THE ONSET OF MARANGONI CONVECTION IN A NON-LINEAR MAGNETIC FLUID WITH A DEFORMABLE FREE SURFACE

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Abstract: - Linear stability theory is applied to the Marangoni convection in a horizontal layer of an incompressible viscous fluid heated from below in the presence of a uniform vertical magnetic field when the fluid is bounded above by a deformable free surface and below by a solid plate. A non-linear relationship between the magnetic field and the magnetic induction suggested by Roberts [13] is applied.

The non-linear relationship has no effect on the development of instabilities through the mechanism of stationary convection but influences the onset of overstable convection and new results were produced depending on this non-linear relation. The numerical results were obtained using the method of expansion of Chebyshev polynomials.

Key-Words: - Marangoni convection - deformable surface - magnetic fluid - Chebyshev method.

1 Introduction

Linear instability theory has attracted considerable interest and has been recognized as a problem of fundamental importance in many areas of fluid dynamics. The phenomena observed by Benard [1,2] when a horizontal layer of incompressible viscous fluid heated from below and contained by two parallel plates have been explained by Rayleigh [3] who considered instability due to the action of buoyancy forces. Rayleigh provided a theoretical explanation for Benard's experimental results, and showed that the numerical value of the non-dimensional parameter R, now commonly called the Rayleigh number, decides whether a layer of fluid is stable or not. Later workers including Jeffreys [4,5], Low [6] and Pellow & Southwell [7] have extended and refined Rayleigh's analysis. Thermal instability theory has been enlarged by the interest in hydrodynamic flows of an electrically conducting fluids in the presence of a magnetic field. Thompson [8], Chandrasekhar [9,10], Nakagawa [11,12] and others have examined Benard problem in the presence of a magnetic field when the relation between the magnetic field H_i and the magnetic induction B_i is linear.

Roberts [13] and Muzikar & Pethick [14] have used a non-linear relation between B_i and H_i to model convection in a neutron star. This non-linear relation have been used by Abdullah & Lindsay [15,16] to discuss the Benard convection in the presence of a vertical and non-vertical magnetic field, and they showed that this non-linear relationship has no effect on the development of instabilities through the mechanism of stationary convection but influences the onset of overstable convection. Abdullah [17,18] used this non-linear relationship to discuss the magnetic Benard problem in the presence of Coriolis forces for a vertical and non-vertical magnetic field respectively. Jan & Abdullah [19] and Abdullah [20] discussed the Benard convection in the presence of porous medium for a linear and non-linear magnetic fluid respectively. Thermosolutal convection in a linear and non-linear magnetic fluid have been discussed by Al-Aidrous & Abdullah [21] and Abdullah [22] respectively.

The earliest work on thermocapillary instability of a fluid layer heated from below with a non-deformable free surface was performed by Pearson [23]. Pearson discussed Benard's experimental results and showed that instability arises due to surface tension rather than buoyancy, and that the non-dimensional parameter M, now commonly called the Marangoni number, must attain a certain minimum critical value for instability to occur. Surface tension driven instability in a horizontal liquid layer with a deformable free surface was examined by Scriven & Sternling [24], Smith [25] and Takashima [26].

All the previous work of thermocapillary instability were limited to the case of stationary convection and the first author to investigate the possibility of an instability setting in as overstability was Takashima [27]. He demonstrated that such an instability was only possible when the free surface was deformable and even then only for certain negative Marangoni numbers, corresponding to the situation in which the temperature of the solid wall below is lower than that of the undisturbed free surface above.

The effect of a uniform vertical magnetic field on the surface tension and buoyancy convection with a deformable free surface was demonstrated by Sarma [28]. The joint effects of magnetic field and rotation was investigated by Sarma [29] on onset of stationary instability when the free surface is deformable. Wilson [30] mentioned that Sarma [28,29] used an incorrect normal stress boundary condition in his analysis and so his results in the case of a non-zero magnetic field and a deformable free surface are incorrect. Wilson [30] investigated the onset of Marangoni convection in the presence of uniform magnetic field. He showed that the presence of the magnetic field has a stabilizing effect on the flow but that certain wave numbers will always remain unstable to steady convection however strong the magnetic field is.

The aim of this paper is to investigate the effect of a non-linear relationship between the magnetic induction and the magnetic field, suggested by Roberts [13], on Marangoni problem when the upper free surface is deformable. Both steady and overstable modes of instability are considered. The numerical computations are performed using the method of expansions of Chebyshev polynomials.

2 Mathematical Formulation

Consider a horizontal thickness three dimensional planar layer of an incompressible, quiescent, thermally and electrically conducting viscous fluid with a deformable free upper surface and solid lower surface. The fluid is subjected to a constant gravitational acceleration in the negative x_3 direction. A constant magnetic field is imposed across the layer in the positive x_3 direction and x_1 and x_2 axes are selected from a right-handed system of Cartesian coordinates in which the magnetic induction has representation $B_i = B_o\,\delta_{i3}$. The geometry of the basic state is shown in Figure 1.

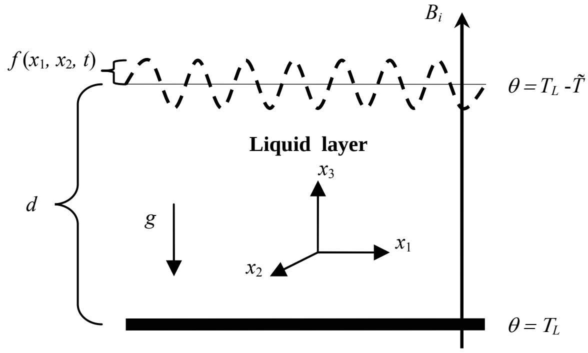

Fig. 1

In order of fully describe the nature of this model we need to discuss the interaction between electromagnetic and mechanical effects. The most general, properly invariant, constitutive relationship between the magnetic field and the magnetic induction has the form

$$H_i = \rho\ \varphi(\rho, B_o)\ B_i\ , \qquad (1)$$

where ρ is the density of fluid, $B_o = \sqrt{(B_i \cdot B_i)}$ is the magnitude of the magnetic induction and φ is the magnetic susceptibility which is related to the partial derivative of the magnetic free energy with respect to B_o (See Roberts [13]). The conventional magnetic permeability, μ, is given by

$$\mu(\rho, B_o) = (\rho\varphi)^{-1}.$$

The strength of the nonlinear permeability is measured in terms of the nondimensional magnetic number σ where

$$\sigma = -\frac{B_o}{\mu(\rho\ , B_o)}\frac{\partial\mu}{\partial B_o}\ .$$

Conventional ideas would indicate that the permeability is a decreasing function of B_o and so $\sigma \geq 0$. The magnetic variables are required to satisfy the Maxwell's equations

$$\begin{aligned} div\ \mathbf{B} &= B_{i,i} = 0\ , \\ curl\ \mathbf{H} &= e_{ijk}\ H_{k,j} = J_i\ , \\ curl\ \mathbf{E} &= e_{ijk} E_{k,j} = -\frac{\partial B_i}{\partial t}, \end{aligned} \qquad (2)$$

where V_i is the velocity, E_i is the electric field, and where the displacement current has been neglected as is customary in this type of problems. The current density J_i is given by

$$\eta J_i = E_i + e_{ijk} V_j B_k , \qquad (3)$$

where η is the magnetic resistivity which is constant. The governing field equations become

$$\frac{DV_i}{Dt} = -(P/\rho_o)_{,i} + \nu \nabla^2 V_i + B_k (\varphi B_i)_{,k} ,$$
$$V_{i,i} = 0, \qquad (4)$$
$$\frac{D\theta}{Dt} = \kappa \nabla^2 \theta,$$

where

$P = p + (\frac{\rho_o}{2})(\varphi B_o^2 + \int \varphi_{,B_k} B_k^2 \, dB_k)$ is the modified pressure, $D(\ )/Dt$ is the convected derivative, ν is the kinematic viscosity, θ is the absolute temperature and κ is the coefficient of thermal diffusivity.

We may observe that equations (2) and equations (4) have a steady state solution in which

$$V_i = 0, \ \theta = T_L - \beta x_3 , \ P = P(x_3),$$
$$B_i = B_o \delta_{i3} , \text{ and, } \ \varphi = \varphi(B_o), \qquad (5)$$

where the adverse temperature gradient $\beta = \tilde{T}/d$ and T_L , (T_L-$\tilde{T}$) are the temperatures on the planes $x_3 = 0$ and $x_3 = d$ respectively.

Suppose that the basic variables are perturbed about their equilibrium values described in equations (5), then we may verify that the linearised version of equations (4) are

$$\frac{\partial \hat{V}_i}{\partial t} = -(\frac{\hat{P}}{\rho_o})_{,i} + \nu \nabla^2 \hat{V}_i + \varphi_{,B_o} B_o^2 \hat{B}_{3,3} \delta_{i3} +$$
$$\varphi B_o \hat{B}_{i,3} ,$$
$$\hat{V}_{i,i} = 0 , \qquad (6)$$
$$\frac{\partial \hat{\theta}}{\partial t} = \kappa \nabla^2 \hat{\theta} + \beta \hat{V}_i ,$$

On taking the (*curl*) of the constitutive law (3) and replacing the electric field by the Maxwell's relation $(2)_3$, the magnetic induction is now readily seen to satisfy the partial differential equation

$$\frac{\partial B_i}{\partial t} = V_{i,j} B_j - V_j B_{i,j} - \eta e_{ijk} J_{k,j} . \qquad (7)$$

By using equation (1), the linearised version of Maxwell's relations $(2)_{1,2}$ and (7) are

$$\hat{B}_{i,i} = 0 ,$$
$$\hat{J}_i = \rho e_{ijk} (\varphi \hat{B}_k + \varphi_{,B_o} B_o \hat{B}_3 \delta_{k3})_{,j} , \qquad (8)$$
$$\frac{\partial \hat{B}_i}{\partial t} = B_o \hat{V}_{i,3} - \eta e_{ijk} \hat{J}_{k,j} .$$

From equations $(8)_{1,2,3}$ we obtain

$$\frac{\partial \hat{B}_i}{\partial t} = B_o \hat{V}_{i,3} - \eta \rho [\varphi_{,B_o} B_o (\hat{B}_{3,i3} - \hat{B}_{3,jj} \delta_{i3}) - \varphi \hat{B}_{i,jj}], \quad (9)$$

thus the governing field equations are (6) and (9). As is the case in many convection problems, vector components parallel to the direction of gravity play a central role and so it is convenient to introduce the variables

$$x_i = \hat{x}_i , \ w = \hat{V}_3 , \ \theta = \hat{\theta}, \text{ and, } b = \hat{B}_3 .$$

Thus the third component of the governing equations become

$$\nabla^2 \frac{\partial w}{\partial t} = \nu \nabla^4 w + B_o [\varphi \nabla^2 b + \varphi_{,B_o} B_o (b_{,11} + b_{,22})]_{,3} ,$$
$$\frac{\partial \theta}{\partial t} = \kappa \nabla^2 \theta + \beta w , \qquad (10)$$
$$\frac{\partial b}{\partial t} = B_o w_{,3} + \eta \rho [\varphi \nabla^2 b + \varphi_{,B_o} B_o (b_{,11} + b_{,22})] .$$

To simplify the analysis we introduce non-dimensional variables. Taking the thickness of fluid layer d as the unit of length. Appropriate scales for the time, velocity, temperature gradient and magnetic induction are d^2/κ , κ/d , βd and B_o respectively. Non-dimensionalizing, the governing equations (10) become

$$\frac{1}{P_r} \nabla^2 \frac{\partial w}{\partial t} = \nabla^4 w + P_m Q [\nabla^2 b + \sigma (b_{,11} + b_{,22})]_{,3} ,$$
$$\frac{\partial \theta}{\partial t} = \nabla^2 \theta + w , \qquad (11)$$
$$\frac{1}{P_m} \frac{\partial b}{\partial t} = \frac{1}{P_m} w_{,3} + [\nabla^2 b + \sigma (b_{,11} + b_{,22})] ,$$

where $p_r = \dfrac{\nu}{\kappa}$ is the mechanical Prandtl number**,** $P_m = \dfrac{\rho \eta \varphi}{\kappa}$ is the magnetic Prandtl number and $Q = \dfrac{B_o^2 d^2}{\rho \nu \eta}$ is the Chandrasekhar number.

Equations (11) will be solved subject to the boundary conditions appropriate to a rigid, thermally conducting lower boundary and an upper interface with deformable surface tension and a general heat radiation condition. At the lower boundary we have

$$V_3 = 0, \; V_{3,3} = 0, \; \text{and}, \; \theta = 0, \tag{12}$$

while at the upper boundary we have

$$\begin{aligned} &V_3 = \frac{\partial f}{\partial t}, \\ &k\theta_{,3} + q\theta - q\beta f = 0, \\ &\rho_o \nu V_{3,33} - \gamma(\theta_{,11} + \theta_{,22}) - \rho_o \nu \nabla_H^2 V_3 + \\ &\qquad \gamma\beta \nabla_H^2 f = 0, \\ &\rho_o[(3\nu\nabla_H^2 - \frac{\partial}{\partial t}) V_{3,3} + \nu V_{3,333}] + (\rho_o g - \\ &\qquad S\nabla_H^2)\nabla_H^2 f + \frac{B_o}{\eta}(\frac{\partial B_3}{\partial t} - B_o V_{3,3}) = 0, \end{aligned} \tag{13}$$

where $\nabla_H^2(\,) = (\,)_{,11} + (\,)_{,22}$, f is the deflection of the free surface, k is the coefficient of heat conduction, q is the rate of change with temperature of the time rate of heat loss per unit area from the upper surface to its upper environment, $-\gamma$ is the rate of change of surface tension with temperature which generally positive, S is the surface tension of the free surface and g is the acceleration due to gravity .

We now consider the electromagnetic conditions. On a perfectly insulating electromagnetic boundary, no currents can flow to the exterior region and the magnetic field is continues across the boundary with the external magnetic field being derived from a scalar potential. We shall associate insulating boundaries with a deformable free surface, i.e. $B_{3,3} + a\,B_3 = 0$ on $x_3 = d$ where a is the dimensionless wave number. On the other hand if the adjoining boundary is a stationary perfect conductor then the normal component of the unsteady magnetic field must be zero and there can be no surface components of electric field. A stationary perfect conducting surface will be associated with rigid boundaries, i.e. $B_{3,3} - aB_3 = 0$ on $x_3 = 0$.

We notice that in the undisturbed state the free surface is located at $x_3 = d$. When motion occurs the free surface will be deformed and then we denote its position by $x_3 = d + f(x_1, x_2, t)$. After perturbation state, the third component of the boundary conditions at free and rigid surfaces become

$$w=0, \; w_{,3}=0, \; \theta=0, \; \text{and}, \; b_{,3} - ab = 0, \tag{14}$$

on $x_3 = 0$, and

$$\begin{aligned} &\frac{\partial f}{\partial t} - w = 0, \\ &k\theta_{,3} + q\theta - q\beta f = 0, \\ &\rho_o \nu w_{,33} - \gamma(\theta_{,11} + \theta_{,22}) - \rho_o \nu \nabla_H^2 w + \\ &\qquad \gamma\beta \nabla_H^2 f = 0, \\ &\rho_o[(3\nu\nabla_H^2 - \frac{\partial}{\partial t} - \frac{B_o^2}{\eta\rho_o}) w_{,3} + \nu w_{,333}] + \\ &\qquad (\rho_o g - S\nabla_H^2)\nabla_H^2 f + \frac{B_o}{\eta}\frac{\partial b}{\partial t} = 0, \\ &b_{,3} + ab = 0, \end{aligned} \tag{15}$$

on $x_3 = d$.

After non-dimentionalization, the boundary conditions (14) and (15) simplify to

$$w=0, \; w_{,3}=0, \; \theta=0, \text{and}, \; b_{,3} - ab = 0, \tag{16}$$

on $x_3 = 0$, and

$$\begin{aligned} &\frac{\partial f}{\partial t} - w = 0, \\ &\theta_{,3} + N_u(\theta - f) = 0, \\ &w_{,33} - M(\theta_{,11} + \theta_{,22}) - \nabla_H^2 w + M\nabla_H^2 f = 0, \\ &C_r[(3\nabla_H^2 - \frac{1}{P_r}\frac{\partial}{\partial t} - Q) w_{,3} + w_{,333}] + (B_{ond} - \\ &\qquad \nabla_H^2)\nabla_H^2 f + C_r Q \frac{\partial b}{\partial t} = 0, \\ &b_{,3} + ab = 0, \end{aligned} \tag{17}$$

on $x_3 = d$ where $M = \dfrac{\gamma\beta d^2}{\rho_o \nu \kappa}$ is the Marangoni number, $N_u = \dfrac{q\,d}{k}$ is the Nusselt (or Biot) number, $B_{ond} = \dfrac{\rho_o\, g\, d^2}{S}$ is the Bond number and $C_r = \dfrac{\rho_o \nu\kappa}{Sd}$ is the Crispation (or Capillary) number.

We aim to investigate the linear stability of the steady solution (5) and with this aim in mind we construct the related eigenvalue problem from equations (11) and the boundary conditions (16) and (17). We now look for a solution of the form

$$\Phi = \Phi(x_3)\exp[\,i\,(a_{x_1} x_1 + a_{x_2} x_2) + \lambda t\,],$$

where $\lambda = \lambda_R + i\,\lambda_{im}$ is the growth rate of the disturbance with time which in general complex and λ_R

and λ_{im} are the real and the imaginary parts of λ thus equations (11) become

$$\begin{aligned} \frac{\lambda}{P_r} \mathrm{L}\, w &= \mathrm{L}^2 w + P_m Q(\mathrm{L}-\sigma a^2)(\mathrm{D}b), \\ \lambda\,\theta &= \mathrm{L}\,\theta + w, \\ \frac{\lambda}{P_m} b &= \frac{1}{P_m} \mathrm{D}w + (\mathrm{L}-\sigma a^2)\, b, \end{aligned} \tag{18}$$

and the boundary conditions (16) and (17) become

$$w=0,\ \mathrm{D}w=0,\ \theta=0, \text{and}, \mathrm{D}b-a\,b=0, \tag{19}$$

on $x_3 = 0$, and

$$\begin{aligned} &\lambda f - w = 0, \\ &\mathrm{D}\theta + N_u(\theta - f) = 0, \\ &(\mathrm{D}^2 + a^2)\ w + a^2 M(\theta - f) = 0, \\ &C_r[((\lambda/P_r) + 3a^2 + Q - \mathrm{D}^2)\,\mathrm{D}w - \lambda\, Q\, b] + \\ &a^2(B_{ond} + a^2)\ f = 0, \\ &\mathrm{D}b + a\,b = 0, \end{aligned} \tag{20}$$

on $x_3 = d$ where $\mathrm{L}(\ ) = (\mathrm{D}^2 - a^2)(\ )$, $\mathrm{D}(\ ) = \mathrm{d}(\ )/\mathrm{d}x_3$ and $a = (a_{x_1}^2 + a_{x_2}^2)^{1/2}$.

3 Results and Discussion

3,1 Stationary convection

When instability appears in the form of stationary convection we put $\lambda = 0$ in equations (18), and if b is eliminated from $(18)_1$ using $(18)_3$ then we obtain

$$(\mathrm{L}^2 - Q\,\mathrm{D}^2)\ w = 0,$$
$$\mathrm{L}\,\theta = -w,$$

which are the same equations obtained by Wilson [30] and the numerical results obtained for this case coincide with those results obtained by him. We notice that the non-linear parameter σ has disappeared from these equations which means that the non-linear relationship between B_i and H_i has no effect in the development of instabilities through the mechanism of stationary convection.

3,2 Overstable convection

The method of expansion of Chebyshev polynomials will be used to solve numerically the eigenvalue problem (18) subject to the boundary conditions (19) and (20) for both stationary and overstability cases. Let $\phi = \mathrm{L}\,w$ then equations (18) become

$$\begin{aligned} 0 &= \mathrm{L}\,w - \phi, \\ \lambda\,\theta &= w + \mathrm{L}\,\theta, \\ \lambda\,((1/P_r)\phi - Q\,\mathrm{D}b) &= -Q\,\mathrm{D}^2 w + \mathrm{L}\,\phi, \\ (\lambda/P_m) b &= (1/P_m)\,\mathrm{D}w + (\mathrm{L} - \sigma a^2)\, b, \end{aligned} \tag{21}$$

since the eigenvalue problem (21) is an eighth order and we have nine boundary conditions in (19) and (20) then we can use equation $(20)_3$ to eliminate the free surface deflection condition. Substituting from free surface deflection in the upper boundary conditions $(19)_{1,2,4}$ these conditions become

$$\begin{aligned} &\lambda\,(2a^2 w + a^2 M\,\theta + \phi) = a^2 M\, w, \\ &0 = -2a^2 N_u\, w + a^2 M\,\mathrm{D}\theta - N_u\,\phi, \\ &\lambda C_r M[(1/P_r)\mathrm{D}w - Q\,b] = -[C_r M(2a^2 + Q)\mathrm{D} + \\ &2a^2(B_{ond} + a^2)]w - a^2 M(B_{ond} + a^2)\,\theta + \\ &[C_r M\,\mathrm{D} - (B_{ond} + a^2)]\,\phi. \end{aligned} \tag{22}$$

The Chebyshev expansion of the basic variables are

$$(w,\theta,\phi,b)(y) = \sum_{m=0}^{\infty} (a^*_{m+1}, b^*_{m+1}, c^*_{m+1}, d^*_{m+1})\ T_m(y)$$

where $y = 2x_3 - 1$. Substitute for the basic variables into equations (21), we obtain an eigenvalue of the form $\mathbf{AX} = \lambda\ \mathbf{UX}$, where

$$\mathbf{A} = \begin{bmatrix} \mathbf{V} & \mathbf{0} & -\mathbf{I} & \mathbf{0} \\ \mathbf{I} & \mathbf{V} & \mathbf{0} & \mathbf{0} \\ -4Q\,\mathbf{C}^2 & \mathbf{0} & \mathbf{V} & \mathbf{0} \\ (2/P_m)\mathbf{C} & \mathbf{0} & \mathbf{0} & \mathbf{V} - \sigma a^2\,\mathbf{I} \end{bmatrix},$$

$$\mathbf{X} = \begin{bmatrix} \boldsymbol{a}^*_{m+1} \\ \boldsymbol{b}^*_{m+1} \\ \boldsymbol{c}^*_{m+1} \\ \boldsymbol{d}^*_{m+1} \end{bmatrix},\ \mathbf{U} = \begin{bmatrix} \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{0} \\ \mathbf{0} & \mathbf{I} & \mathbf{0} & \mathbf{0} \\ \mathbf{0} & \mathbf{0} & (1/P_r)\mathbf{I} & -2\,Q\,\mathbf{C} \\ \mathbf{0} & \mathbf{0} & \mathbf{0} & (1/P_m)\mathbf{I} \end{bmatrix},$$

$\mathrm{V}(\ ) = (4\mathrm{C}^2 - a^2)(\ )$ and $\mathrm{C} = \mathrm{d}(\ )/\mathrm{d}y$. And where $\mathbf{I}$ is the identity matrix. This system is solved numerically using the Nag routine (F02BJF) .

We now examine the overstable case, and that corresponds to the case when $Im(\lambda) = 0$, when the free surface is deformable, i.e. $C_r \neq 0$. Since the marginal stability curves for overstable convection are more complicated than those of steady convection then a great reliance has to be placed on numerical calculations.

In obtaining our results we first calculate the Marangoni number for various assigned values of the wave number, a , for specific values of P_r , P_m , N_u , B_{ond} , C_r , Q and σ. Then we make use of those results to calculate the critical Marangoni numbers M_c When $Q = 0$, σ has no effect.

Figures 2(a) and 2(b) show the marginal stability curves and the corresponding values of λ_{im} plotted as functions of the wave number, a, in the case when $C_r = P_r = P_m = 1$ and $B_{ond} = N_u = 0$. The marginal stability curves form infinitely many loops, two of them are shown in the figures, which lie entirely in the negative M–domain and extend to $M \to -\infty$ as $a \to 0$. Since the region inside the loops correspond to unstable disturbances we can identity the critical maximum value of $M \approx 1012.716$ at $a \approx 0.2821$ and $\lambda_{im} \approx 6.570$.

Increasing Q from zero show the effect of the non-linear parameter, σ. In fact when $Q = 1$, $\sigma \neq 0$ the value of the Marangoni number decreases which means that the non-linear relation between B_i and H_i has a stabilizing effect. This effect is prominent as the value of σ increases. The critical maximum values of M for σ = 0, 2, 4, 6, 8 and 10 are listed in table 1.

The marginal stability curve when Q = 10, $C_r = 0.0001$, $P_r = P_m = 1$ and $B_{ond} = N_u = 0$. is just a single closed loop with finite maximum and minimum values. The critical maximum and minimum values of M for σ = 0, 2, 4, 6, 8 and 10 are listed in table 2.

Figures 3(a) and 3(b) show the complete numerically calculated, marginal stability curves and the corresponding values of λ_{im} plotted as functions of a in case $Q = 100$, $C_r = 0.0001$, $P_r = P_m = 1$ and $B_{ond} = N_u = 0$ for σ = 0, 6 and 10. Here the marginal stability curve form two distinct closed loops each with a finite maximum and minimum values and so there are two ranges of values of M in which unstable disturbances exist.. In each case of σ we can identity the critical maximum and minimum of the marginal stability curves. These results are listed in table 3.

Increasing Q further so that Q =1000 and $C_r = 0.0001$, the marginal stability curves disappear altogether and all disturbances are stable for any value of the non-linear parameter σ.

4 Conclusion

The effect of a non-linear relation, suggested by Roberts [13], between the magnetic field and the magnetic induction is investigated when the upper free surface is a deformable and the lower surface is a solid plate. This non-linearity has no effect on the development of instabilities through stationary convection but effect the development of instabilities through oscillatory convection. Moreover it appears that the nature of the neutral state in this problem is of a stationary and oscillatory pattern. The numerical results were obtained using the method of expansion of Chebyshev polynomials. The results obtained show more accurate values over a large range of the magnetic parameter and extend similar results obtained by Wilson [30].

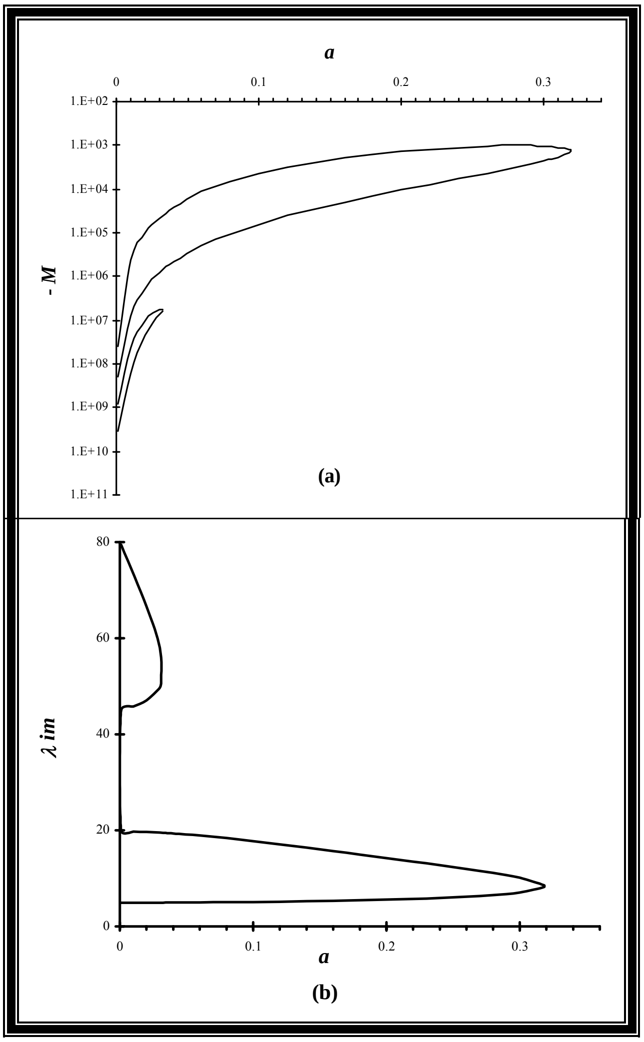

Fig. 2: Typical marginal stability curves for overstable convection and the corresponding value λ_{im} for any value of σ when $Q = 0$, $C_r = P_r = P_m = 1$ and $B_{ond} = N_u = 0$.

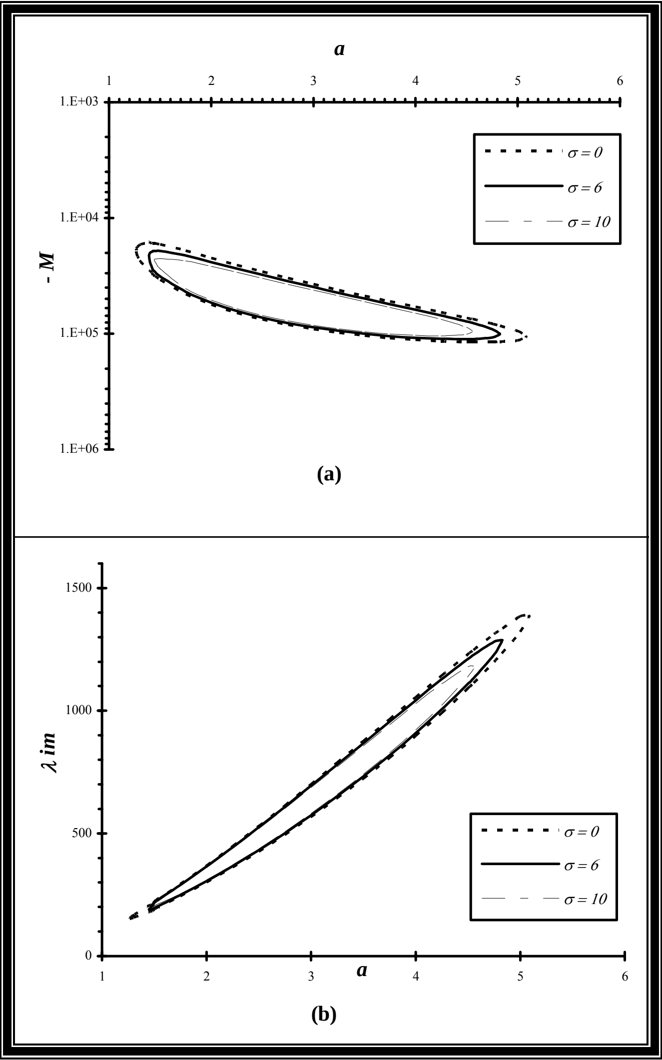

Fig. 3: Typical marginal stability curves for overstable convection and the corresponding value λ_{im} for any value of σ when Q = 100, C_r= 0.0001, $P_r = P_m$ = 1 and $B_{ond} = N_u$ = 0.

σ =	$M_c \approx$	$a \approx$	$\lambda_{im} \approx$
0	-1787.342	0.2374	7.300
2	-1805.057	0.2367	7.318
4	-1825.025	0.2359	7.336
6	-1847.094	0.2349	7.354
8	-1871.107	0.2338	7.371
10	-1896.905	0.2320	7.387

Table 1: The critical maximum values of M and the corresponding values of $\lambda_{im}$for different values of σ when Q =1, $C_r = P_r = P_m$ = 1 and $B_{ond} = N_u$ = 0.

σ =	$M_c \approx$		$a \approx$	$\lambda_{im} \approx$
0	*Max*	-1987.733	0.456	22.4835
	Min	-150450.097	5.05292	1526.9388
2	*Max*	-2025.741	0.457	22.7008
	Min	-150311.398	5.05291	1526.5410
4	*Max*	-2067.625	0.459	22.9274
	Min	-150137.470	5.05293	1526.0293
6	*Max*	-2113.376	0.460	23.1628
	Min	-149929.582	5.5297	1525.4019
8	*Max*	-2162.951	0.461	23.4068
	Min	-149689.448	5.05293	1524.6336
10	*Max*	-2216.307	0.462	23.6593
	Min	-149418.675	5.05291	1523.759

Table 2: The critical maximum and minimum values of M and the corresponding values of λ_{im} for different values of σ when Q = 10, C_r = 0.0001, $P_r = P_m$ = 1 and $B_{ond} = N_u$ = 0.

σ =	$M_{c\,(big\ loop)} \approx$		$a \approx$	$\lambda_{im} \approx$
0	*Max*	-16455.716	1.3836	169.19241
	Min	-117442.718	4.5801	1254.7311
2	*Max*	-17123.875	1.4041	174.6240
	Min	-115757.961	4.5362	1235.6978
4	*Max*	-18030.532	1.4328	181.9342
	Min	-113598.223	4.4785	1210.7211
6	*Max*	-19213.237	1.4709	191.4690
	Min	-110934.897	4.4236	1185.1845
8	*Max*	-20727.799	1.5206	203.8207
	Min	-107673.866	4.3764	1160.393
10	*Max*	-22657.858	1.5857	219.9828
	Min	-103981.795	4.2148	1098.1191
σ =	$M_{c\,(small\ loop)} \approx$		$a \approx$	$\lambda_{im} \approx$
0	*Max*	-4746.330	0.6065	49.7729
	Min	-11025.177	0.5010	49.7146
2	*Max*	-5109.800	0.6063	49.0493
	Min	-11410.922	0.5010	50.5710
4	*Max*	-5505.036	0.6062	49.8023
	Min	-11830.493	0.5010	51.4568
6	*Max*	-5936.017	0.6061	50.5848
	Min	-12285.809	0.5010	52.3721
8	*Max*	-6404.848	0.6060	51.3969
	Min	-12778.975	0.5010	53.3170
10	*Max*	-6913.905	0.6058	52.2392
	Min	-13312.347	0.5010	54.2921

Table 3: The critical maximum and minimum values of M and the corresponding values of λ_{im} for different values of σ when Q = 100, C_r = 0.0001, $P_r = P_m$ = 1 and $B_{ond} = N_u$ = 0.

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