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Empirical Test of the Currency Option Pricing Model

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Abstract: - This paper analyze the theoretical value of American and European–style call (GBP call against MYR) on an underlying asset with amount of 1 units of sterling (GBP). Besides that, the marginal effects of the six variables are testing for its contributions on American-style call premium. Binomial Option Pricing model, of Cox, Ross, and Rubinstein (1979), are applied to measure both style of currency options. We find that the call premiums of American option tend to have higher value compare with the European option. In addition, the spot exchange rate, domestic interest rate, time to maturity and volatility have proven of its positive relationship to the American call premium.

Key-Words: - Binomial Option Pricing model, Black Scholes Model, American/European call option, currency option, dynamic programming.

1 Introduction

An *option* is a contract that gives one party the right, but not the obligation, to buy (for a *call option*) or sell (for a *put option*) a specific amount of an asset at a specified price during a specified period of time. Each option has a buyer (*holder*), and a seller (*writer*).

Currency option is an option to exchange two currencies. The holder of a currency option has the right, but not the obligation to exchange a fixed amount of one currency for another at a fixed rate of exchange on a date in future, with the currencies, amounts, rate and date are all predetermined.

There are two types of options, i.e. American and European–style option. American option is an option which may be exercised at any business day up to and including the expiry date (any time during the life of the option). Whereas European option is an option which may only be exercised at expiration (maturity).

2 Theory

2.1 The Black Scholes Model

Fischer Black and Myron Scholes derived an *option pricing formula* in 1973 to price European options on non dividend paying asset formed the foundation for much theory in derivatives finance.

The theoretical values for a call price c or put price p are:

$$c = sN(d_1) - xe^{-r(T-t)}N(d_2) \quad (1)$$

$$p = xe^{-r(T-t)}N(-d_2) - sN(-d_1) \quad (2)$$

where:

$$d_1 = \frac{\log(s/x) + (r + \sigma^2/2)(T-t)}{\sigma\sqrt{(T-t)}} \quad (3)$$

$$d_2 = d_1 - \sigma\sqrt{(T-t)} \quad (4)$$

1. s = current/underlying stock price.
2. x = strike price.
3. r = continuously compounded risk free interest rate.
4. $(T - t)$ = the time in years until the expiration of the option.
5. σ = the annual volatility for the underlying stock.
6. N = the standard normal cumulative distribution function.

2.2 The Binomial Option Pricing Model

Cox, Ross and Rubinstein (1979) constructed the *Binomial model* as a simple and intuitive discrete stock-market model, which converges to the BS setup. The Binomial model can be used to price both American and European options.

The Binomial model divides the time to an option's expiry into a large number of *intervals*, or steps. A *tree* of stock prices is initially produced working forward from the present to expiration.

The option value can be derived by *backward induction*, calculating the value of the option at each step.

By dynamic programming, we can easily handle a model with 100 or so steps, which gives a sufficient level of accuracy for calculating a theoretical fair value.

2.2.1 Binomial Model for American Option

Assumes that the price of the underlying can go up (u) or down (d) by fixed multiples: $u = e^{\sigma\sqrt{T-t}}$ (5)

and $d = e^{-\sigma\sqrt{T-t}} = \frac{1}{u}$ (6)

The probability of the stock price increasing at the next time periodis:

$$p = \frac{e^{(r-D)(T-t)} - d}{(u-d)} \quad (7)$$

where D is dividend.

We can now, via backwards induction, determine the price of an American call ($C_{i,j}$) or put ($P_{i,j}$):

$$C_{i,j} = Max\ \{Su^j d^{i-j} - X,\ e^{-r(T-t)}[pC_{i+1,j+1} + (1-p)C_{i+1,j}]\} \quad (8)$$

$$P_{i,j} = Max\ \{X - Su^j d^{i-j},\ e^{-r(T-t)}[pP_{i+1,j+1} + (1-p)P_{i+1,j}]\} \quad (9)$$

2.2.2 Binomial Model for European Option

For European call C and put P options, the binomial model is given as:

$$C = e^{-r\sqrt{T-t}}\ \Sigma \frac{n!}{i!(n-i)!} p^i (1-p)^{n-i} [Su^i d^{n-i} - X] \quad (10)$$

$$P = e^{-r\sqrt{T-t}}\ \Sigma \frac{n!}{i!(n-i)!} p^i (1-p)^{n-i} [X - Su^i d^{n-i}] \quad (11)$$

where i define as the number of up moves.

With a significant number of time nodes, the binomial method begins to converge, and the convergence of this value ultimately becomes the BS value.

3 Methodology

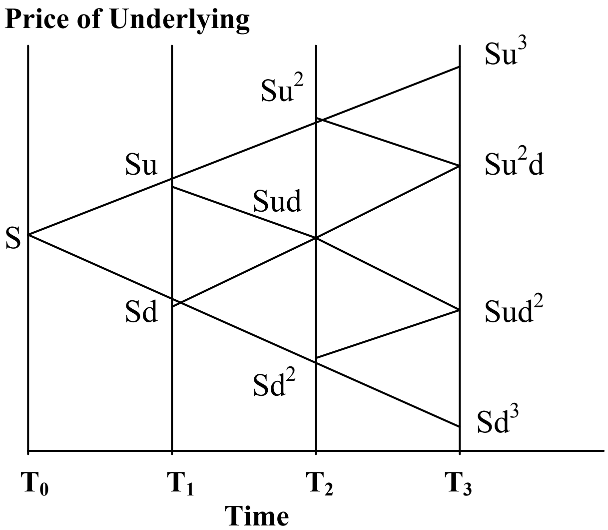
The data that used for this analysis:

1. *Option* = MYR put/GBP call
2. *Deliverable* = 1 units of GBP
3. *Strike exchange rate*, K: RM 6.6000/£
4. *Spot exchange rate*, S_0: RM 6.7678/£
5. *Domestic interest rate* (Malaysia), R_d = 2.773%
6. *Foreign interest rate* (United Kingdom), R_f = 3.83%
7. *Days to expiration*, τ = 60 days
8. *Volatility*: annual volatility of exchange rate (year 2003), $\sigma = 0.2$

S_0, R_d, R_f are inputted on 31.12.2003. The data of MYR/GBP are collected from 2.1.2003 to 31.12.2003 for the calculation of σ. This data are obtained from Central Bank of Malaysia and Central Bank of England. The type of option, amount of deliverable, K and τ can be predetermined.

3.1 The Binomial Model for American Currency Call Option

Figure 1: Binomial Model Lattice



The spot rate goes up and down by a multiplicative factor u and d where:

$$\text{Up Jump: } u = e^{(R_d - R_f)\frac{\tau}{N} + \sigma\sqrt{\frac{\tau}{N}}} \quad (12)$$

$$\text{Down Jump: } d = e^{(R_d - R_f)\frac{\tau}{N} - \sigma\sqrt{\frac{\tau}{N}}} \quad (13)$$

3.1.1 One-Period Model

The spot exchange rate on expiration day (t_1), would be either

Go up: $S_T = {}^1S_u = u\, S_0$ or (14)

Go down: $S_T = {}^1S_d = d\, S_0$ (15)

The value of the call at expiry is:

${}^1C_u = \text{Max}\ [0, {}^1S_u - K]$ (16)

${}^1C_d = \text{Max}\ [0, {}^1S_d - K]$ (17)

From the risk-neutral argument of Cox and Ross (1976), C_0 defined as:

$$C_0 = \text{Max}\ [S_0 - K, \frac{p\ {}^1C_u + (1-p)\ {}^1C_d}{e^{R_d \frac{\tau}{N}}}] \quad (18)$$

$$\text{where } p = \frac{e^{(R_d - R_f)\frac{\tau}{N}} - d}{(u-d)} \quad (19)$$

3.1.2 Two-Period Model

The spot exchange rate on expiration day (t_2), would be either

$S_T = {}^2S_{uu} = S_0\, u^2$ or (20)

$S_T = {}^2S_{du} = S_0 \cdot u \cdot d$ or (21)

$S_T = {}^2S_{dd} = S_0\, d^2$ (22)

The three expiration values are:

${}^2C_{uu} = \text{Max}\ [{}^2S_{uu} - K, 0]$ (23)

${}^2C_{ud} = \text{Max}\ [{}^2S_{du} - K, 0]$ (24)

${}^2C_{dd} = \text{Max}\ [{}^2S_{dd} - K, 0]$ (25)

$${}^1C_u = \text{Max}\ [{}^1S_u - K, \frac{p\ ({}^2C_{uu}) + (1-p)\ ({}^2C_{ud})}{e^{R_d \frac{\tau}{N}}}] \quad (26)$$

$${}^1C_d = \text{Max}\ [{}^1S_d - K, \frac{p\ ({}^2C_{ud}) + (1-p)\ ({}^2C_{dd})}{e^{R_d \frac{\tau}{N}}}] \quad (27)$$

$$C_0 = \text{Max}\ [S_0 - K, \frac{p\ {}^1C_u + (1-p)\ {}^1C_d}{e^{R_d \frac{\tau}{N}}}] \quad (28)$$

3.1.3 General Case

$$C_{u^j d^{N-j}} = Max\ \{u^j d^{N-j} S_0 - K, e^{-R_d \frac{\tau}{N}} [pC_{u^{j+1} d^{N-j}} + (1-p) C_{u^j d^{N-j+1}}]\} \quad (29)$$

3.2 The Binomial Model for European Currency Call Option

3.2.1 One-Period Model

$$C_0 = \frac{p\ {}^1C_u + (1-p)\ {}^1C_d}{e^{R_d \frac{\tau}{N}}} \quad (30)$$

3.2.2 Two-Period Model

$$ {}^{1}C_u = \frac{p\ ({}^{2}C_{uu}) + (1-p)\ ({}^{2}C_{ud})}{e^{R_d \frac{\tau}{N}}} \quad (31)$$

$$ {}^{1}C_d = \frac{p\ ({}^{2}C_{ud}) + (1-p)\ ({}^{2}C_{dd})}{e^{R_d \frac{\tau}{N}}} \quad (32)$$

$$ C_0 = \frac{p^2\ ({}^{2}C_{uu}) + 2p(1-p)\ ({}^{2}C_{ud}) + (1-p)^2\ ({}^{2}C_{dd})}{(e^{R_d \frac{\tau}{N}})^2} \quad (33)$$

3.2.3 *N*–Period Model

We defined:

1. N = number of steps to a point;
2. j = number of up moves to a point;
3. $N - j$ = number of down moves to a point.

$$ C_0 = (e^{-R_d \frac{\tau}{N}})^N \ \Sigma \frac{N!}{j!(N-j)!}\ p^j (1-p)^{N-j} C_{u^j d^{N-j}} \quad (34)$$

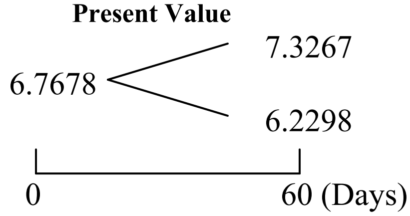
where $C_{u^j d^{N-j}} = Max\ [0, u^j d^{N-j} S_0 - K]$ (35)

4 Discussion

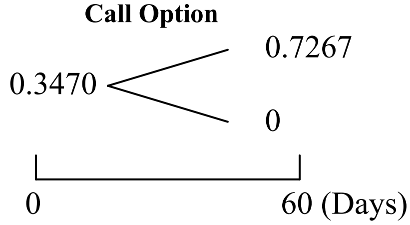
4.1 The Binomial Model for American Currency Call Option

4.1.1 One-Period Model

Present Value



Call Option



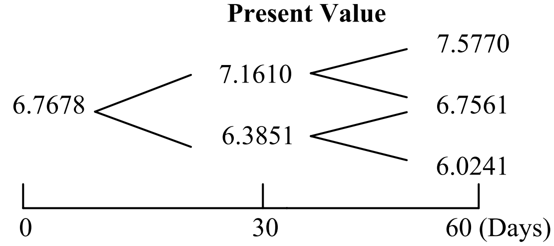
At the period 1, the spot rate will be either rises to RM7.3267/£ with probability 0.4797 or falls to RM6.2298/£ with probability 0.5203.

The call option gives the right, but not the obligation to buy 1 sterling 60 days from today for a price of RM6.6000/£. At time zero, we would be willing to pay the *premium* RM0.3470 to the seller. The premium is all the writer of an option receives for taking virtually unlimited risk. On the other hand, the premium is all the holder loss.

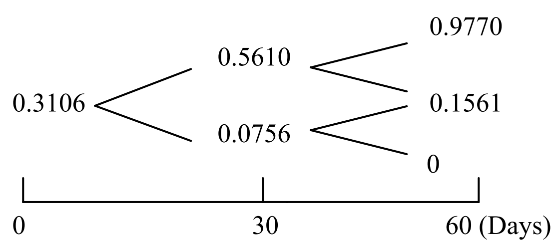
On expiration, for spot price less than or equal to RM6.6000/£, the value of the call is zero. The buyer of the call suffers a loss equal to RM0.3470 premium. If spot prices greater than RM6.6000/£, the expiration value is positive (RM0.7267). The *expiration profit* will be equal to the net difference between the option's terminal value and its current premium, i.e. RM0.3797 (0.7267 – 0.3470).

4.1.2 Two-Period Model

Present Value



Call Option



The price of the option is RM0.3106. If the spot rate is RM7.5770/£ at expiry, this reflect the fact that we can exercise the option at 60 days to purchase £1 for RM6.6000/£ and then immediately sell the £1 at the spot price RM7.5770/£ to create a profit of RM0.9770/£. The net benefit is equal to 0.9770 – 0.3106 = RM0.6664.

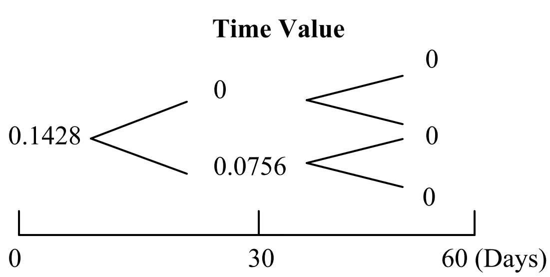
The same procedure is worked if the spot rate is RM6.7561/£ at expiry. However, its profit is less favourable. We only obtain RM0.1561 if the option is exercised. This price cannot balance the premium that we paid (RM0.3106). We will still exercise the option rather than lose the entire premium.

Meanwhile, if the spot rate at expiry is RM6.0241/£, we can purchase GBP at RM6.0241 in the market rather than RM6.6000/£ through the option. Therefore, if we decide not to exercise the option, then the option will expire worthless and we will have simply lost the premium of RM0.3106. The writer of the option will have gained the premium.

Prior to expiration, the option worth exploring a numbers of observed relationship between the *intrinsic value* (spot price – strike price) and *time value* (price paid – intrinsic value). The option will worth either RM0.5610 or RM0.0756 if we decide to exercise earlier. At 30 days, if the spot rate is RM7.1610/£, the option will worth RM0.5610. It is entirely intrinsic value. Because the option is *deep in-the-money* (sufficient high value of spot rate compare to strike rate), the time value tends towards zero.

If the spot price is RM6.3851/£, it is *out-of-the-money* options (strike rate RM6.6000/£ greater than spot rate), that value RM0.0756 is entirely time value.

Figure below show the decay of time value over time remaining to maturity.



It is a major concern that very few calls are ever exercised before the last few days of their life. This is true because when we exercise a call early, we forfeit the remaining time value on the call and just collect the intrinsic value. Towards the end of the life of a call, the remaining time value is very small. The time values become zero at maturity. At expiration, the option is solely intrinsic value.

4.2 The Binomial Model for European Currency Call Option

By the equation (30) and (33), we can calculate the European option value for one and two-period. These values are displayed in Table 1.

4.3 Comparison Between American and European Foreign Currency Options

Table 1: American and European Call Option Values

Period	American Call Option	European Call Option
One–Period	0.3470	0.3470
Two–Period	0.3106	0.3071

Both American and European option value are theoretically equal in one–period model. An American-style option provides an investor with a greater degree of flexibility than a European style option as American option can be exercised at any point before maturity date. Therefore, for number of jumps or periods more than two, the premium for an American option is higher than the premium for a European option.

4.4 Marginal Effect of a Parameter Change on Option Prices

We use LINGO to evaluate the America call option value. Conditional on the other parameters remaining constant, we assigned every parameter to change at 1%

(except *K* which changes at 1% and 2%). The results of five-period Binomial model are displayed in Table 2.

The value of call option rises typically as *S*, R_d, τ and σ rise. The call values fall when *K* and R_f rises. All the changes confirming their theoretical argument about the sign.

Among *S*, *K*, R_d, R_f, τ and σ, at 1% rise and 1% falls, *S* brings the most effect on the call option.

As the strike price getting higher, the difference between spot and strike price are decrease typically. Indeed, the call options are getting lower. Calls are worth more when the strike price is high.

It appear that the option premium is primarily affected by the *S*, *K*, τ and σ. Affecting the premium to a lesser degree are factors such as R_d and R_f (its percentage change are the smallest).

5 Conclusions

The American option tends to have a higher option premium (compare with the European option) due to its flexibility to exercise the option any time before expiration. In practice, early exercise on an American call option rarely optimized as the remaining time value on the call will be lost.

The effects of changes in the variables on the fair values of American-style call premium have been examined. The *S*, R_d, τ and σ have positive relationship to the premium. The factors of R_d and R_f less affect the option premium. The evidence from these tests provided more support for the theories.

References:

[1] Black, Fischer, and Myron Scholes, The Pricing of Options and Corporate Liabilities, *Journal of Political Economy*, Vol. 81, 1973, pp. 637–654.

[2] Bodurtha, James N., and Georges R. Courtadon, Tests of the American Option Pricing Model on the Foreign Currency Options Market, *Journal of Financial and Quantitative Analysis*, Vol. 22, 1987, pp. 153–168.

[3] Cox, John C., Stephen A. Ross, and Mark Rubinstein, Option Pricing: A Simplified Approach. *Journal of Financial Economics*, Vol. 7, 1979, pp. 229–263.

Table 2: Marginal Effect of a Parameter Change on American Call Option Prices

Effects	**S**	**K**	R_d	R_f	τ	σ	**N**	C_0	**Changes**
Original	6.7678	6.6000	2.7730	3.8300	60	0.200	5	0.3049811	–
Changes of *S*	6.8355 (↑)	6.6000	2.7730	3.8300	60	0.200	5	0.3404784	↑11.6%
	6.7001 (↓)	6.6000	2.7730	3.8300	60	0.200	5	0.2701615	↓11.4%
Changes of *K*	6.7678	6.7320 (↑)	2.7730	3.8300	60	0.200	5	0.2414642	↓20.8%
	6.7678	6.6660 (↑)	2.7730	3.8300	60	0.200	5	0.2732227	↓10.4%
	6.7678	6.5340 (↓)	2.7730	3.8300	60	0.200	5	0.3374308	↑10.6%
	6.7678	6.4680 (↓)	2.7730	3.8300	60	0.200	5	0.3863462	↑26.7%
Changes of R_d	6.7678	6.6000	2.8007 (↑)	3.8300	60	0.200	5	0.3051007	↑0.04%
	6.7678	6.6000	2.7453 (↓)	3.8300	60	0.200	5	0.3048615	↓0.04%
Changes of R_f	6.7678	6.6000	2.7730	3.8683 (↑)	60	0.200	5	0.3048009	↓0.06%
	6.7678	6.6000	2.7730	3.7917 (↓)	60	0.200	5	0.3051614	↑0.06%
Changes of τ	6.7678	6.6000	2.7730	3.8300	60.6 (↑)	0.200	5	0.3060481	↑0.35%
	6.7678	6.6000	2.7730	3.8300	59.4 (↓)	0.200	5	0.3039084	↓0.35%
Changes of σ	6.7678	6.6000	2.7730	3.8300	60	0.202 (↑)	5	0.3072419	↑0.74%
	6.7678	6.6000	2.7730	3.8300	60	0.198 (↓)	5	0.3027204	↓0.74%