

Evaluation for Transmission Loss Based on Share Proportion Principle and Sensitivity Analysis

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Abstract: - This paper presents a method to evaluate the economic value of transmission loss based on the share proportion principle and the sensitivity analysis. We first formulate an optimal power flow problem to derive the operation conditions, the nodal prices of each bus and the transmission prices between two buses, and then trace the transmission loss during operation based on the share proportion principle of the graph theory. The sensitivity analysis has been used to analyze the wheeling path from each generator to any load, and the value of transmission loss is also evaluated. The proposed method is applied to the IEEE 30-bus standard system, which confirms our theoretical results. The proposed method can be used not only to improve the efficient usage of the power grid and congestion management, but also to provide valuable economic signals for the generation or transmission investment.

Key-Words: - Transmission loss, Optimal power flow, Share proportion principle, Sensitivity analysis, Wheeling path

1 Introduction

The transmission system is open to access by all power market participants. In an electricity market, it is desirable to account the value of transmissions in a simple, unambiguous and transparent way for all market participants. To keep the system operation impartially, pricing for the cost of transmission losses has become important increasingly in the interest of fair competition with the deregulation of power systems. Despite significant amounts of effort from academic circles and industry, the problem of pricing becomes increasingly complicated due to the rapid advance of deregulation in the power industry. The complexity is caused not only by its internal nonlinearity, but also by the characteristics of market structures and related economic policies [1].

In system operation, the generators and consumers measure their real-time outputs and consumptions, respectively. It is obvious that the generators' productions should be the sum of load consumptions and transmission loss. Hence, someone in the power market has to pay for the loss. A fair and adequate method for pricing the cost of transmission loss may help the market participants makes appropriate and efficient decisions for power trade.

This paper aims to propose an algorithm to

evaluate the equivalent value of the transmission loss, by applying a decomposition technique for transmission prices [3]. An operation problem of an electric power system as an optimal power flow (OPF) problem is defined firstly. Then the allocation of transmission losses in an open system operating under both pool and bilateral contracts is analyzed. According to the operation solutions, the transmission losses are traced based on the share proportion principle of the graph theory, and such decomposition is unique and continuously. This approach allows considering loss supply as a marketable product, since each bilateral contract can choose its more appropriate loss supplier. Many methodologies which determine the wheeling path have been developed so far, such as the postage stamp methodology and the contract path methodology [1, 8]. However, since in the aforementioned methods, the use of transmission lines and time varying load flows are not reflected sufficiently, shrewd improvements against these problems are desired for the practical use of the methods. In this paper, the extended sensitivity analysis is proposed to identify the wheeling path so that it is possible to fix the proper and fair wheeling rate according to the degree of responsibility of each power flow. Finally, the value of the transmission losses can be derived based on the above calculation

of OPF and decomposition of wheeling path.

In Section 2, an operation problem is formulated as a nonlinear optimization problem, which has the same form as the conventional OPF. A general explanation is described by deriving the nodal price of each bus and transmission price between any two buses from OPF based-model. Section 3 proposes a theoretical method to determine the amount of transmission loss distributed to each load and to show the contribution of generators to the transmission loss based on the share proportion principle of graph theory. In Section 4 the transmission path is identified by extended sensitivity analysis. The economic value of transmission loss is evaluated. Section 5 is a numerical simulation for IEEE 30-bus system.

2 An Operation Problem Formulation

For a n buses system with m generators, let $P = (P_1, \dots, P_n)$ and $Q = (Q_1, \dots, Q_n)$, where P_i and Q_i represent real and reactive power demands of the bus- i respectively. Define voltage variables in the power system operation to be $X = (V_1, \theta_1, \dots, V_n, \theta_n)$, which are magnitudes and phases of each bus voltage. $P_G = (P_{G1}, \dots, P_{Gm})$ and $Q_G = (Q_{G1}, \dots, Q_{Gm})$ represent the generators outputs, which can be regarded as the functions of X, P, Q .

Therefore, the operation problem of a power system for the given loads (P, Q) can be formulated as the following optimization problem [2],

$$\text{Min } f(X, P_G, Q_G, P, Q) \quad \text{for } X, P_G, Q_G \quad (1)$$

$$\text{s.t. } G(X, P_G, Q_G, P, Q) = 0 \quad (2)$$

$$H(X, P_G, Q_G, P, Q) \leq 0 \quad (3)$$

where f is a scalar that represents the operating cost, and can be expressed as $f = \sum_{i=1}^m f_i$ where f_i is the operation cost of the i -th generator. $G = (g_1(X, P_G, Q_G, P, Q), \dots, g_{n_1}(X, P_G, Q_G, P, Q))^T$ and $H = (h_1(X, P_G, Q_G, P, Q), \dots, h_{n_2}(X, P_G, Q_G, P, Q))^T$ have n_1 and n_2 equations respectively, and are column vectors. G is a vector, equality constraints, such as energy balance for generators and loads. H is a vector too, inequality constraints, which includes all variable limits and function limits, such as upper and lower bounds of transmission lines, generation outputs, etc.

Obviously the formulation given by (1)-(3) is a typical OPF problem as far as the demands (P, Q) are given. There are many efficient approaches, which can be used to obtain an optimal solution, such as successive linear programming, successive quadratic programming and the Newton method.

From the viewpoint of economics, for a set of given demand (P, Q) , the nodal prices of real power for bus- i are expressed below for $i = 1, \dots, n$,

$$\pi_i = \frac{\partial f}{\partial P_i} + \lambda \frac{\partial G}{\partial P_i} + \rho \frac{\partial H}{\partial P_i} \quad (4)$$

Therefore, the transmission price of real power is defined as follow,

$$\pi_{i-j} = \pi_i - \pi_j \quad (5)$$

where π_i is nodal price of real power at the bus- i , π_{i-j} is transmission price of real power at transmission line $i-j$. $\lambda = (\lambda_1, \dots, \lambda_{n_1})$ and $\rho = (\rho_1, \dots, \rho_{n_2})$ are the dual variables, and are usually explained as shadow prices. Notice that $\lambda \cdot G(X, P_G, Q_G, P, Q) = \sum_{i=1}^{n_1} \lambda_i \cdot g_i(X, P_G, Q_G, P, Q)$ and $\rho \cdot H(X, P_G, Q_G, P, Q) = \sum_{i=1}^{n_2} \rho_i \cdot h_i(X, P_G, Q_G, P, Q)$.

The above formulation is conventional OPF, and the results are fundamental for the evaluation for transmission loss in real-time operation, such as, output of each generator P_{Gi} , power flow of each transmission line P_{ij} , nodal prices π_i and transmission price π_{ij} .

3 The Amount of Transmission Loss

In real-time operation, consumers measure their actual consumption, while generator meters measure their actual productions. The transmission loss is the difference between generator outputs and load consumptions. If there is not any transmission loss, more power should be conveyed from generators to the loads than realistic operation. The gross power supplied to each load is symbolized as $P_d = (P_{d1}, \dots, P_{dn})$, which is the sum of real power demand and transmission loss $P_s = (P_{s1}, \dots, P_{sn})$ distributed to each load, i.e.,

$$\sum_{i=1}^m P_{Gi} = \sum_{i=1}^n P_{di} = \sum_{i=1}^n (P_i + P_{si}) \quad (6)$$

Next, we establish the matrix of power flow which is symmetrical with half positive and half

negative values based on OPF calculation. For any bus, the transmission lines can be divided into two groups, injecting lines and emitting lines. In this section, α_i^{in} and α_i^{out} represent the two aggregates which inject into and flow out from bus-*i*, respectively.

3.1 Share Proportion Principle

From the viewpoint of graph theory, the power network is kind of graph with direction, while the buses and transmission lines are nodes and branches, respectively. The share proportion principle can be used to analyses power flow at each bus.

Fig. 1 shows an example without any loss for the share proportion principle. Node-*i* connects with 3 injecting branches and 2 emitting branches, i.e., $\alpha_i^{in} = \{A, B, C\}$ and $\alpha_i^{out} = \{E, F\}$. Without any loss, $20 + 30 + 50 = 60 + 40 = 100$, i.e., the 3 branches supply the power to node-*i* by 20%, 30% and 50%, respectively. It is reasonable to hypothesize that each emitting branch shares same proportion as the node connected with. For instance, for branch-E, 20% comes from branch-A, left 30% and 50% come from branch-B and C. In the other word, the injected power is mixed at nod-*i*, and sent out as the same rate to any emitting branches.

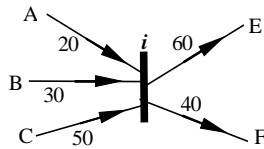


Fig. 1 An example without any loss for the share proportion principle

Fig. 2 shows an example with sources and losses. Similarly as Fig. 1, node-*i* connects with 3 injecting branches and 2 emitting branches, but the flows come from two sources G1 and G2, i.e.,

$$\alpha_i^{in} = \begin{Bmatrix} A_{G1} & B_{G1} & C_{G1} \\ A_{G2} & B_{G2} & C_{G2} \end{Bmatrix} \text{ and } \alpha_i^{out} = \begin{Bmatrix} E_{G1} & F_{G1} \\ E_{G2} & F_{G2} \end{Bmatrix},$$

Here, The notations with footmark G1 and G2 represent the components come from G1 and G2. The 3 branches supply the power, which are produced by G1 and G2. The same proportion is shared in the input and output of node-*i*, i.e.,

$$\frac{E_{Loss}}{F_{Loss}} = E/F = 59/39, \\ \frac{A_{G1} + B_{G1} + C_{G1}}{A + B + C} = \frac{E_{G1} + F_{G1}}{E + F} = \frac{E_{G1-Loss} + F_{G1-Loss}}{E_{Loss} + F_{Loss}},$$

$$\frac{A_{G2} + B_{G2} + C_{G2}}{A + B + C} = \frac{E_{G2} + F_{G2}}{E + F} = \frac{E_{G2-Loss} + F_{G2-Loss}}{E_{Loss} + F_{Loss}}.$$

It means that the injected power is mixed at nod-*i*, and sent out as same rate to any emitting branch, or consumed as the transmission losses.

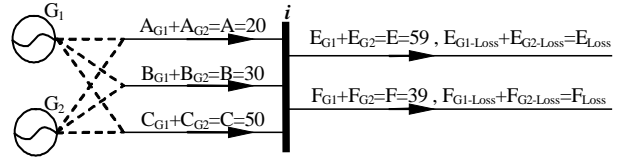


Fig. 2 An example with sources and losses for the share proportion principle

3.2 Tracing Power Flow by Down-stream Looking

In this section, the formulation for tracing power flow from generators to loads is given, and the transmission loss is distributed to each load.

$P^g = (P_1^g, \dots, P_n^g)$ represents the gross power injected into or flowed out from each bus, P_{ij}^g is the gross power flow on the transmission line *i*-*j*. P_{di} is gross power demand, which has been defined by (6), and the gross power at bus-*i* is defined in (7),

$$P_i^g = P_{Gi} + \sum_{j \in \alpha_i^{in}} |P_{ij}^g| = P_{di} + \sum_{j \in \alpha_i^{out}} |P_{ij}^g| \quad (7)$$

Let P_i^{in} and P_i^{out} be the real power injected into and flow out from bus-*i*, i.e., $P_i^{in} = P_{Gi} + \sum_{j \in \alpha_i^{in}} |P_{ij}|$,

$$P_i^{out} = P_i + \sum_{j \in \alpha_i^{out}} |P_{ij}|.$$

Because the percentage of transmission loss in the power injected into bus-*i* is quite small in realistic system operation, the rate of power flow based on above OPF calculation result can be used approximately.

For the injection to bus-*i*, $j \in \alpha_i^{in}$,

$$\frac{P_{ij}^g}{P_i^g} = \frac{P_{ij}}{P_i^{in}} \quad (8)$$

By substituting (8) into (7), (9)-(10) can be derived,

$$P_i^g - \sum_{j \in \alpha_i^{in}} \frac{|P_{ij}| \cdot P_i^g}{P_i^{in}} = P_{Gi} \quad (9)$$

$$M \cdot P^g = P_G \text{ or } P^g = M^{-1} \cdot P_G \quad (10)$$

M is a sparsity matrix, and its elements are shown as follows,

$$[M]_{ij} = \begin{cases} 1 & i = j \\ -\frac{|P_{ij}|}{P_i^{in}} & j \in \alpha_i^{in} \\ 0 & \text{others} \end{cases}$$

For the flow out bus- i , $j \in \alpha_i^{out}$,

$$\frac{P_{ij}^s}{P_i^s} = \frac{P_{ij}}{P_i^{out}} \quad (11)$$

By substituting (11) into (7), (12)-(13) can be derived,

$$P_{di} = P_i^s - \sum_{j \in \alpha_i^{out}} \frac{|P_{ij}| \cdot P_i^s}{P_i^{out}} \quad (12)$$

$$P_d = N \cdot P^g \quad (13)$$

N is also a sparsity matrix, and its elements are shown as follow,

$$[N]_{ij} = \begin{cases} 1 & i = j \\ -\frac{|P_{ij}|}{P_i^{out}} & j \in \alpha_i^{out} \\ 0 & \text{others} \end{cases}$$

According to (6), (10) and (13), the transmission loss P_s distributed to each load can be obtained by (14),

$$P_s = N \cdot M^{-1} \cdot P_G - P \quad (14)$$

(14) seems to have given a full description for tracing transmission loss even without any further analysis. However it is still unknown exactly which generator supplies, how much power is supplied for the transmission loss and what kind of wheeling path the lost power is passed.

3.3 Decomposition for Transmission Losses

To decompose the components of transmission loss, the share proportion principle is used again to trace the power flow from each generator side to any load side.

Start from the bus connected with a generator but without any injection line, it is obvious that the power flows which flow out from this bus are provided hundred percent by this generator. And all transmission lines keep the rates provided by generators. Let P_{si-Gk} and P_{ij-Gk} be the contributions of generator- k for transmission loss P_{si} and for the power flow which is burdened by line- ij , respectively.

For bus- i , the proportion of composition for transmission loss can be shown as (15), $i = 1, \dots, n$, $k \in (1, \dots, m)$,

$$\frac{P_{si-Gk}}{P_{si}} = \frac{P_{Gk} + \sum_{j \in \alpha_i^{in}} |P_{ij-Gk}|}{P_i^{in}} \quad (15)$$

If $k = i$, the generator injects all production to bus- i . When $k \neq i$, P_{Gk} should be zero in (15). The procedure for tracing transmission loss can be described in Fig 3.

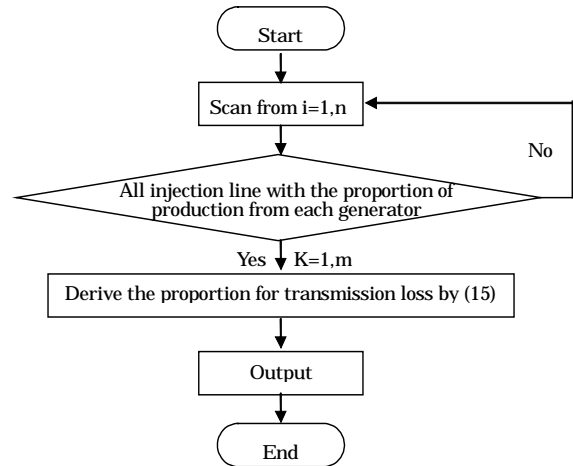


Fig. 3 Flowchart of decomposition for the transmission losses

4 Evaluation for Transmission loss based on Sensitivity Analysis

In this section, the sensitivity analysis is derived to distribute the wheeling path for the transmission loss in the grid theoretically. Firstly, the sensitivity method in power system is formulated. Then, a practical and efficient method is proposed to identify the wheeling paths of transmission loss from each generator to any customers. Finally, the transmission loss is evaluated based on sensitivity analysis results.

4.1 Sensitivity Analysis in Power System

Let the voltage vector V_i at node- i be expressed in polar form and the admittance $Y_{ij} = R_{ij} + jB_{ij}$ in rectangular form. Then power flow equation is a set of $2n$ real simultaneous equations, for $i = 1, 2, \dots, n$,

$$G_{2i-1} = P_i - P_{Gi} + V_i \sum_{j=1}^n V_j [R_{ij} \cos(\theta_i - \theta_j) + B_{ij} \sin(\theta_i - \theta_j)] = 0$$

$$G_{2i} = Q_i - Q_{Gi} + V_i \sum_{j=1}^n V_j [R_{ij} \sin(\theta_i - \theta_j) - B_{ij} \cos(\theta_i - \theta_j)] = 0$$

For simplicity, the variables and parameters involved in above power flow equations will be classified into two vectors. They are dependent variable vector Z with t dimensions and

controllable variable vector U with k dimensions comprising the operating variables in system analysis and control.

Using two vectors Z, U defined above, the power flow is expressed by the following simple equation vector,

$$G(Z, U) = 0 \quad (16)$$

where G is a $2n$ -dimensional column vector function. The pair of vectors Z_0 and U_0 satisfies (16), i.e., $G(Z_0, U_0) = 0$.

Assume that by changing the operating condition of regulating devices, the controllable variable vector U has experienced a change ΔU from U_0 . If dependent variable vector Z changes from Z_0 to $Z_0 + \Delta Z$ in accordance with the change ΔU , then there is $G(Z_0 + \Delta Z, U_0 + \Delta U) = 0$.

If ΔU is very small, then the variance ΔZ is generally small. By applying Taylor's series expansion with (Z_0, U_0) as the reference state and neglecting higher order terms in ΔZ and ΔU , we have

$$G(Z_0, U_0) + G_Z(Z_0, U_0)\Delta Z + G_U(Z_0, U_0)\Delta U = 0 \quad (17)$$

where $G_Z = \partial G / \partial Z$, $G_U = \partial G / \partial U$ are the Jacobian matrix of G with respect to the dependent variable vector Z, U . Since the first term of (17) is eliminated,

$$\Delta Z = -G_Z^{-1} \cdot G_U \cdot \Delta U \quad (18)$$

Let $S \equiv -G_Z^{-1} \cdot G_U$, (18) can be rewritten in the form of (19),

$$\Delta Z = S \cdot \Delta U \quad (19)$$

or more concretely in following form,

$$\begin{pmatrix} \Delta z_1 \\ \Delta z_2 \\ \vdots \\ \Delta z_t \end{pmatrix} = \begin{pmatrix} S_{11} & S_{12} & \cdots & S_{1k} \\ S_{21} & S_{22} & \cdots & S_{2k} \\ \cdots & \cdots & \cdots & \cdots \\ S_{t1} & S_{t2} & \cdots & S_{tk} \end{pmatrix} \begin{pmatrix} \Delta u_1 \\ \Delta u_2 \\ \vdots \\ \Delta u_k \end{pmatrix}$$

The $t \times k$ coefficient matrix S in (19) is called the sensitivity matrix of power flow with respect to the controllable variable vector U .

4.2 Identifying Wheeling Paths of Transmission loss

We derive the sensitivity constants which are required for power flows of each transmission line for identifying the wheeling paths.

For a system with n buses, m generators and l transmission lines, let F be the power current. By the use of the two vectors X and P , the line flows are conveniently expressed by the simple function

vector $F(X, P)$, where F is an l -dimensional column vector function.

Let us assume that when the state of the system is changed from an initial state (X_0, P_0) to a state $(X_0 + \Delta X, P_0 + \Delta P)$ by the operation of some regulating devices, line flows are also changed by $\Delta F(X_0, P_0)$. Similar to (17), the Taylor series expansion with (X_0, P_0) as the reference state yields,

$$\Delta F(X_0, P_0) = F_X(X_0, P_0)\Delta X + F_P(X_0, P_0)\Delta P \quad (20)$$

Let S_F be the sensitivity matrix representing the change in power flow ΔF due to a change ΔP in transmission loss. In this paper, P_s is regarded as the change of load demand ΔP , i.e., $S_F = \Delta F / P_s$. Partial derivatives, $F_X = \partial F / \partial X$ and $F_P = \partial F / \partial P$ are obtained quite easily by simple calculation $\partial X / \partial P$ which is already known as the elements of the sensitivity matrix (19).

From (20), the following relation is obtained.

$$S_F = \frac{\Delta F(X_0, P_0)}{P_s} = F_X(X_0, P_0) \cdot \frac{\partial X}{\partial P} + F_P(X_0, P_0) \quad (21)$$

Let F_{ij} be the power flow on the transmission line $i - j$. Generally, (21) can be used to calculate the line currency.

$$F_{ij} = \frac{V_i \cdot (V_i - V_j) + 2V_i V_j \cdot \sin^2[(\theta_i - \theta_j)/2]}{(R_{ij}^2 + B_{ij}^2)/R_{ij}} + \frac{V_i V_j \sin(\theta_i - \theta_j)}{(R_{ij}^2 + B_{ij}^2)/B_{ij}} \quad (22)$$

Then the related wheeling path of transmission loss is identified by examining the magnitude of sensitivity factors from generator to load.

4.3 Evaluation for Transmission Loss

Based on the above analysis results, the economic value of transmission loss can be evaluated easily. We separate the values into two parts; one is for power production, and the other is for power transmission. Here it is assumed that power consumers pay for the transmission loss, which also can be shared by other power market participators, or be regarded as a kind of service provided by utility.

For a n -bus system with m generators and l transmission lines,

i) Pricing of power production for transmission loss:

$$\text{The payment of bus-}i: \sum_{j=1}^m (P_{si-gj} \cdot \pi_j)$$

$$\text{The income of generator-}j: \pi_j \cdot \sum_{i=1}^n P_{si-gj}$$

ii) Pricing of power transmission for transmission loss:

There two conditions have to be considered.

(a) All lines belong to one grid company.

The payment of bus- i : $\sum_{j=1}^m (P_{si-gj} \cdot \pi_{ij})$

The income of the grid company:

$$\sum_{i=1}^n \sum_{j=1}^m (P_{si-gj} \cdot \pi_{ij})$$

(b) The lines belong to plural grid companies.

If the loads and generators belong to different companies, the wheeling paths have to be traced to derive the amount of power flow through the boundary bus or buses. For example, there are two grid companies in a system. Company A holds a part of lines and all generators, and B is the owner of the rest lines and all loads. P_{si-gj}^t and P_{si-gj}^r represent the transmission losses pass bus- t and bus- r which are boundary of two companies, $t, r \in (1, \dots, n)$. Note that $P_{si-gj} = P_{si-gj}^t + P_{si-gj}^r$.

The bus- i for company A:

$$\sum_{j=1}^m (P_{si-gj}^t \cdot \pi_{jt}) + \sum_{j=1}^m (P_{si-gj}^r \cdot \pi_{jr})$$

The bus- i for company B:

$$\pi_{it} \sum_{j=1}^m P_{si-gj}^t + \pi_{ir} \sum_{j=1}^m P_{si-gj}^r$$

5 Case Study

5.1 System Description

The IEEE 30-bus system is used to illustrate the proposed methodology. As shown in Fig. 7 or Fig. 8, the system has 30 buses, 19 loads and 41 transmission lines.

In this paper, as same as the power demand of each bus P, Q , all limits are regarded as given conditions. It is assumed that the voltage value for all buses is bounded between 0.95 and 1.05; the power flow in each line is restricted between -0.7 and 0.7. All the values are indicated by p.u.

5.2 Numerical Simulation

The load demands are the given conditions. The generator outputs are presented based on OPF calculation results, and the allocated transmission loss to either generators or loads is shown in Tables 1 and 2. The generator outputs are 238.516MW, in which 6.816MW is used by transmission loss.

In Table 2, if there were not transmission loss in system operation, more power should be transported to the loads. The gross power at each load is the sum of real demand and loss distributed on that bus, i.e., the gross power of bus-3 is $2.4MW + 30.7KW$. “%”

indicates the percentage of loss in load demand. Due to less than 5%, they are small enough to fit for sensitivity analysis. Table 2 also shows the allocation by tracing the source of transmission loss, i.e., the transmission loss of bus-10 is 87.65KW, in which 4.98KW is supplied by generator G1, 3.7KW, 74.78KW and 4.19KW are supplied by generator G2, G22 and G27, respectively. It is clear that G22 provides much more than other generators, because G22 is much closer to bus-10 than the others.

Table 1. Results of Generator Output

(MW)	Total output	For load demand	For loss
G1	73.67	71.3714	2.2986
G2	61.656	59.5277	2.1283
G22	53.19	52.2303	0.9597
G27	50	48.5705	1.4295
Total	238.516	231.7	6.8160

Table 2. Distribution of the Transmission Loss

Bus No.	Load demand MW	Loss on each load KW	From G1 KW	From G2 KW	From G22 KW	From G27 KW	%
3	2.4	30.70	30.70	0	0	0	1.28
4	7.6	116.97	89.00	27.97	0	0	1.54
5	94.2	3520.46	1433.87	1693.27	0	393.32	3.74
7	22.8	942.68	364.49	271.03	0	307.17	4.14
10	5.8	87.65	4.98	3.70	74.78	4.19	1.51
12	11.2	235.92	162.02	51.52	21.19	1.19	2.11
14	6.2	150.31	87.61	27.86	18.95	15.88	2.42
15	8.2	273.24	79.19	25.18	62.40	106.46	3.33
16	3.5	76.84	4.36	3.24	65.56	3.68	2.20
17	9	197.60	11.22	8.34	168.58	9.46	2.19
18	3.2	96.51	18.71	6.91	46.88	24.02	3.02
19	9.5	185.41	10.53	7.83	158.18	8.87	1.95
20	2.2	33.25	1.89	1.40	28.36	1.59	1.51
21	17.5	172.39	0	0	172.39	0	0.99
23	3.2	131.39	0	0	43.30	88.09	4.11
24	8.7	300.86	0	0	99.15	201.71	3.46
26	3.5	85.94	0	0	0	85.94	2.46
29	2.4	40.09	0	0	0	40.09	1.67
30	10.6	137.81	0	0	0	137.81	1.30
Total	231.7	6816	2298.56	2128.27	959.71	1429.47	2.94

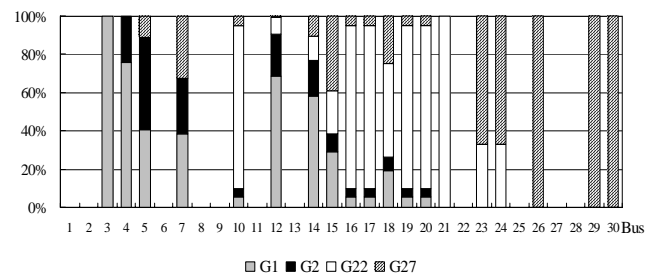


Fig.4. Percentage of transmission loss from generators to loads

Fig. 4 represents the percentage of transmission loss from generators to each load. In a grid network without special contract, the generators provide the power to closer load priority. The transmission loss takes place during the transportation of power demand. The closer generator takes the more transmission loss for any load, for example, G1

burdens all transmission loss for load-3, and G27 burdens all transmission loss for load-27, 29 and 30.

The payment is divided into two parts; one is for generator output, and the other is for wheeling service. We have assumed that the power consumers pay for the transmission loss. Because the generator and load use the grid at the same time, both of them should share the payment, which can be determined by negotiations.

Fig.5 presents the payment of transmission loss for generator outputs. The total payment of generator output for transmission loss is \$375.7, which is shared by G1 \$128.73, G2 \$121.82, G22 \$48.64 and G27 \$76.5. The load-5 and load-7 have to pay much more than others, owing to their large amount of power demand and long electric distances.

If all transmission lines belong to a single grid company, there is not necessary to identify the wheeling paths for transmission loss, because the congestion has been generally alleviated by OPF calculation, and the transmission price is determined by the difference between two nodal prices of the buses connected with generator and load.

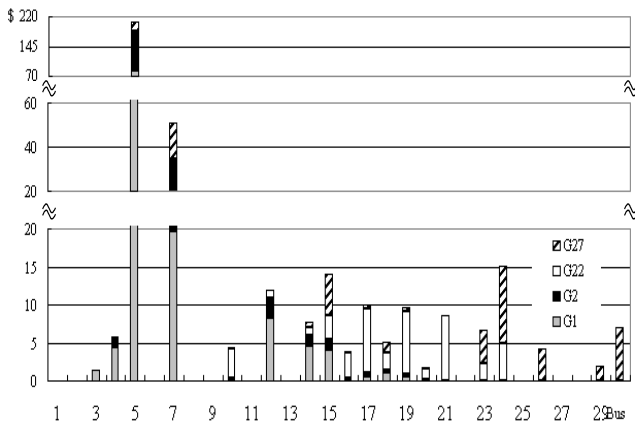


Fig.5. Payment of transmission loss for generators output by loads

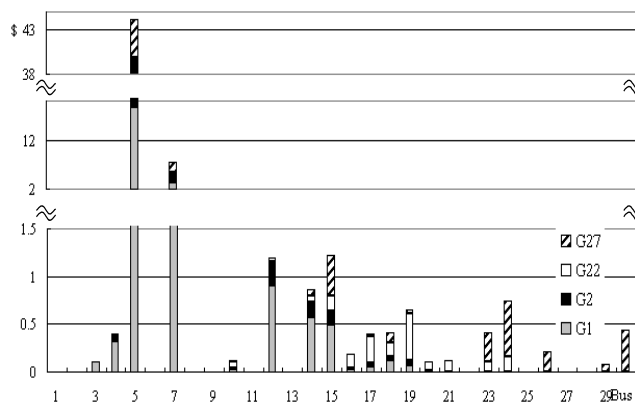


Fig.6. Payment of transmission loss for wheeling by loads

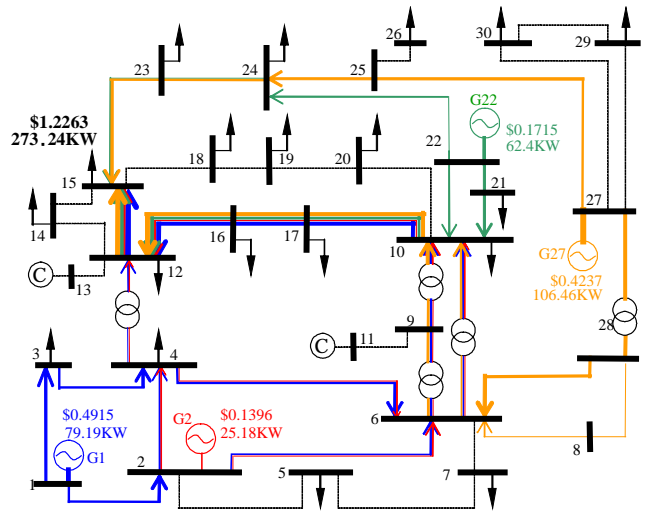


Fig.7. Wheeling path and amount of transmission loss to bus-15

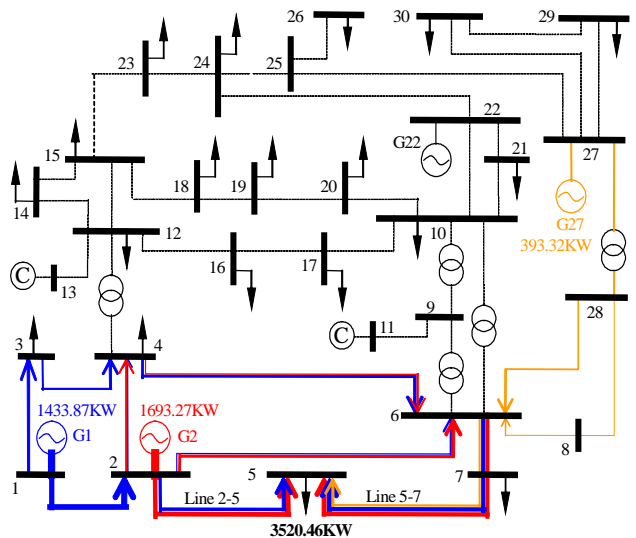


Fig.8. Wheeling path and amount of transmission loss to bus-5

Fig. 6 shows the payment of transmission loss for wheeling. The total payment of wheeling for transmission loss is \$59.215, which is no difference, when there is only one or plural owners of transmission lines in this 30-bus system. As shown in Fig. 7, the load-15 has to pay \$1,226.3 for transportation of the loss from four generators, G1 \$0.4915, G2 \$0.1396, G22 \$0.1715 and G27 \$0.4237, respectively. The transmission route is shown in Fig. 7.

Next it is assumed that there are two line-owners in the target system, i.e., grid company A and B. Company B holds the lines 2-5 and 5-7, and the other lines all belong to company A. In other words, the boundary of two companies is bus-2 and bus-7. According to OPF result, load-5 is the only user of lines 2-5 and 5-7, which has to pay for both

companies A and B. Fig.8 shows the wheeling path and mount of transmission loss to bus-5, where only 3 generators' productions are effected by transmission loss to load-5.

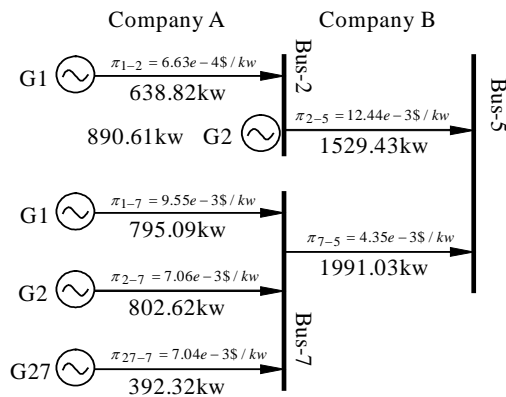


Fig.9. Illustration of Distribution transmission loss to bus-5

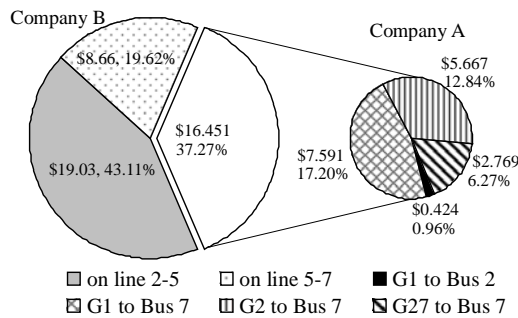


Fig.10. Payment of load-5 to company A and B for transmission loss

The total payment of load-5 for transmission loss is \$44.14. How to share this payment depends on the wheeling paths for transmission loss. Figs. 9 and 10 illustrate how two companies share the payment. As shown in Fig. 9, the company A carries power from G1 638.82KW to bus-2, and total 1991.03KW from G1, G22 and G27 to bus-7, while the company B transport the loss power to bus-5 contiguously. The G2 output for this transmission loss is 1693.27KW, in which 890.61KW is injected into company B area directly, and 802.62KW is injected into company B area after they are transported to bus-7 by company A. In the area of company A, 392.32KW is transported from G27 to bus-7 by lines 27-28, 28-6, 28-8, and 8-6, which have the transmission price $\pi_{27-7} = 7.04e-3$ \$/KW. The charge for this transportation is \$2.769, 6.27% in total payment of load-5 for transmission loss. For company B, 1529.43KW and 1991.03KW are transported by lines 2-5 and 7-5, which have the

wheeling prices $\pi_{2-5} = 12.44e-3$ \$/KW and $\pi_{7-5} = 4.35e-3$ \$/KW. The payment of load-5 for transmission loss is shared by company A \$16.45 and company B \$27.69. The payment for loss is decided by the electric distance and the amount of power.

6 Conclusion

A pricing technique has been proposed to evaluate the equivalent values of the transmission loss by the sensitivity analysis and the share proportion principle. It has shown that transmission loss can theoretically be evaluated, or traced from the viewpoint of economics. To verify the effectiveness of the proposed method, the simulations have been conducted on the IEEE 30-bus test system, which confirms the effectiveness of the proposed method. The theoretical results enable us to evaluate the values of transmission losses for market players in the power system easily and effectively. As the future studies for implementation of the proposed approach, the time varying demand, the adaptation of the transaction framework to reactive power and voltage and the volatility of electricity prices in power markets should be taken into consideration.

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