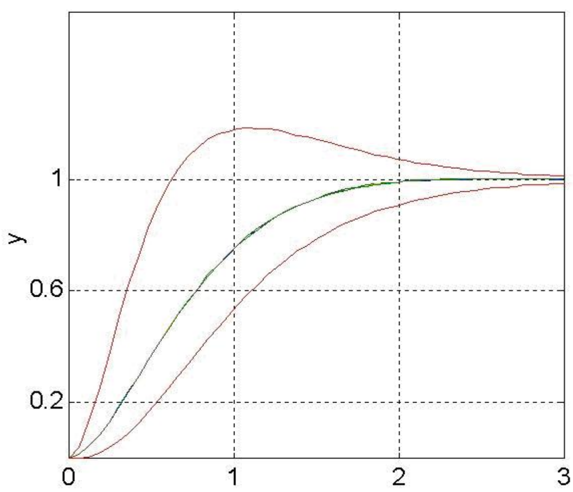

Fig.6. Closed loop system output for all uncertainties and variable changes in Robust Feedback Linearization.

Conclusion

Verifying Fig.(6) show that using Robust Feedback Linearization, system output placed in the desired parameter bound rate. According to this research it can be concluded that it is possible to make the controller to be more robust and reliable in a good range of parameter uncertainties and disturbances on nonlinear systems. Other important advantages of this new method listed as follows:

1. This new method is useable and benefit in different types of systems.
2. It includes all parameter's uncertainties in design procedure.
3. Achieving a fixed structure controller.
4. High robustness in structure of controller.

But because of fixed structure of controller the main disadvantages of this new method can be summarized in high amount of control energy as shown in Fig.(5).In such conditions using Nero-Fuzzy theory, it is possible to design better controller to find a better position in loop shaping step.

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