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MATRICIAL ESTIMATION FOR START TIMES WITH STOCHASTIC BEHAIVOR BY PERIODIC REAL TIME TASKS IN A CONCURRENT SYSTEM

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Abstract. Using a multivariable dynamic model for describing startup times of real Time Tasks supposing the following considerations: The system is stationary, first order, with jitter and external perturbations bounded with a normal distribution without correlation that closely represent periodical behavior of real time tasks. To bear closer the task model in a concurrent system, systems internal dynamics are required, those are represented through the parameter matrix in function of output vectors in the regressive model as is for perturbations, but knowing their value is work for a multivariable estimator. Results of an example performed on a real-time platform are presented, considering periodic and concurrent tasks; an instrumental variable algorithm is used because of his good convergence time and his relatively easy implementation.

Keywords*: Startup time, estimation, real time, periodic tasks, instrumental variable.*

1. Introduction

Periodic tasks are usually found in several applications like airplanes and control process where uniform monitoring is required. Modeling them is not simple because each of those needs an adequate representation. A set of periodic concurrent tasks may be represented using a multivariable state model where in explicit way the model is function of the internal dynamic and previous states of the system's output information.
These matrix and perturbations are going to give the start times characteristics through system evolution. A parameter matrix estimator is needed in order to adjust the model for reconstruction, tracing and prediction in real time.

2. Start Times Model in PRTT (Periodic Real Time Tasks)

As seen in [1], [4], [5], [6], [10], [14] and [15], PRTT model is represented by a stochastic, stationary, first order and first grade type difference equation; Considering that external processor perturbations are not correlated and obey a normal distribution function.

Proposition 1. (Absolute Arrival Time for RTT). The vector of absolute arrival times L_k of an instance set with index k is described by:

$$L_k = L_{k-1} + \Pi_k . \qquad (1)$$

Proposition 2. (Inter-arrival Time for RTT). The Inter-arrival time vector Π_k of an instance with index k of a RTT J_i set is describes as:

$$\Pi_k = A_k(\Pi_{k-1} - W_{k-1}) + U_k + W_k . \qquad (2)$$

Where: A_k *is the system parameter matrix with unknown dynamics bounded in agreement with [7], [11], [12] y [14];* Π_k *is the Inter-arrival times vector of instances with index k;* W_k *is the external processor perturbations vector, represented through random variables with Gaussian distribution;* U_k *is the Inter-arrival times vector of reference.*

Proposition 3. (Start Times for RTT). The starting times vector S_k of a feasible task with index k of a set of RTT J_i is described with:

$$S_k = S_{k-1} + \Pi_k + V_k - V_{k-1} . \qquad (3)$$

Comment 1. If an inter-start times vector Π'_k is modeled it is just needed to add the *jitter* V_k as internal perturbation to the state equation such that:

$$\Pi'_k = X'_k + W_k , \qquad (4)$$

$$X'_k = A_k X'_{k-1} + U_k + V_k , \qquad (5)$$

$$\Pi'_k = A_k(\Pi'_{k-1} - W_{k-1}) + U_k + W_k + V_k . \qquad (6)$$

Proposition 4. (Periodic Tasks in Real Time). A set of PRTT is that J_i where all their instances have inter-arrival times vectors Π_k close to a periodic vector T_k.

$A_k \qquad \{\lambda_{i,j,k}\} \subset [0,1] \qquad \lambda_{i,j,k} \ \forall\ i,j,k \in Z^+$

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$U_k \quad \{u_{i,k} = T_i - a_{i,k}T_j\} \quad \forall\ ilk \in Z^+$

3. Real Time Parameter estimator (RTPE)

In order to trace the parameter matrix dynamic of the concurrent PRTT a Real Time Estimator is needed, which is defined in agreement to [3], [7],[10], [14], [15], [16] as:

Definition 1. (Real Time Parameter Estimator RTPE). All RTPE is a digital filter with the following conditions:

a. Extraction and emission of observable information (input and outputs), where $\{u(k)_i \in \mathbf{U}(k)\}$ & $\{y(k)_j \in \mathbf{Y}(k)\}$, with $i, j, k \in \mathbf{Z}^+$, in criteria of [2], [3], [7], [8], [11], [12] and [13],
b. To give correct answers on the mater to the process considering some reestablished criteria as those shown in [2],[11], [12] and [13],
c. Being expressed recursively (see [3], [7], [10], [13], [17]),
d. Convergence value will be bounded in an infinite interval in which it will be varying the convergence value.
e. Matrix operation usage in agreement with process dynamic restrictions.

Proposition 5. (Convergence in all multivariables RTPE). All RTPE as a parameter estimator has an error functional bounded (see: [2],[7], [9], [16] and [17]), such that :

$$m^* = \arg\min \inf_{k \geq m} P\{|\hat{a}_k - a| \leq \Delta\} = 1 \qquad (7)$$

Where Δ is the error bound defined by noise variance, m is the convergence interval and m^* is the set of intervals where the RTPE has converged.

4. Estimation of Periodic Real Time Task Start Times with stochastic jitter using the Instrumental Variable technique.

As an example a set of 2 concurrent PRTT is analyzed. The set of relative arrival times Π_k is used as the observable signal and the parameter matrix A_k is going to be estimated, such that:

$$E\{W_{k+1}\Pi_k^T\} = [0], \quad E\{W_k\Pi_k^T\} = \Theta_{w_k}^2, \quad E\{V_k\Pi_k^T\} = \Theta_{v_k}^2,$$
$$E\{W_k(W_k)^T\} = \Theta_{w_k}^2, \quad E\{V_k(V_k)^T\} = \Theta_{v_k}^2, \qquad (8)$$
$$E\{V_k(W_k)^T\} = [0].$$

The estimator is expressed as:

$$\hat{A}_k := (\hat{A}_{k-1}B_{k-1} + \Pi_k Z_k^T)B_k^{-1}. \qquad (9)$$

This representation obeys the definitions of a RTPE.

The estimation error is defined as in [2], [3], [7] and [9]:

$$\Delta_k = |\hat{A}_k - A| \qquad (10)$$

In agreement with [9] the estimation error is described as:

$$\Delta_k := ((-A\Theta_{w_k}^2 + \Theta_{v_k}^2)(I - A^2))$$
$$(\Theta_{w_k}^2(2A^2 + I + A^2) + \Theta_{v_k}^2(I + 2A))^{-1} \qquad (11)$$

As the error functional in accordance with [9] and [15]:

$$\mathrm{J}_k = E(\Delta_k(\Delta_k)^T). \qquad (12)$$

The following data was considered as results of the estimation algorithm validation:

$$A = \begin{bmatrix} 0.5 & 0.2 \\ 0.3 & 0.4 \end{bmatrix}, \quad \Theta_{v_k}^2 = \begin{bmatrix} 0.95 & 0.93 \\ 0.97 & 0.94 \end{bmatrix}, \qquad (13)$$
$$\Theta_{w_k}^2 = \begin{bmatrix} 0.95 & 0.93 \\ 0.97 & 0.94 \end{bmatrix}.$$

For the experimental implementation of the RTPE the following was considered:

a. The maximal system deadline $\mathbf{D}_{k,\ max}$, is equal to period $\mathbf{T}_k$.
b. The Start time ($\mathbf{S}_k$- $\mathbf{L}_k$) was obtained such that $\mathbf{S}_k = \mathbf{L}_k + 0.0015$ ms.

c. The sampling period T_k (Real Time temporizator impulse) for task activation is 10 ms.
d. Minimal deadline $\mathbf{D}_{k\,min}$= 1 ms.
e. Convergence deadline of the PRTT set is: d= 4 s.

For an experiment of the RTPE Fig. 1 was obtained with the following results:

m_{max}= 354 intervals, t_{c_max}= 3,54 s. The convergence time is t_c =3.54 s.

a. m= [354, 213, 241, 318] intervals,
b. t_c= [3.54, 2.13, 2.41, 3.18] s,
c. d= 4 s. (400 intervals).

Fig.1 y Fig. 2 are just examples of a RTPE behavior used as a parameter estimator trough the instrumental variable technique.

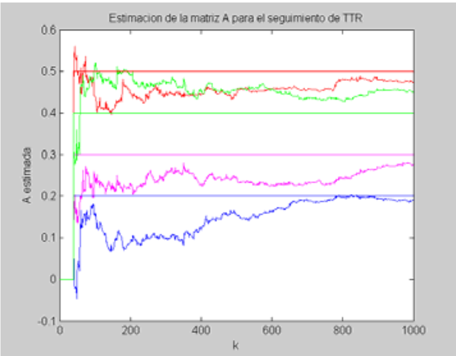

Fig.1 *Graphics of parameter matrix estimation "A" using the RTPE..*

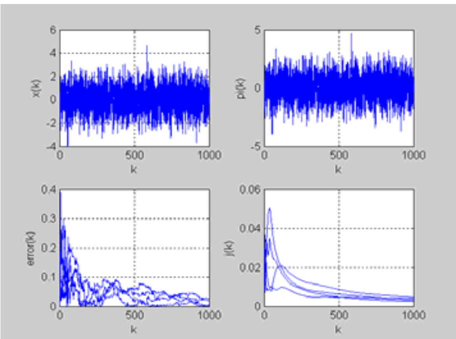

Fig. 2 *Internal state X_k, observable signal Π_k, estimation error and functional j_k for the RTPE.*

5. Conclusions

The presented Start times model considers internal dynamics of the system for Periodic Real Time Tasks in a concurrent system. Definitions were made for tasks depending on their arrival and start times and were characterized in accordance with his parameter or parameter matrix, his entrance and computing equipment's internal and external perturbations. The model showed to be capable of characterize several Real Time Task behaviors and obey characteristics mentioned by several authors. Important things to mention in this kind of real time filtering are: task characteristics, synchronicity, sampling periods and convergence time. For start time estimation, the convergence times where bounded by a Δ estimation error bound defined by the second moment of probability of the internal and external perturbations; convergence periods were acceptable. Instrumental Variable technique was used because of his good response time and his convergence time obeys Real Time System conditions, approaching to real parameters rapidly and in deadlines.

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