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# Proving the STATCOM Controllability at Arbitrary Operating Point Based on Per-unit Model

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*Abstract*：-Based on a per-unit model of STATCOM (Static Synchronous Compensator), STATCOM controllability for an arbitrary operating point is proved in this paper. First the nonlinear per-unit model of STATCOM is derived. Then, using the method of Jacobian matrix, the per-unit model of the STATCOM is linearized around a given operating point. Last, based on the linearized per-unit model, the conclusion that the STATCOM can be controlled at arbitrary operation point is obtained. The conclusion is consistent with the one based on the nominal model of STATCOM, and ensure the viewpoint that the STATCOM controllability at an arbitrary operating point is acceptable.

*Key Words*：- STATCOM, Controller, Controllability, Per-unit model, Linearization, Arbitrary operation point

## 1 Introduction

Transmission System Operators are governed by operational security standards that are applied in real time. During system disturbances, the System Operators must rely on the installed protection and control equipment, prior to human intervention. New power electronic solutions bring rapid and repeatable responses to disturbances, which will help System Operators to guarantee a stable system [1].

The STATCOM is one of the new generation flexible AC transmission systems (FACTS) devices with a promising future application, which is recognized to be one of the key advanced technologies of future power system. The technology of STATCOM improving system transmission capability has been successfully applied in power systems in developed countries, such as Japan, USA and Britain [2].

In china, electrical resources mainly centralize in the west region, while electric power consumers centralize in southern foreland where developed. The inconsistence between the geography distribution of power resources and that of consumers demands long distant and high capable power transmission. The 500KV power grid has been developed due to the requirement long distant and high capable power transmission, and long transmission distance and large power supply area are the major characteristics. At present 500 KV transmission lines play important roles in Chinese power system. The installation of STATCOM is suggested to support voltage at the middle of distant power transmission lines by the North China Power System (NCPS).

The successful application of STATCOM home and abroad means the STATCOM is mature in technique. At present the researches on STATCOM mainly concentrate on modeling and controller design to STATCOM. In fact, the STATCOM itself is an automation control system. As an automation control system, its researches include the analysis and controller design of the system [3]. Before designing a STATCOM controller, there are several open loop system properties that must be known. Controllability, observability and stability are the properties of the system presentation that must be established before any attempt in controller design is done. And paper [4] points out that system control performance can be determined by controllability, observability, etc. Plenty of work has been done on the controller design while little work has been done on the analysis of the controllability, stability and observability of STATCOM [5, 6]. Paper [7] only gave a qualitative analysis of the relationship between controllability and the controller design. Paper [8] proved the observability and stability. Although paper [8] also proved the controllability of STATCOM controller, it was limited to a given operation point. Paper [9] proved the controllability for arbitrary operation point of STATCOM based on the nominal model. To ensure the conclusion obtained from paper [9], this paper proceeds proving the controllability based on a per-unit model of STATCOM.

The main content of this paper is organized as follows: first derives the per-unit model of STATCOM. Then the per-unit model is linearized using the method of Jacobian matrix and the controllability for an arbitrary operating point of STATCOM based on per-unit model is proved. Finally, the major contribution of this paper is summarized.

## 2 Mathematical Model Derivation and Linearization

### 2.1 A Per-unit Mathematical Model Derivation [10]

We suppose the following is true during the modeling:

(1)All the losses of the device (including that of the inverter itself and the transformers) are represented by a terminal equivalent resistor R;

(2)The leakage inductance of transformers and the lead inductance are represented by a terminal equivalent inductor L;

(3)The output voltage of the inverter is a three-phase positive-sequence system, and its magnitude is in proportion with the capacitor voltage;

(4)The output high order harmonics are negligible due to the adoption of a harmonic elimination technique.

The model based on above assumptions is suitable to investigate the device performance when system voltage includes some low-order harmonics.

With the above assumption, a three-phase circuit of a STATCOM is derived as Fig.1.

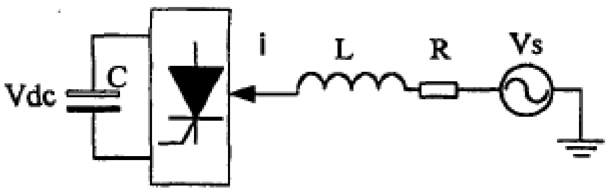


Fig.1 One line diagram of equivalent circuit of STATCOM

In p-q frame, a mathematical model of STATCOM based on real values is obtained as (1)

$$\frac{d}{dt}\begin{bmatrix} i_p \\ i_q \\ v_{dc} \end{bmatrix} = \begin{pmatrix} -\frac{R}{L} & \omega_0 & -\frac{1}{X_L}\cos\sigma \\ -1 & -\frac{R}{L} & -\frac{R}{L}\sin\sigma \\ \frac{K}{C}\cos\sigma & \frac{K}{C}\sin\sigma & 0 \end{pmatrix} \begin{bmatrix} i_p \\ i_q \\ v_{dc} \end{bmatrix} + \frac{1}{L}\begin{pmatrix} v_s \\ 0 \\ 0 \end{pmatrix} \quad (1)$$

It is obvious that all variables are represented by real physical values. In order to get a per-unit model, equation (1) is rewritten as (2)

$$\frac{d}{\omega_0 \cdot dt}\begin{bmatrix} i_p \\ i_q \\ kv_{dc} \end{bmatrix} = \begin{pmatrix} -\frac{R}{\omega_0 L} & \omega_0 & -\frac{1}{\omega_0 L}\cos\sigma \\ -1 & -\frac{R}{\omega_0 L} & -\frac{R}{\omega_0 L}\sin\sigma \\ \frac{K^2}{\omega_0 C}\cos\sigma & \frac{K^2}{\omega_0 C}\sin\sigma & \frac{1}{k}\cdot\frac{dk}{\omega_0 \cdot dt}0 \end{pmatrix} \cdot \begin{bmatrix} i_p \\ i_q \\ kv_{dc} \end{bmatrix} + \frac{1}{L}\begin{pmatrix} v_s \\ 0 \\ 0 \end{pmatrix} \quad (2)$$

The AC side inductor L can be expressed as its base frequency reactance $X_l = \omega_0 L$. Since the output line-to-line voltage of the DC/AC inverter is K times the DC side voltage, the inverter can be regarded as a voltage converter with ratio k. we can apply the impedance transformation method to the DC/AC inverter, $C_{eq} = C / K^2$ is defined as a reflected equivalent capacitor for the DC/AC side capacitor, and $X_c = \frac{1}{\omega_0 C_{eq}} = \frac{K^2}{\omega_0 C}$ is the reflected equivalent reactance of DC side capacitor. With these equivalent variables, DC side physical variables are transformed to AC side. In a three-phase AC system, following base values can be chosen.

$S_{base}$ = rating of STATCOM,

$V_{base}$ = rated system voltage,

$I_{base} = \frac{S_{base}}{\sqrt{3}V_{base}}$,

$Z_{base} = \frac{V_{base}}{\sqrt{3}I_{base}} = \frac{V_{base}^2}{S_{base}}$

Taking into account the transformation matrix, a set of base values in d-q frame is introduced.

$S_{basepq} = S_{base}$

$V_{basepq} = V_{base}$

$I_{basepq} = \sqrt{3}I_{base}$

$Z_{basepq} = Z_{base}$

The base value for time is defined as $t_{base} = 1/\omega_0$.

With the above base values, a per-unit model of STATCOM in a synchronous p-q reference frame can be given as (3)

$$\frac{d}{dt}\begin{bmatrix} i_p \\ i_q \\ v_{dc} \end{bmatrix} = \begin{pmatrix} -\frac{R}{X_l} & 1 & -\frac{1}{X_L}\cos\sigma \\ -1 & -\frac{R}{X_l} & -\frac{R}{L}\sin\sigma \\ X_c\cos\sigma & X_c\sin\sigma & \frac{1}{k}\frac{dk}{dt} \end{pmatrix} \begin{bmatrix} i_p \\ i_q \\ v_{dc} \end{bmatrix} + \frac{1}{X_L}\begin{pmatrix} v_s \\ 0 \\ 0 \end{pmatrix} \quad (3)$$

Under such base values, the DC/AC inverter will not change the per-unit values of DC side voltage and capacitor reactance. So, $V_{dc}$, $X_c$ in (3) can also be regarded as the per-unit values calculated directly from the DC side base value system.

There are two kinds of STATCOM classified by the control method, one changes $\sigma$ to regulate the output reactive power and the modulation index is fixed, it is so called PAM (pulsed Amplitude Modulation) method, the other one change $\sigma$ and K at the same time to regulate the output reactive power and keep the DC voltage constant. Equation (3) is the generalized form of the per-unit model of STATCOM. For the first type of STATCOM, a constant K leads to a more concise form (4).

$$\frac{d}{dt}\begin{bmatrix} i_p \\ i_q \\ v_{dc} \end{bmatrix} = \begin{pmatrix} -\frac{R}{X_l} & 1 & -\frac{1}{X_L}\cos\sigma \\ -1 & -\frac{R}{X_l} & -\frac{R}{L}\sin\sigma \\ X_c\cos\sigma & X_c\sin\sigma & \frac{1}{k}\frac{dk}{dt} \end{pmatrix} \begin{bmatrix} i_p \\ i_q \\ v_{dc} \end{bmatrix} + \frac{1}{X_L}\begin{pmatrix} v_s \\ 0 \\ 0 \end{pmatrix} \quad (4)$$

According to the above per-unit mathematical model of STATCOM, a per-unit equivalent circuit of STATCOM in the p-q synchronous frame is given in Fig.2.

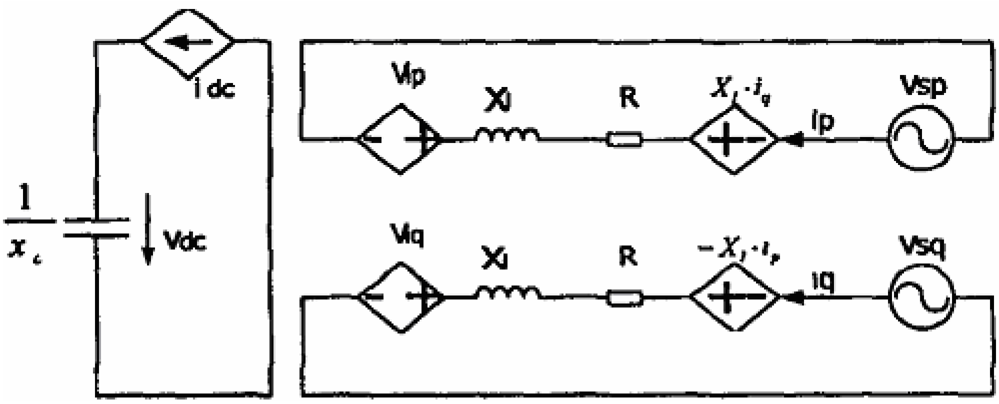


Fig.2 The per-unit equivalent circuit of STATCOM in p-q reference frame

## 2.2 Model Linearization

From the STATCOM mathematic model (4), it can be easily seen that STATCOM is a nonlinear system. To analyze the performance of STATCOM easily, equation (4) must be linearized to linear equations using the method of Jacobian matrix.

For the nonlinear system (4), its vector function form can be expressed as $\dot{\vec{X}} = \vec{f}(\vec{x}\,,\ \vec{u})$, which means:

$$\dot{X} = \vec{f}(\vec{x}\,,\ \vec{u}) = \frac{d}{dt}\begin{bmatrix} i_p \\ i_q \\ v_{dc} \end{bmatrix}$$

$$= \frac{1}{X_L}\begin{pmatrix} v_s \\ 0 \\ 0 \end{pmatrix} + \begin{bmatrix} i_p \\ i_q \\ v_{dc} \end{bmatrix}$$

$$\begin{pmatrix} -\frac{R}{L} & 1 & -\frac{1}{X_L}\cos\sigma \\ -1 & -\frac{R}{L} & -\frac{R}{L}\sin\sigma \\ X_c\cos\theta & X_c\sin\sigma & 0 \end{pmatrix}$$

here, $i_p$、$i_q$、$v_{dc}$ is the state variables and $\delta$ is the control variable.

because $\dot{\vec{X}} = \vec{f}(\vec{x}, \vec{u})$ is the function to be linearized and $f(x) \in R^{3\times1}$, then its Jacobin matrixes are in the form as:

$$\frac{\partial \vec{f}}{\partial \vec{X}} = \begin{bmatrix} \frac{\partial f_1}{\partial x_1} & \frac{\partial f_1}{\partial x_2} & \frac{\partial f_1}{\partial x_3} \\ \frac{\partial f_2}{\partial x_1} & \frac{\partial f_2}{\partial x_2} & \frac{\partial f_2}{\partial x_3} \\ \frac{\partial f_3}{\partial x_1} & \frac{\partial f_3}{\partial x_2} & \frac{\partial f_3}{\partial x_3} \end{bmatrix}, \frac{\partial \vec{f}}{\partial \vec{u}} = \begin{bmatrix} \frac{\partial f_1}{\partial u} \\ \frac{\partial f_2}{\partial u} \\ \frac{\partial f_3}{\partial u} \end{bmatrix}$$

by using above method of Jacobin matrix, STATCOM per-unit model can be linearized around the operating points of $\vec{X}^0$ and $\vec{U}^0$, where,

$$\vec{X}^0 = \begin{bmatrix} i_{p_0} \\ i_{q_0} \\ V_{dc_0} \end{bmatrix}, \vec{U}^0 = (\sigma_0)$$

And the Jacobin matrixes are:

$$A = \frac{\partial \vec{f}}{\partial \vec{X}}\bigg|_{(\vec{X}^0, \vec{U}^0)}$$

$$= \begin{pmatrix} -\frac{R}{L} & 1 & -\frac{1}{X_L}\cos\sigma_0 \\ -1 & -\frac{R}{L} & -\frac{R}{L}\sin\sigma_0 \\ X_c\cos\sigma_0 & X_c\sin\sigma_0 & 0 \end{pmatrix} \quad (5)$$

$$B = \frac{\partial \vec{f}}{\partial \vec{u}}\bigg|_{(\vec{X}^0, \vec{U}^0)} = \begin{bmatrix} v_{dc0}\frac{R}{L}\sin\sigma_0 \\ -\frac{1}{X_L}\cos\sigma_0 \\ -X_c i_{p_0}\sin\sigma_0 + X_c i_{q0}\cos\sigma_0 \end{bmatrix} \quad (6)$$

## 3 Controllability Proving Based on Per-unit Model

For the convenience of proving and without influencing the proving results, we suppose the following component parameters are:

$R = 1$, $L = 1$, $X_L = 1$, $X_c = 1$

And the arbitrary operating point is expressed by $m$, $n$, $\alpha$ separately ($m > 0$, $n > 0$, $0 \le \alpha \le \frac{\pi}{2}$), that is:

$i_{p_0} = 0$, $i_{q_0} = m$, $v_{dc_0} = n$, $\sigma_0 = \alpha$ ($i_{p_0}$ and $i_{q_0}$ are orthogonal)

From (5) and (6), the coefficient matrixes are:

$$A = \begin{bmatrix} -1 & 1 & -\cos\alpha \\ -1 & -1 & -\sin\alpha \\ \cos\alpha & \sin\alpha & 0 \end{bmatrix},$$

$$B = \begin{bmatrix} n\sin\alpha \\ -n\cos\alpha \\ -l\sin\alpha + m\cos\alpha \end{bmatrix}$$

Then, controllability matrix $M$ is

$$M = \begin{pmatrix} B & AB & A^2B \end{pmatrix} =$$

$$\begin{bmatrix} n\sin\alpha & -n\sin\alpha - n\cos\alpha - m\cos^2\alpha & 2n\cos\alpha + m\cos^2\alpha - m\sin\alpha\cos\alpha \\ -n\cos\alpha & -n\sin\alpha + n\cos\alpha - m\sin\alpha\cos\alpha & 2n\sin\alpha + m\cos^2\alpha + m\sin\alpha\cos\alpha \\ m\cos\alpha & 0 & -n - m\cos\alpha \end{bmatrix}$$

And

$$\|M\| = n\sin\alpha \cdot (n^3\sin\alpha + mn^2\sin\alpha\cos\alpha$$
$$+3nm^2\cos^2\alpha\sin\alpha$$
$$+m^3\cos^3\alpha\sin\alpha + 3n^2 m\sin\alpha\cos\alpha)$$

For $m > 0$, $n > 0$, $0 \le \alpha \le \frac{\pi}{2}$, then, $\|M\| > 0 \ne 0$, that is $rankM = 3$

$rankM = 3$ means that the M matrix has full rank, that is STATCOM is controllable at arbitrary operating point .

## 4 Conclusions

STATCOM controllability at arbitrary operation point is proved based on per-unit model. Proved result is consistent with the one based on the nominal model，which means the conclusion that STATCOM can be controlled at arbitrary operation point is correct. It also gives the theory base for STATCOM controller design.

## Acknowledgment

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*References:*

[1] Baker, M.H., Gemmell, B.D., Horwill, C., Hanson, D.J., STATCOM helps to guarantee a stable system, *Transmission and Distribution Conference and Exposition, 2001 IEEE/PES,* vol.2, 2001, pp:1129 - 1132

[2] Qingguang Yu, Pei Li, Wenhua Liu, Xiaorong Xie, Overview of STATCOM technologies, *Proceedings of the 2004 IEEE International Conference on Electric Utility Deregulation, Restructuring and Power Technologies,* Vol.2, 5-8 April 2004,pp:647- 652

[3] Gao Guoshen and Yu Wenxiao, Automation. control principle, *Huanan University of Technology press (China),* 2003.

[4] Larsen EV, Sanchez-Gasca J J, Chow J H. Concept for Design of FACTS Controller to Damp Power Swings, *IEEE Trans on PWRS,* Vol.10,No.2,1995, pp:948-956.

[5] Sekoguchi, M., Konishi, H. etc. Nonlinear optimal control applied to STATCOM for power system stabilization, *IEEE/PES Transmission and Distribution Conference and Exhibition,* Vol.1, 2002, pp:342 - 347

[6] Yonezawa, H., Shimato, etc. study of a STATCOM application for voltage stability evaluated by dynamic PV curves and time simulations, *IEEE Power Engineering Society Winter Meeting,* Vol.2, 2000, pp:1471-1476

[7] Li Chun, Jiang Qirong, Wang Zhonghong, Lichengye, Input Signal Selection For the Damping Controller of ±20 Mvar STATCOM, *Power System Automation (China),* Vol.12,2000, pp:50-53

[8] Li Qunxing. A Comparison of Pole Assignment & LQR Design Methods for Multivariable Control for STATCOM, *The Florida State University College of Engineering,* 2003.

[9] Zhang Liuchun, Xiao Dengming, Zhang Dong, Proving the Controllability for Arbitrary Operating Point of STATCOM, *the WSEAS Int. conf. on Mathematical Methods and Computational Techniques in Electrical Engineering (MMACTEE'05)* (Accepted).

[10]Dong Shen, Xu Liang, Yingduo Han, A Modified Per-unit STATCOM Model and Analysis of Open Loop Response Time, *IEEE Power Engineering Society Winter Meeting.* Vol. 4, 2000, pp: 2624- 2629.