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Multi-Rate Methods for Simulating Electronic Circuits Driven by Envelope-Modulated Signals

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Abstract: - In electronics many kinds of circuits with widely separated rates of variation are frequently found. Such situation usually takes a considerable dimension in telecommunications, like radio frequency or microwave applications. These are typical cases where multi-rate behavior may be found, due for example to envelope modulated excitations. In this paper some efficient methods for simulating circuits excited by this type of stimulus are presented and tested in an illustrative example.

Key-Words: - Electronic circuit simulation, envelope-modulated signals, separated time scales.

1 Introduction

Dynamical behavior of electronic circuits can in general be described by ordinary differential equations (ODEs) in time, involving electric voltages, currents and charges and magnetic fluxes. For a general nonlinear circuit, Kirchhoff's laws lead to the system

$$p(y(t)) + \frac{dq(y(t))}{dt} = x(t), \qquad (1)$$

where $x(t) \in \mathbb{R}^n$ and $y(t) \in \mathbb{R}^n$ stand for the excitation and state-variable vectors, respectively. $p(y(t))$ represents memoryless linear or nonlinear elements, while $q(y(t))$ models dynamical linear or nonlinear elements (capacitors or inductors).

When the excitation $x(t)$ is a multi-rate stimulus the simulation process of such circuits is often a very challenging issue, especially if they are highly nonlinear. In the particular case in which we are interested (envelope modulated excited circuits) it happens that while envelopes are slowly varying signals, carriers are usually very high frequency sinusoids. Thus, obtaining the numerical solution of (1) is difficult because once we have signals with widely separated rates of variation we are forced to make discretizations on long time grids with extremely small time steps.

2 Multidimensional Problem

In this section we will begin by presenting a recent generic formulation for solving electronic circuits with widely separated time scales. As we can see for example in [2] or [6], the key idea behind this strategy is to use multiple time variables, which enable multi-rate signals to be represented more efficiently. In our case, we need two time variables and we will adopt the following procedure: for the slowly-varying parts (envelope) of the expressions of $x(t)$ and $y(t)$, t is replaced by t_1; for the fast-varying parts (carrier) t is replaced by t_2. This results in bivariate representations for the excitation and the solution, and we will denote these two argument functions by $\hat{x}(t_1, t_2)$ and $\hat{y}(t_1, t_2)$.

This way the ordinary differential (ODE) system (1) can be converted to the multi-rate partial differential (MPDE) system

$$p(\hat{y}(t_1, t_2)) + \frac{\partial q(\hat{y}(t_1, t_2))}{\partial t_1} + \frac{\partial q(\hat{y}(t_1, t_2))}{\partial t_2} = \hat{x}(t_1, t_2). \qquad (2)$$

If somehow we want the original univariate solution $y(t)$ for $0 \le t \le t_s$, then we must solve (2) on the rectangular region $[0, t_s] \times [0, T_2]$ of t_1, t_2 space (where T_2 is the carrier period) with the following initial and boundary conditions:

$$\hat{y}(0, t_2) = g(t_2), \qquad (3)$$

$$\hat{y}(t_1, 0) = \hat{y}(t_1, T_2). \qquad (4)$$

$g(\cdot)$ is any given initial-condition function and (4) appears due to the periodicity of the problem in t_2 dimension. The univariate solution $y(t)$ may then be recovered from $\hat{y}(t_1, t_2)$, simply by setting $y(t) = \hat{y}(t, t \bmod T_2)$. This is a result that can be seen in detail for example in [2] or [6].

3 Numerical Solution of the MPDE

We will now present some efficient methods for solving (2)-(4), based on the bivariate strategy introduced in the previous section. The first three ones operate purely in the time domain. The last one is used to solve the MPDE for t_1 dimension in the time domain and for t_2 dimension in the frequency domain.

3.1 Finite Differences Method

Let us consider the set of grid points (t_{1_i}, t_{2_j}) defined on the rectangle $[0,t_s]\times[0,T_2]$ by

$$0 = t_{1_0} < t_{1_1} < \cdots < t_{1_{K_S}} = t_s, \quad h_{1_i} = t_{1_i} - t_{1_{i-1}}, \quad (5)$$

$$0 = t_{2_0} < t_{2_1} < \cdots < t_{2_{K_2}} = T_2, \quad h_{2_j} = t_{2_j} - t_{2_{j-1}}. \ (6)$$

By discretizing the partial differentiation operators of the MPDE collocated on the grid, we obtain a system of nonlinear algebraic equations that can be numerically solved using for example Newton-Raphson method. For instance, consider the finite difference approximation given by the backward Euler rule

$$\left.\frac{\partial q(\hat{y})}{\partial t_1}\right|_{(t_1,t_2)=(t_{1_i},t_{2_j})} \approx \frac{q(\hat{y}_{i,j}) - q(\hat{y}_{i-1,j})}{h_{1_i}},$$

$$\left.\frac{\partial q(\hat{y})}{\partial t_2}\right|_{(t_1,t_2)=(t_{1_i},t_{2_j})} \approx \frac{q(\hat{y}_{i,j}) - q(\hat{y}_{i,j-1})}{h_{2_j}},$$

with $\hat{y}_{i,j} = \hat{y}(t_{1_i}, t_{2_j})$. This leads for each level i, from $i = 1$ to $i = K_S$, to the scheme

$$\hat{p}_{i,j} + \frac{\hat{q}_{i,j} - \hat{q}_{i-1,j}}{h_{1_i}} + \frac{\hat{q}_{i,j} - \hat{q}_{i,j-1}}{h_{2_j}} - \hat{x}_{i,j} = 0, \quad (7)$$

$$j = 1,\ldots,K_2,$$

where $\hat{p}_{i,j} = p(\hat{y}_{i,j})$, $\hat{q}_{i,j} = q(\hat{y}_{i,j})$ and $\hat{x}_{i,j} = \hat{x}(t_{1_i}, t_{2_j})$. This way, knowing the initial solution on $i = 0$ $(t_1 = 0)$ given by $\hat{y}_{0,j} = g(t_{2_j})$, we can find the solution on each next level i by iteratively solving (7).

3.2 Method of Lines

Consider the semi-discretization of $[0,t_s]\times[0,T_2]$ defined by (6). Thus, by discretizing the MPDE (2) only in t_2, we obtain an ordinary differential system in t_1 dimension, that can be time-step integrated with an initial value solver (e.g. Runge-Kutta). If we use, once again, finite difference approximations based on the backward Euler rule then we have

$$p(\hat{y}_j(t_1)) + \frac{dq(\hat{y}_j(t_1))}{dt_1} + \frac{q(\hat{y}_j(t_1)) - q(\hat{y}_{j-1}(t_1))}{h_{2_j}} =$$

$$= \hat{x}_j(t_1), \quad j = 1,\ldots,K_2, \quad (8)$$

where $\hat{y}_j(t_1) = \hat{y}(t_1, t_{2_j})$ and $\hat{x}_j(t_1) = \hat{x}(t_1, t_{2_j})$. Now, according to the chain rule

$$\frac{dq(\hat{y}_j(t_1))}{dt_1} = \left.\frac{dq}{dy}\right|_{y=\hat{y}_j(t_1)} \hat{y}_j'(t_1),$$

(8) can be described in the classical form

$$\hat{y}' = f(t_1, \hat{y}), \quad 0 \le t_1 \le t_s,$$

$$\hat{y}(0) = \hat{y}_0,$$

which in this case results in

$$\hat{y}_1'(t_1) = \left[\hat{x}_1(t_1) - p(\hat{y}_1(t_1)) - \frac{q(\hat{y}_1(t_1)) - q(\hat{y}_{K_2}(t_1))}{h_{2_1}}\right] \left[\left.\frac{dq}{dy}\right|_{y=\hat{y}_1(t_1)}\right]^{-1}$$

$$\vdots$$

$$\hat{y}_{K_2}'(t_1) = \left[\hat{x}_{K_2}(t_1) - p(\hat{y}_{K_2}(t_1)) - \frac{q(\hat{y}_{K_2}(t_1)) - q(\hat{y}_{K_2-1}(t_1))}{h_{2_{K_2}}}\right] \left[\left.\frac{dq}{dy}\right|_{y=\hat{y}_{K_2}(t_1)}\right]^{-1}$$

with $\hat{y}_0 = \left(g(t_{2_1}),\ldots,g(t_{2_{K_2}})\right)^T$.

3.3 Shooting

Consider now the semi-discretization of the rectangle $[0,t_s]\times[0,T_2]$ defined by (5). By discretizing the MPDE (2) only in t_1 dimension, we obtain for each level t_{1_i} an ordinary differential system in t_2, with periodic boundary conditions. If we use again the backward Euler rule then we have for each i, from $i = 1$ to $i = K_S$, the boundary value problem

$$p(\hat{y}_i(t_2)) + \frac{q(\hat{y}_i(t_2)) - q(\hat{y}_{i-1}(t_2))}{h_{1_i}} + \frac{dq(\hat{y}_i(t_2))}{dt_2} =$$

$$= \hat{x}_i(t_2), \quad (9)$$

$$\hat{y}_i(0) = \hat{y}_i(T_2), \quad (10)$$

where $\hat{y}_i(t_2) = \hat{y}(t_{1_i}, t_2)$ and $\hat{x}_i(t_2) = \hat{x}(t_{1_i}, t_2)$. This means that once $\hat{y}_{i-1}(t_2)$ is known, the solution on next level, $\hat{y}_i(t_2)$, is obtained by solving (9)-(10). Here we propose to solve (9)-(10) using classical shooting [1], [4].

Shooting is an iterative solver that uses an initial value technique to solve a boundary value problem. In our case we have periodic boundary conditions and the problem can be formulated in the following way: what initial condition, or left boundary $\hat{y}_i(0)$, should be selected for time-step integration, that would lead to a final condition, or right boundary, satisfying $\hat{y}_i(T_2) = \hat{y}_i(0)$? Shooting is, in fact, a procedure that consists in guessing the initial estimate, by comparing and wisely updating the initial condition after successive time-step integrations. More details can be viewed in [1] or [4].

3.4 Mixed Frequency-Time Method

Let us return again to (9)-(10). We will now propose to solve each one of these boundary value problems using harmonic balance (HB) [1], [4]. HB is a classical solver commonly used in RF and microwave circuit simulation. As it can be seen in detail for example in [4], this method uses a linear combination of sinusoids to build the solution, by expanding waveforms in Fourier series.

In this case, for each time-step t_{1_i} we will have an HB equation

$$\mathbf{P}\left(\hat{\mathbf{Y}}_i\right)+\frac{\mathbf{Q}\left(\hat{\mathbf{Y}}_i\right)-\mathbf{Q}\left(\hat{\mathbf{Y}}_{i-1}\right)}{h_{1_i}}+j\mathbf{\Omega}\mathbf{Q}\left(\hat{\mathbf{Y}}_i\right)=\hat{\mathbf{X}}_i, \quad (11)$$

where $\hat{\mathbf{X}}_i$ and $\hat{\mathbf{Y}}_i$ are vectors with the Fourier coefficients for the excitation and the solution and $j\mathbf{\Omega}=\mathrm{diag}(-jK\omega_0,\ldots,jK\omega_0)$, with K the number of significant harmonics, $\omega_0=2\pi/T_2$ and j the imaginary unit. So we have a mixed mode technique that handles the envelope (t_1 dimension) in the time domain and the carrier (t_2 dimension) in the frequency domain.

All resolution details of (11) are omitted here for brevity.

4 Experimental Results

4.1 Sample Application

In order to test the efficiency of the methods presented in the previous section, we propose the nonlinear single node circuit of Fig.1.

Beyond the power source i_S, this circuit is composed by a linear conductance, a nonlinear capacitance and a nonlinear current source. For the current we considered a nonlinear voltage-dependent current source

$$i_{NL}\left(v_O(t)\right)=I_0\tanh\left(\alpha v_O(t)\right)$$

and for the capacitance we considered

$$q_{NL}\left(v_O(t)\right)=\tau_F\, i_{NL}\left(v_O(t)\right).$$

These are models usually encountered in doped semiconductors.

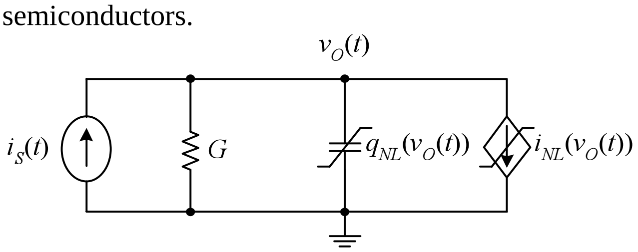

Fig.1. Nonlinear circuit example

The nodal analysis of the circuit leads to the following ordinary differential equation:

$$Gv_O(t)+\frac{dq_{NL}\left(v_O(t)\right)}{dt}+i_{NL}\left(v_O(t)\right)=i_S(t).$$

4.2 Numerical Simulation Results

The circuit was simulated in *MATLAB*® from $t=0$ to $t=1000$ ms, for $G=1\,\mathrm{m}\Omega^{-1}$, $I_0=0.4$ mA, $\alpha=1\mathrm{V}^{-1}$, $\tau_F=2\cdot10^{-3}$ s and an excitation

$$i_S(t)=e(t)\sin(2\pi f_c t)\ \mathrm{mA},$$

with a carrier frequency $f_c=1$ kHz and an envelope $e(t)=5\sin(2\pi f t)$, $f=0.5$ Hz.

The values of G, I_0, α and τ_F were chosen in way to obtain a considerably nonlinear problem and the overall results of this simulation are presented in Table 1 where, for comparison, we also included a non multi-rate method (classical univariate time-step integration). The bivariate solution is shown in Fig.2 and the univariate solution is plotted in Fig.3.

Method	time[1] (sec)	error $\lVert\cdot\rVert_\infty$	error $\lVert\cdot\rVert_{L^2}$
Finite differences	1.82	0.0583	0.0407
Method of lines	2.49	0.0664	0.0491
Shooting	2.32	0.0378	0.0129
Mixed (freq. - time)	0.18	0.0256	0.0072
Univariate	15.93	0.0328	0.0199

Table 1. Numerical simulation results

As we can see, the multi-rate methods presented in Section 3 exhibit significant advantages in speed over the non multi-rate method. In fact, while in the MPDE based methods we have total computation times ranging from 0.18 to 2.49 seconds, in the univariate time-step integration we have 15.93 seconds.

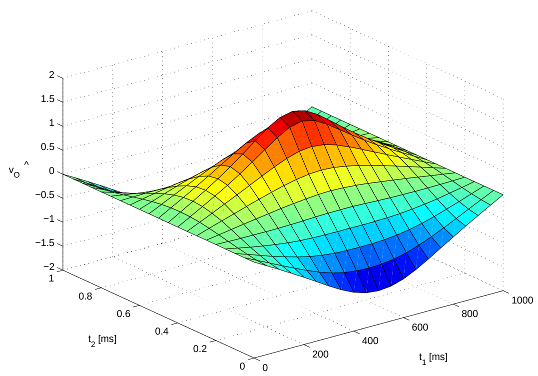

Fig.2. Bivariate solution, $\hat{v}_O(t_1,t_2)$

[1] Computation time (AMD Athlon 1.8 GHz, 256MB RAM).

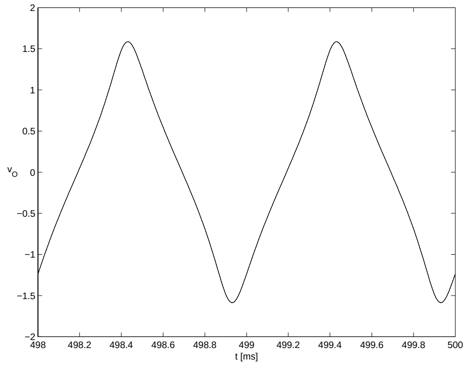

Fig.3. Univariate solution $v_O(t)$, in [498,500]ms

We have tested other values of G, I_0, α and τ_F, leading to weakly nonlinear and quasi-linear problems, and the results were similar to the ones presented in Table 1: multi-rate methods were always much faster than the univariate one and an excellent speedup was exhibited by the mixed (frequency-time) method. However, the extreme efficiency of this mixed method cannot be generalized. In fact, if for a fixed excitation we successively increase the value of α or decrease the values of G and I_0, the circuit nonlinearities become stronger and the method looses efficiency. We have simulated the problem with $G = 0.74\,\text{m}\Omega^{-1}$, $I_0 = 0.155\,\text{mA}$, $\alpha = 2\,\text{V}^{-1}$ and $\tau_F = 5\cdot 10^{-3}\,\text{s}$, and we obtained a total of 4.98 seconds for the computation time of the mixed method, while for example in the finite differences method this time was 1.99 seconds. Furthermore, if we try to decrease the values of G or I_0 the solution cannot even be found by the mixed method.

5 Conclusions

Multi-rate methods have demonstrated to be much more efficient than the classical univariate method. It is so because they are based in a powerful strategy that uses two time variables to describe multi-rate behavior. Efficiency is achieved without compromising accuracy and considerable speedups are obtained. The mixed (frequency-time) method is extremely efficient for solving weakly nonlinear or quasi-linear circuits, but may become inefficient for solving strongly nonlinear circuits. In fact, under strong nonlinearities frequency methods become even useless because they require a large number of harmonics in Fourier expansions.

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