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Dry Friction Influence on the Stability of a Mechanical System with Two Degree of Freedom

DOMENICO GUIDA and LUCIO DURSO
Department of Mechanical Engineering
University of Salerno
Ponte Don Melillo, 84084 Fisciano (SA)
ITALY

Abstract: - The influence of the dry friction on the dynamics of a system with two degrees of freedom is proposed. The model consists of a body of mass m1, attached to a driving support by means of a spring and a damper, moving relatively to its counterpart of mass m_2. Maps of stability are proposed for assigned values of the parameters. These maps can be used to check the stability of equilibrium position..

Key-Words: - Self Excited Vibrations, Dry Friction, Limit Cycle, Stick-Slip, Friction Instability, Structural Stability.

1 Introduction

In many mechanical systems the dry friction can induce self-excited vibrations which often are unwanted.

In [2,8] it has been analyzed a system with one degree of freedom in presence of a friction force characteristic. The method used for the analysis has mostly been a geometrical type method. Suitable conditions have been fixed upon the phase trajectory in the discontinuity points of friction characteristic.

In [9] the static friction influence upon the two degrees system dynamical behavior is analyzed. The system is shown in Figure 1 and it is composed of two masses m_1 and m_2 ; the first one is undergo to an elastic and viscous force field. The second one is attached to a fixed wall by a spring and a viscous damper with constantsk$_2$ and σ_2, respectively. The friction force to the interface is represented in Figure 2. The most important result obtained in this paper were that also under conditions of stability of the equilibrium position, self excited vibrations of the slides can reveal themselves. It was shown that the system exhibits self excited vibrations increasing the ratio static friction/ support speed. So, it can be asserted that such ratio value is the basic parameter to analyze the bifurcation conditions of the system.

In this paper a method for the calculating the values of the parameters that correspond to bifurcation conditions is proposed. Moreover maps of stability are proposed for assigned values of the parameters. These maps can be used to check the stability of equilibrium position..

2 Mathematical Model

Let X_1 and X_2, respectively, the displacement of the slides of mass m_1 and m_2 in the reference frame system indicated in Figure 1. The motion equations can be written so as indicated in the following relations:

$$\begin{cases} m_1\ddot{X}_1 + \sigma_1\left(\dot{X}_1 - v\right) + k_1\left(X_1 - vt\right) + F\left(\dot{X}_1 - \dot{X}_2\right) = 0 \\ m_2\ddot{X}_2 + \sigma_2\dot{X}_2 + k_2X_2 - F\left(\dot{X}_1 - \dot{X}_2\right) = 0 \end{cases} \qquad (1)$$

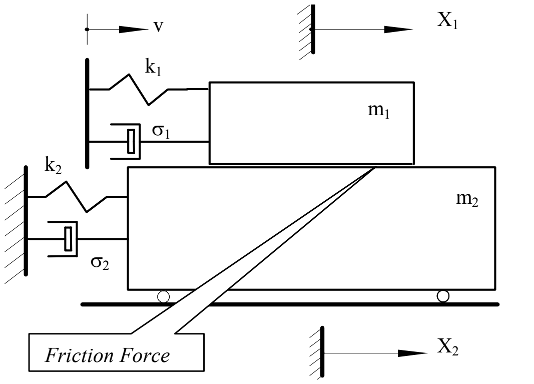

Figure 1 - System Model

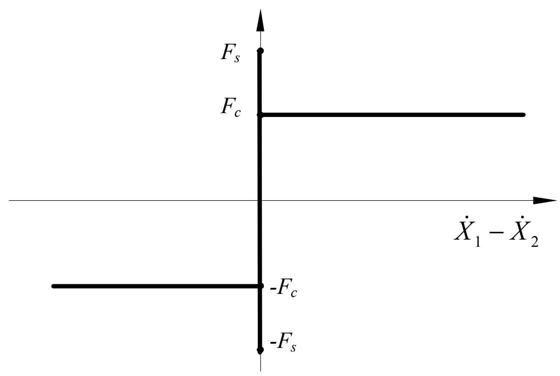

Figure 2 – Friction Force Characteristic

The friction characteristic is assumed to be piecewise linear function as shown in Figure 2. This function is analytically expressed by the followings relationships:

$$F\left(\dot{X}_1 - \dot{X}_2\right) = \begin{cases} F_c & \dot{X}_1 - \dot{X}_2 > 0 \\ F_s & \dot{X}_1 - \dot{X}_2 = 0^+ \\ F \quad |F| < F_s & \dot{X}_1 - \dot{X}_2 \equiv 0 \\ -F_s & \dot{X}_1 - \dot{X}_2 = 0^- \\ -F_c & \dot{X}_1 - \dot{X}_2 < 0 \end{cases} \tag{2}$$

Putting:

$$\begin{aligned} & x_1 = X_1 - vt \qquad \tau = \omega_1 t \\ & x_2 = X_2 \qquad (\cdot) = d/d\tau \\ & \frac{\sigma_1}{m_1\omega_1} = 2s_1 \qquad \frac{\sigma_2}{m_2\omega_2} = 2s_2 \\ & \frac{F_c}{m_1\omega_1 v} = f_{c1} \qquad \frac{F_s}{m_1\omega_1 v} = f_{s1} \\ & \omega_1^2 = \frac{k_1}{m_1} \qquad \omega_2^2 = \frac{k_2}{m_2} \end{aligned} \tag{3}$$

$$\frac{m_1}{m_2} = r \qquad \frac{\omega_1}{\omega_2} = \zeta \qquad \frac{\beta}{m_1\omega_1} = 2\mu_1$$

$$\left(x_1 \frac{\omega_1}{v}\right) = \eta_1(\tau) \qquad \left(x_2 \frac{\omega_1}{v}\right) = \eta_2(\tau)$$

the equations (3) can be rewritten as follows:

$$\begin{cases} \ddot{\eta}_1 + 2s_1\dot{\eta}_1 + \eta_1 + \\ \dfrac{1}{m_1\omega_1 v} F\{v[(\dot{\eta}_1 - \dot{\eta}_2)] + 1\} = 0 \\ \ddot{\eta}_2 + 2s_2\dot{\eta}_2 + \dfrac{1}{\zeta^2}\eta_2 + \\ \dfrac{r}{m_1\omega_1 v} F\{-v[(\dot{\eta}_1 - \dot{\eta}_2)] + 1\} = 0 \end{cases} \tag{4}$$

By the integration of the (4) it is possible to determine the dynamic behavior of the system for assigned initial conditions. For assigned values of the parameters only two types of dynamical behavior are possible. The former correspond to stable equilibrium position, in these conditions the phase trajectory converge to position equilibrium Figure 3.

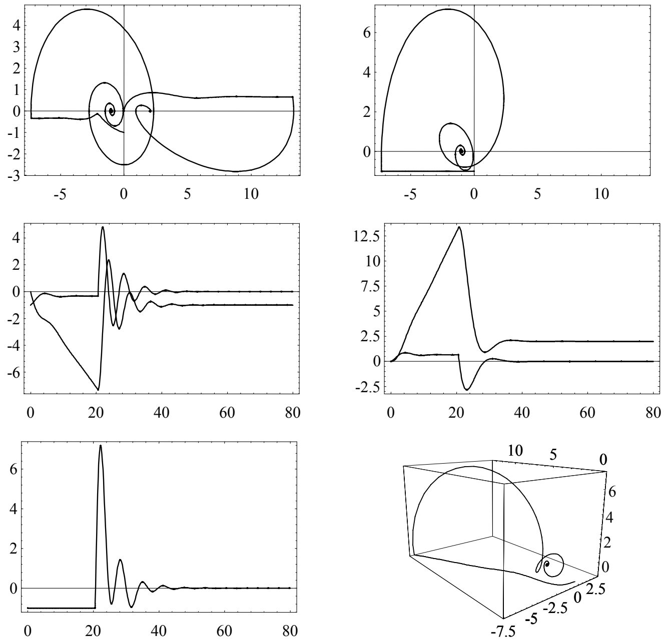

Figure 3 - r=0.5; =2; s1=0.2; s2=0.3; fc_1=1; fs_1=7.4
Fig. 3[1,1]=Phase trajectories on the plane $\{\eta_1,\dot{\eta}_1\}$ and $\{\eta_2,\dot{\eta}_2\}$
Fig. 3[1,2]=Phase trajectories on the plane $\{\eta_2,(\dot{\eta}_1-\dot{\eta}_2)\}$
Fig. 3[2,1]=Solution $\{\tau,\eta_1\},\{\tau,\dot{\eta}_1\}$
Fig. 3[2,2]=Solution $\{\tau,\eta_2\},\{\tau,\dot{\eta}_2\}$
Fig. 3[3,1]=Solution $\{\tau,\dot{\eta}_1-\dot{\eta}_2\}$
Fig. 3[3,2]=Solution $\{\eta_1,\eta_2,(\dot{\eta}_1-\dot{\eta}_2)\}$

In the other case the trajectory intersects the pints in which the following relationship is verified:

$$\dot{\eta}_1 - \dot{\eta}_2 = -1 \tag{5}$$

In Figure 4 is shown the dynamical behavior of the systems analyzed in critical conditions, that is, the system exhibits a stable cycle limit for assigned parameter values.
In this paper has been used a numerical procedure for calculating the foregoing parameters. The results are shown in the Figure 5, 6, 7 and 8.

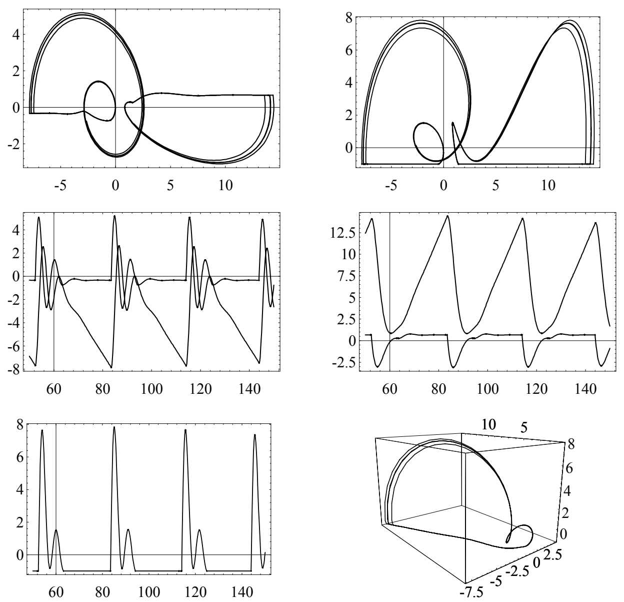

Figure 4 - r=0.5; =2; s1=0.2; s2=0.3; fc1=1; fs1=7.8

Fig. 4[1,1]= Phase trajectories on the plane $\{\eta_1, \dot{\eta}_1\}$ and $\{\eta_2, \dot{\eta}_2\}$

Fig. 4[1,2]=Phase trajectories on the plane $\{\eta_1, (\dot{\eta}_1 - \dot{\eta}_2)\}$ and $\{\eta_2, (\dot{\eta}_1$ -

Fig. 4[2,1]=Solution $\{\tau, \eta_1\}, \{\tau, \dot{\eta}_1\}$

Fig. 4[2,2]=Solution $\{\tau, \eta_2\}, \{\tau, \dot{\eta}_2\}$

Fig. 4[3,1]=Solution $\{\tau, \dot{\eta}_1 - \dot{\eta}_2\}$

Fig. 4[3,2]=Solution $\{\eta_1, \eta_2, (\dot{\eta}_1 - \dot{\eta}_2)\}$

In Figure 5 are indicated the critical values $\zeta - f_s$ for assigned values of r, s_1 and s_2 while in Figure 5 are indicated the critical values $r - f_c$ for assigned values of ζ, s_1 and s_2. In Figures 7 and 8 are shown the stability maps $s_1 - f_s$, $s_2 - f_s$ for assigned values of parameters ζ, r and f_c.

3 Conclusion

The influence of the on the dynamical behavior of a two degree of freedom system is proposed. In a previous work the authors analyzed the dynamical behavior of the system and found that it can reveal vibrations for assigned values of parameters which define itself.

In this paper has been used a numerical procedure for calculating the foregoing parameters. The results are shown in maps of immediate use.

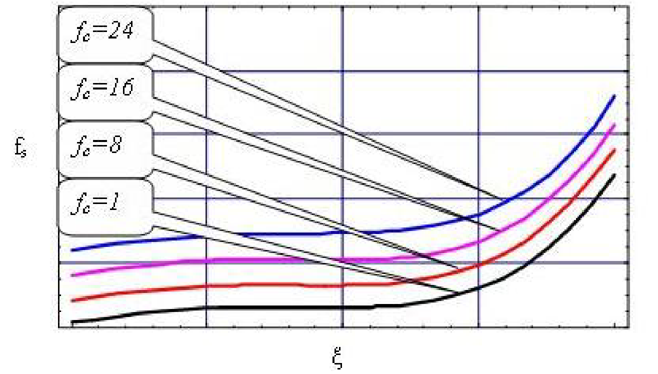

Figure 5 - Stability map $\zeta - f_s$ for: r=0.5, s_1=0.2, s_2=0.3

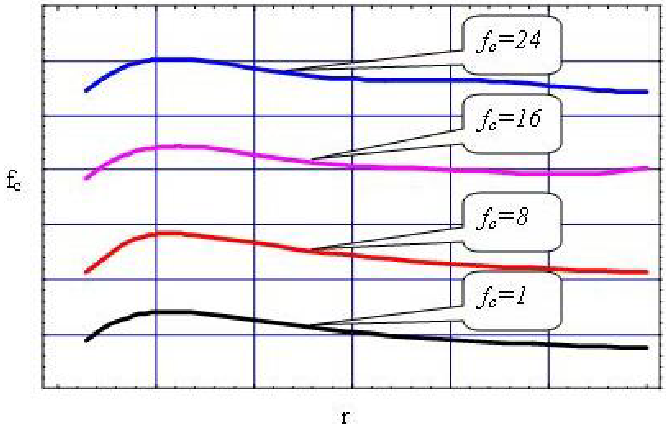

Figure 6 - Stability map $r - f_c$ for: ζ=2, s_1=0.2, s_2=0.3

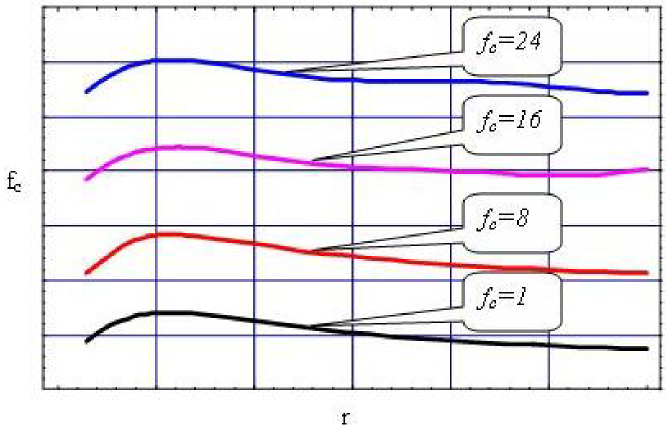

Figure 7 - Stability map $s_1 - f_s$ for: ζ=2, r=0.5, s_2=0.3

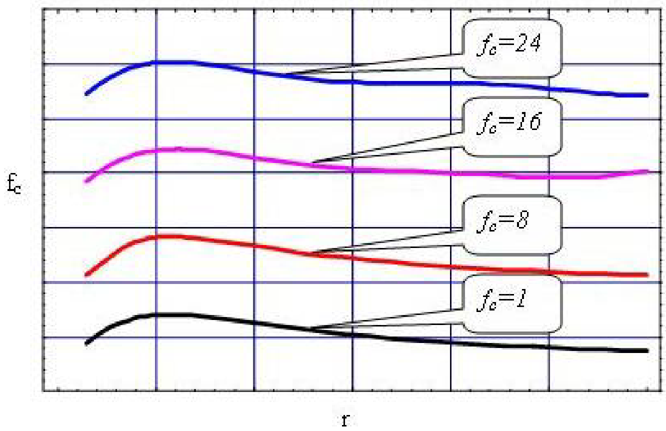

Figure 8 - Stability map $s_2 - f_s$ for: ζ=2, r=0.5, s_1=0.2

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