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Analysis and Simulation of a Core-Type Single-Phase Transformer with Non-uniform Cross-section

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Abstract: - A core-type single-phase transformer model is presented in this work, which takes into account the non-linearity of the core material. The model is based on the analysis of the electric and the magnetic circuits on the level of the state equations. Simulation results with a non-uniform cross-section of the magnetic core are presented that show the influence of the magnetic circuit on the electrical quantities.

Key-Words: - Magnetic circuit, magnetic core, non-linear circuits, simulation, state equations, transformers.

1 Introduction

Transformer modeling is very complicated task due to difficulty in the description of the magnetic core characteristics such as the saturation effect, the hysteresis and eddy current losses, or to the various geometrical design configurations. The core-type or the shell-type topologies of the single-phase transformers, the three-legged or five-legged topologies and the different windings connections of the three phase transformers contribute to the large number of transformer types. Furthermore, for the cases where the skin and eddy current effects in coils and the capacitive effects have to be included, the representation is more complicated. Thus, various methods of analysis have been developed that may roughly be classified into several groups.

A first group is based on the electromagnetic field analysis in the entire volume of the magnetic core using numerical analysis as the finite elements method [1-4]. These analyses are suitable when designing large transformers but are impractical for transient and steady state calculations since they would give very time-consuming simulations.

A second group is based on the principle of duality between electric and magnetic circuits and most efforts, [5-7] deal with an implementation on the Electromagnetic Transients Program (EMTP). According to this principle, the magnetomotive forces are replaced with current sources and the magnetic reluctances with inductances. The inductances are linear when they represent the air fluxes and non-linear when they refer to the fluxes inside various parts of the core. Also, in [8] a complete transformer model is presented where the non-linearity of the core material is included through a magnetization curve. These descriptions do not follow mathematical approaches but they are based on an electrical equivalent circuit.

The models of a third group are based on a mathematical description of the magnetic equations and their coupling with the electrical equations [13-16] and take into account the actual core topology and the windings configuration. The main difference between them is the way in which the complete set of equations is derived. In [17] an incremental transformer model is suggested where the core characteristics are represented either a piece-wise linear curve or a magnetization curve.

Other efforts are focused on the representation of the electric part as in [18] where the modeling is based on coupled coils or on the magnetic part as in [20] where the core material is considered as parallel R-L frequency-depended components. Also, the aim in [19] is the introduction of the hysteresis effect according to Jiles-Atherton model [21-23] in models that are implemented with the EMTP program. In [8-12] there is a successful implementation of complete transformer models in the PSPICE program.

The aim of this paper is the analysis and the simulation of a core-type transformer with non-uniform cross-section based on [17]. The mathematical procedure is based on the analysis of the electric and the magnetic parts according to its geometrical features. Also, this model takes into account the non-linear characteristic curve of the core material and the mathematical formulation is described on the level of state equations.

2 Transformer Model

The single-phase core-type transformer with uniform cross-section shown in Fig.1 will be examined. The corresponding magnetic circuit is as shown in Fig.2. The magnetic potential f is expressed through the magnetic resistance R_m and the core flux Φ as

$$f=R_m\Phi$$

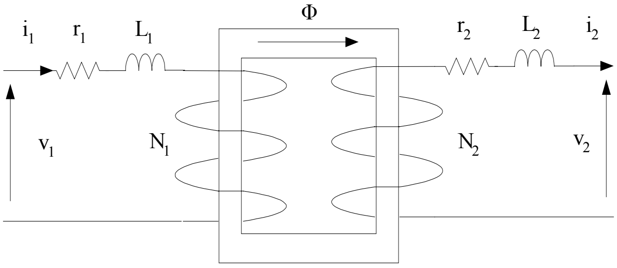


Fig.1 Single-phase core type transformer

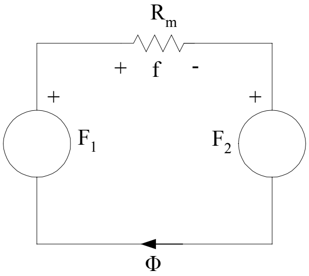


Fig.2 The magnetic circuit of the core type transformer shown in Fig.1

The total magnetomotive force F is equal to the difference of the magnetomotive forces of the primary and the secondary windings and can be expressed as

$$F=N_1i_1-N_2i_2 \qquad (1)$$

where N_1, i_1 and N_2, i_2 are the number of turns and the currents for the primary and the secondary windings respectively.

The core flux of the transformer is, in general, a non-linear function of the total magnetomotive force and one can write

$$\frac{d\Phi}{dt}=\frac{\partial\Phi}{\partial f}\frac{\partial f}{\partial F}\frac{dF}{dt} \qquad (2)$$

At first, the partial derivative of the core flux with respect to the magnetic potential will be derived. The magnetic potential and its differential can be written as

$$f=\Phi R_m=\Phi\frac{1}{\mu}\frac{L_c}{A_c}$$

$$df=\frac{L_c}{A_c}\frac{\mu d\Phi-\Phi d\mu}{\mu^2}$$

where $\mu=b(h)/h$ is the magnetic permeability of the core material, b the magnetic flux density of the core, h the magnetic field strength, L_c is the mean length path of the magnetic lines inside the core and A_c is the cross-section of the core.

The derivative of the permeability μ with respect to the magnetic filed strength h inside the core is determined as

$$\frac{d\mu}{dh}=\frac{h\frac{db}{dh}-b}{h^2}$$

$$\frac{d\mu}{dh}=\frac{\mu_d-\mu}{h}$$

where the term μ_d is defined as

$$\mu_d=\frac{db}{dh} \qquad (3)$$

and is called the incremental permeability.

Then, the differential of the magnetic potential is

$$df=\frac{L_c}{A_c}\left(\frac{d\Phi}{\mu}-\frac{\Phi}{\mu^2}\frac{\mu_d-\mu}{h}dh\right)$$

$$df=\frac{L_c}{A_c}\left(\frac{d\Phi}{\mu}-\frac{A_c\mu_d}{\mu}dh+A_cdh\right)$$

Also, the magnetic flux Φ is expressed as $\Phi=bA_c$ and consequently

$$d\Phi=A_cdb=A_c\mu_ddh$$

Thus, $df=\frac{d\Phi}{\mu_d}\frac{L_c}{A_c}$ and the derivative of the magnetic potential with respect to the core flux results as

$$\frac{df}{d\Phi}=\frac{1}{\mu_d}\frac{L_c}{A_c}=R_{md} \qquad (4)$$

where R_{md} is called the incremental magnetic resistance.

Substituting equations (1) and (4) into (2) and taking into account that $\frac{\partial f}{\partial F}=1$, due to the magnetic circuit shown in Fig.2, the time derivative of the core flux results as

$$\frac{d\Phi}{dt}=\frac{N_1}{R_{md}}\frac{di_1}{dt}-\frac{N_2}{R_{md}}\frac{di_2}{dt} \qquad (5)$$

The equations of the electrical part can be written in matrix form as

$$\begin{pmatrix} v_1 \\ v_2 \end{pmatrix} = \begin{pmatrix} r_1 & 0 \\ 0 & r_2 \end{pmatrix}\begin{pmatrix} i_1 \\ i_2 \end{pmatrix} + \begin{pmatrix} L_1 & 0 \\ 0 & L_2 \end{pmatrix}\frac{d}{dt}\begin{pmatrix} i_1 \\ i_2 \end{pmatrix} + \frac{d}{dt}\begin{pmatrix} \psi_1 \\ \psi_2 \end{pmatrix} \tag{6}$$

where v_1, v_2 are the voltages of the transformer terminals, r_1, r_2 the windings resistances, L_1, L_2 the leakage inductances and ψ_1, ψ_2 the flux linkages.

The time derivatives of the flux linkages ψ_1 and ψ_2 are expressed via the turns of the windings N_1 and N_2 and the flux Φ inside the transformer core in matrix form as

$$\frac{d}{dt}\begin{pmatrix} \psi_1 \\ \psi_2 \end{pmatrix} = \begin{pmatrix} N_1 \\ -N_2 \end{pmatrix}\frac{d\Phi}{dt} \tag{7}$$

Substituting equation (5) into (7) and the result into (6) the electric equations can be written as

$$\begin{pmatrix} v_1 \\ v_2 \end{pmatrix} = \begin{pmatrix} r_1 & 0 \\ 0 & r_2 \end{pmatrix}\begin{pmatrix} i_1 \\ i_2 \end{pmatrix} + \begin{pmatrix} L_1+N_1^2/R_{md} & -N_1N_2/R_{md} \\ -N_1N_2/R_{md} & L_2+N_2^2/R_{md} \end{pmatrix}\frac{d}{dt}\begin{pmatrix} i_1 \\ i_2 \end{pmatrix} \tag{8}$$

The relations (5) and (8) constitute the complete set of the state equations for the transformer considered.

The magnetic core configuration of the core-type transformer with non-uniform cross-section is shown in Fig.3 where the portion of length L_{c1} with cross-section A_{c1} is declared as part-1 and the portion with length L_{c2} and cross-section A_{c2} is declared as part-2.

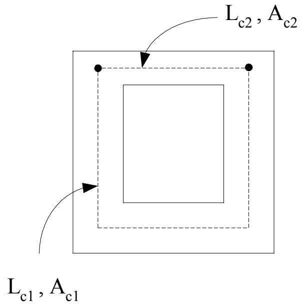


Fig.3 The magnetic core configuration with non-uniform cross-section where $A_{c2}<A_{c1}$.

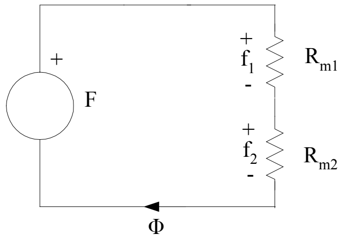


Fig.4 The magnetic circuit for the case with non-uniform cross-section.

The corresponding magnetic circuit for this case is shown in Fig.4 where F is the total magnetomotive force and R_{m1} and R_{m2} the magnetic resistances of the parts 1 and 2 respectively.

From the analysis of the magnetic circuit in Fig.4 the time derivative of the core flux reveals as

$$\frac{d\Phi}{dt} = \frac{N_1}{R_{md1}+R_{md2}}\frac{di_1}{dt} - \frac{N_2}{R_{md1}+R_{md2}}\frac{di_2}{dt} \tag{9}$$

where $R_{md1}=\dfrac{1}{\mu_{d1}}\dfrac{L_{c1}}{A_{c1}}$ and $R_{md2}=\dfrac{1}{\mu_{d2}}\dfrac{L_{c2}}{A_{c2}}$.

Substituting equation (9) into (7) and the result into (6) the electric equations can be written as

$$\begin{pmatrix} v_1 \\ v_2 \end{pmatrix} = \begin{pmatrix} r_1 & 0 \\ 0 & r_2 \end{pmatrix}\begin{pmatrix} i_1 \\ i_2 \end{pmatrix} + \begin{pmatrix} L_1+\dfrac{N_1^2}{R_{md1}+R_{md2}} & \dfrac{-N_1N_2}{R_{md1}+R_{md2}} \\ \dfrac{-N_1N_2}{R_{md1}+R_{md2}} & L_2+\dfrac{N_2^2}{R_{md1}+R_{md2}} \end{pmatrix}\frac{d}{dt}\begin{pmatrix} i_1 \\ i_2 \end{pmatrix} \tag{10}$$

In equations (8) and (10), the self and mutual inductances for the windings depend on the incremental permeability μ_d, which is included in the incremental resistance. The dynamic term μ_d is updated in each time step and expresses the slope of the characteristic curve of the core material. Consequently, this term is responsible for the influence of the linear or the non-linear attribute of the core material to the behavior of the transformer. In this work the characteristic curve of the core material is selected to be a magnetization curve as shown in Fig.5.

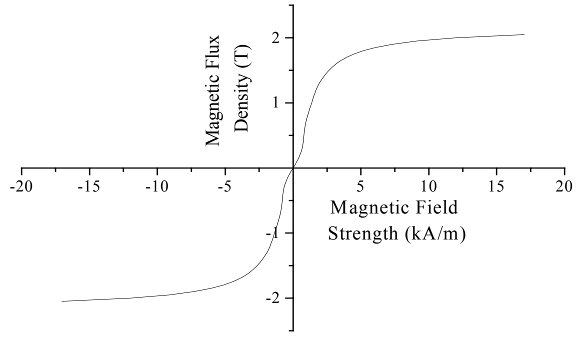


Fig.5 Magnetization curve.

3 Simulation Results

Based on the proposed core type transformer model, simulation results have been conducted for the two above-analyzed configurations of the magnetic core. Data values for these cases are included in Table 1.

In Fig.6, waveforms for the transient and in Fig.7, waveforms for the steady states for the uniform cross-section case are shown. The primary and the secondary currents as well, the core flux Φ and the trajectory on the *b-h* plane are presented. In Fig.8 and in Fig.9, the corresponding waveforms for the non-uniform cross-section case are shown where instead of the magnetic flux Φ, the *b-h* trajectories for the part-1 and the part-2 are presented. Comparing the results, it is evident that the non-uniform cross-section area of the core further affects the primary and the secondary currents waveforms. This is due to the magnitude of the core flux density of part-2, with the smaller cross-section, which passes to the saturated region on the *b-h* plane as Fig.8d and Fig.9d show.

TABLE 1
Data values for the two cases of the transformers parameters.

Windings		
	Primary	**Secondary**
Resistance [Ω]	64.2	0.0457
Leakage Inductance [mH]	243.6	0.1732
Number of Turns	4165	111
Load at Secondary		
Resistance [Ω]	2.7	
Inductance [mH]	5	
Input Voltage		
Amplitude [kV]	12.247	
Frequency [Hz]	50	
Magnetic Core		
	Case 1	**Case 2**
Length, Part 1 [cm]	97.5	97.5
Length, Part 2 [cm]	32.5	32.5
Cross-Section, Part-1 [cm^2]	52.8	52.8
Cross-Section, Part-2 [cm^2]	52.8	35.2

4 Conclusion

A core-type transformer model is presented in this work, which takes into account the core material non-linearity using the magnetization curve. This model is based on the decoupling and the direct solution of the electric and the magnetic circuits.

Two cases are considered where in the first the cross-section area of the core is uniform and in the second the cross-section is non-uniform along the core magnetic length path. Simulation results show the influence of the geometrical characteristics of the magnetic circuit on the electrical quantities. Thus, the model that is presented is a powerful tool for the representation of core type single-phase transformers and for a sensitivity analysis to various design parameters.

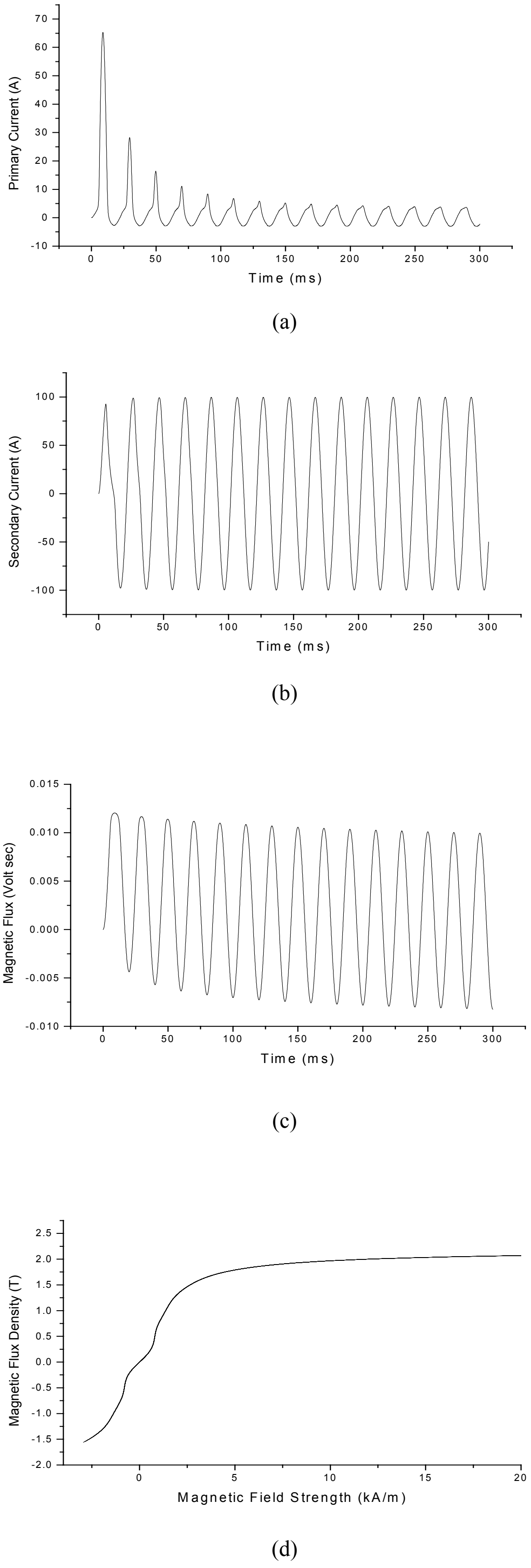
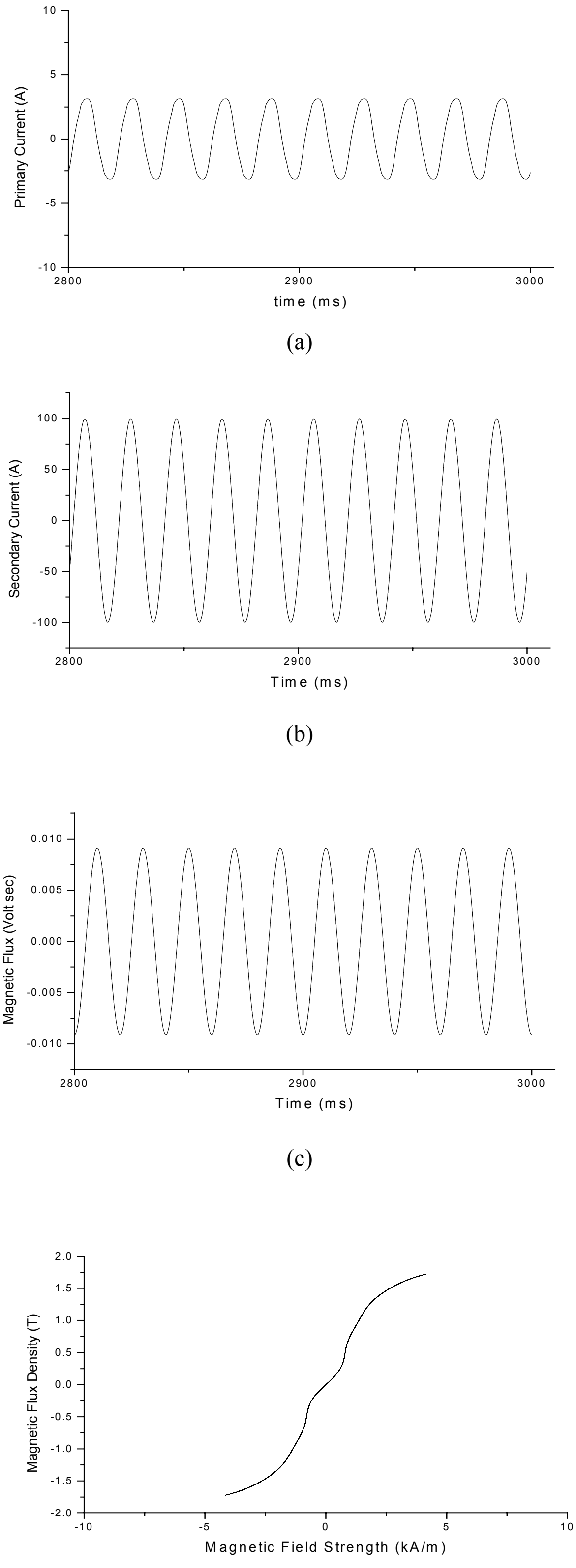


Fig.6 Transient waveforms for the transformer in the case of a uniform cross-section. a) Primary current, b) secondary current, c) magnetic flux, d) *b-h* trajectory.



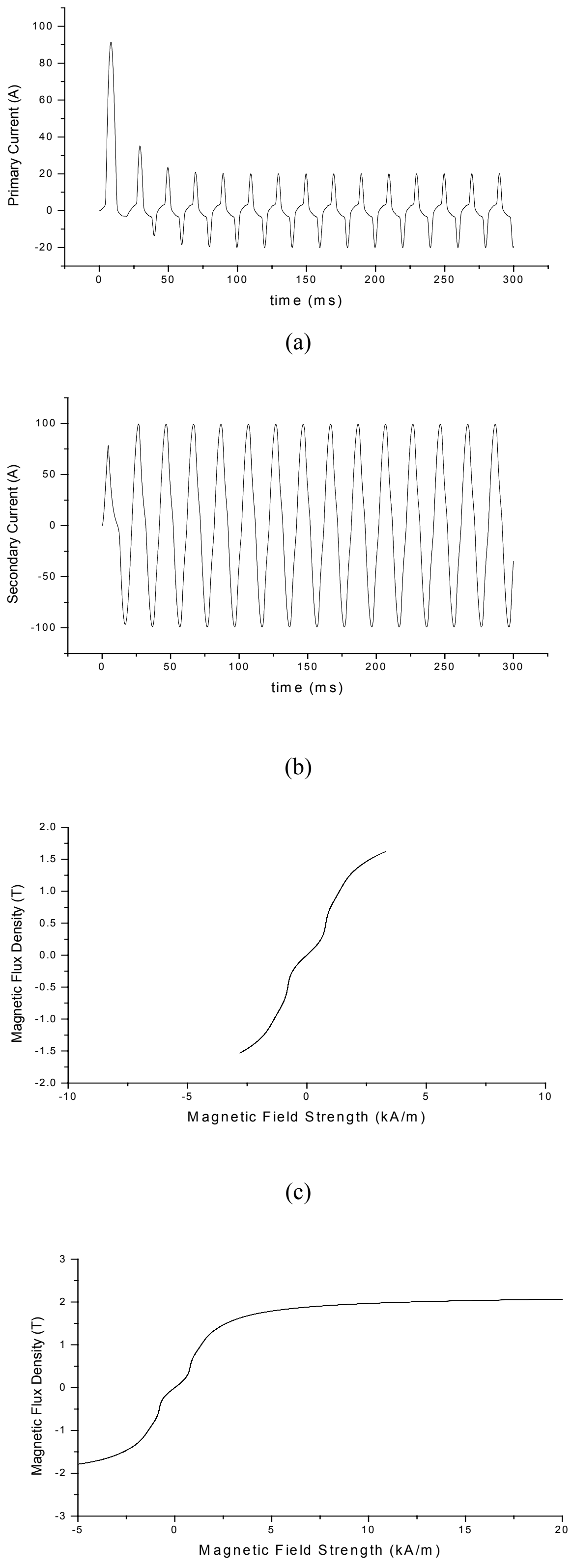
(a)

(b)

(c)

(d)

Fig.7 Steady state waveforms for the transformer in the case of a uniform cross-section. a) Primary current, b) secondary current, c) magnetic flux, d) *b-h* trajectory**.**



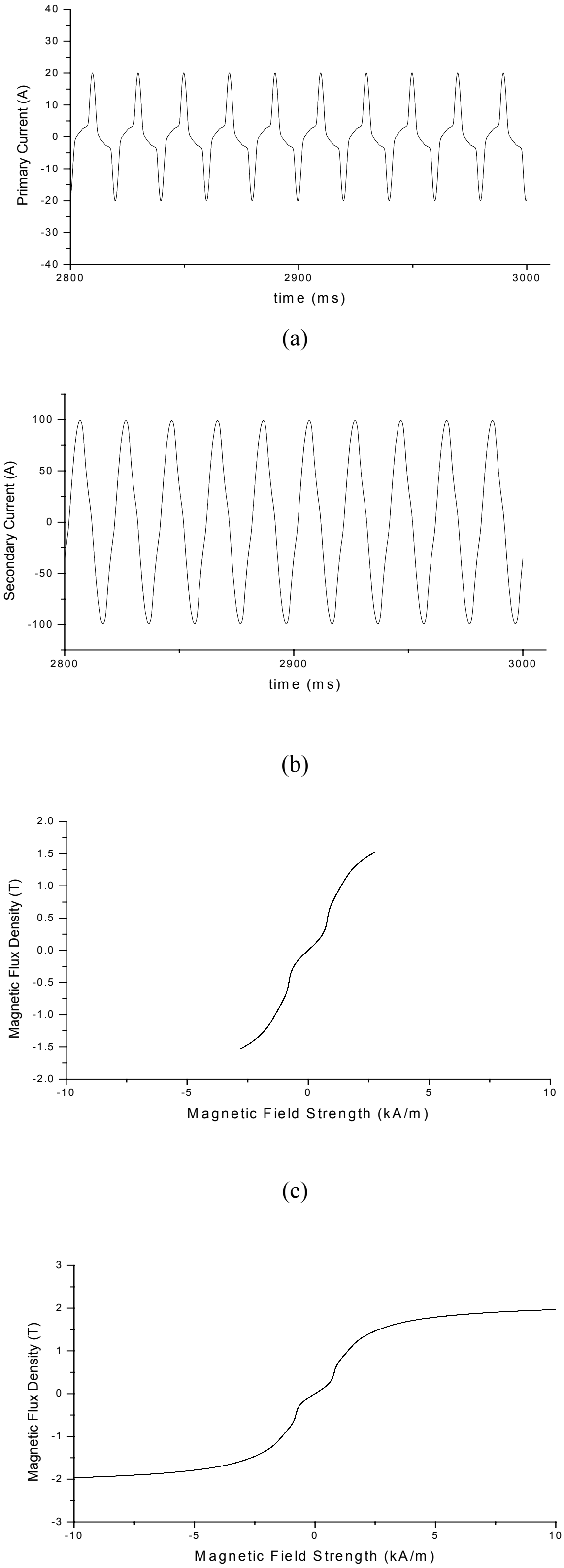
(a)

(b)

(c)

(d)

Fig.8 Transient waveforms for the transformer in the case of a non-uniform cross-section. a) Primary current, b) secondary current, c) *b-h* trajectory for part 1, d) *b-h* trajectory for part 2.



(a)

(b)

(c)

(d)

Fig.9 Steady state waveforms for the transformer in the case of a non-uniform cross-section. a) Primary current, b) secondary current, c) *b-h* trajectory for part 1, d) *b-h* trajectory for part 2.

References:

[1] P. Silvester and M. V. K. Chari, "Finite Element Solution of Saturable Magnetic Field Problems, IEEE Transactions on Power Apparatus and Systems, Vol.PAS-89, No.7 ,Sept./Oct. 1970, pp.1642-1651.

[2] M. V. K. Chari, J. D'angelo, M. A. Palmo and D. K. Sharma, "Application of Three-Dimensional Electromagnetic Analysis Methods to Elecrtical Machinery and Devices", IEEE Transactions on Energy Conversion, Vol.EC-1, No.2, June 1986, pp. 145-157.

[3] N. A. Demerdash, T. W. Nehl, F. A. Fuand and O. A. Mohammed, "Three Dimensional Finite Element Vector Potential Formulation of Magnetic Fields in Electrical Apparatus", IEEE Transactions on Power Apparatus and Systems, Vol.PAS-100, No.8, August 1981, pp. 4104-4111.

[4] N. A. Demerdash, T. W. Nehl, F. A. Fuand and O. A. Mohammed, "Experimental Verification and Application of the Three Dimensional Finite Element Magnetic Vector Potential Method in Electrical Apparatus", IEEE Transactions on Power Apparatus and Systems, Vol.PAS-100, No.8, August 1981, pp. 4112-4122.

[5] X. Chen and S. S. Venkata, " A Three-Phase Winding Core Type Transformer Model for Low Frequency Transient Studies", IEEE Transaction on Power Delivery, Vol.12, No.2, April 1997, pp. 775-782.

[6] Narag and R. H. Brierley, "Topology Based magnetic Model for Steady State and Transient Studies for Three-Phase Core Type Transformers", IEEE Transaction on Power Systems, Vol.9, No.3, August 1994, pp. 1337-1349.

[7] A. Mork, "Five Legged Wound Core Transformer Model: Derivation, Parameters, Implementation and Evaluation", IEEE Transaction on Power Delivery, Vol.14, No.4, October 1999, pp. 1519-1525.

[8] F. De Leon and A. Semlyen, "Complete Transformer Model for Electromagnetic Transients", IEEE Transactions on Power Delivery, Vol.9, No.1, January 1994, pp. 231-239.

[9] J. Pedra, L. Sainz, F. Corcoles, R. Lopez and M. Salichs, "PSPICE Computer Model Of A Nonlinear Three Phase Three Legged Transformer", IEEE Transaction on Power Delivery, Vol.19, No.1, January 2004, pp. 200-207.

[10] L. Guasch, F. Corcoles, J. Pedra and L. Sainz,, "Effects Of Symmetrical Voltage Sags On Three Phase Three Legged Transformers", IEEE Transaction on Power Delivery, Vol.19, No.2, April 2004, pp. 875-883.

[11] J. Pedra, F. Corcoles, L. Sainz and R. Lopez, " Harmonic Nonlinear Transformer Modelling", IEEE Transactions on Power Delivery, Vol.19, No.2, April 2004, pp. 884-890.

[12] J. Pedra, L. Sainz, F. Corcoles and L. Guasch, "Symmetrical And Unsymmetrical Voltage Sag Effects On Three Phase Transformers", IEEE Transaction on Power Delivery, Vol.20, No.2, April 2005, pp. 1683-1691.

[13] D. Dolinar, J. Pihler and B. Grcar, "Dynamic Model Of A Three Power Transformer", IEEE Transaction on Power Delivery, Vol.8, No.4, October 1993, pp. 1811-1819.

[14] X. S. Chen and P. Neudorfer, "Digital Model For Transient Studies Of A Three-Phase Five Legged Transformer", IEE Proceedings-C, Vol.139, No.4, July 1992, pp. 351-358.

[15] D. N. Ewart, "Digital Computer Simulation Model of A Steel Core Transformer", IEEE Transaction on Power Delivery, Vol.PWRD-1, No.3, July 1986, pp. 174-183.

[16] H. L. Nakra and T. H Barton, "Three Phase Transformer Transients", IEEE Transactions on Power Apparatus and Systems, Vol.93, No.4, 1974, pp. 1810-1818.

[17] C. Hatziantoniu, G. D. Galanos and J. Milias-Argitis, "An Incremental Transformer Model for The Study of Harmonic Overvoltages In Weak AC/DC Systems", IEEE Transactions on Power Delivery, Vol.3, No.3, July 1988, pp. 1111-1121.

[18] X. Chen, "A Three Phase Multi Legged Transformer Model In ATP Using The Directly Formed Inverse Inductance Matrix", IEEE Transaction on Power Delivery, Vol.11, No.3, July 1996, pp. 1554-1562.

[19] U. D. Annakkage, P. G. McLaren, E. Dirks, R. P. Jayasinghe and A. D. Parker, "A Current Transformer Model Based On Jiles-Atherton Theory Of Ferromagnetic Hysteresis", IEEE Transactions on Power Delivery, Vol.15, No.1, January 2000, pp. 57-61.

[20] J. Avila-Rosales, F. L. Alvarado, "Non-linear Frequency Depended Transformer Model For Electromagnetic Transient Studies In Power Systems", IEEE Transactions on Power Apparatus and Systems, Vol.PAS-101, No.11, November 1982, pp. 4281-4288.

[21] D. C. Jiles and D. L. Atherton, "Theory Of Ferromagnetic Hysteresis", Journal of Applied Physics, Vol. 55, No. 6, 15 March 1984, pp. 2115-2120.

[22] D. C. Jiles and J. B. Thoekle, "Theory Of Ferromagnetic Hysteresis: Determination Of Model Parameters From Experimental Hysteresis Loops, IEEE Transaction on Magnetics, Vol. 25, No. 5, September 1989, pp. 3928-3930.

[23] D. C. Jiles, J. B. Thoekle and M. K. Devine, "Numerical Determination Of Hysteresis Parameters For The Modeling Of Magnetic Properties Using The Theory Of Ferromagnetic Hysteresis", IEEE Transaction on Magnetics, Vol. 28, No.1, January 1992, pp. 27-35.