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Ranking Income Distributions in Poland

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Abstract: - Comparing income distributions from a point of view of social welfare we must take into consideration both the level of average income and the level of inequality. Income inequality can be compared by means of the well known Lorenz curves but the results will be ambiguous when the Lorenz functions for the considered population intersect. Quantile functions and generalized Lorenz curves are more useful tools for ranking income distributions but they are also more restrictive. In many situations it is necessary to make additional assumptions concerning social preferences which can be reflected in a social welfare function. In the paper we present the results of the application of the methods mentioned above to the analysis of income distributions in Poland. As a theoretical distribution the Dagum type-I model was used.

Key-Words: - Income distribution, Income inequality, Social welfare, Stochastic dominance.

1 Introduction

In the period of transformation from a centrally-planned to a market economy the problem of evaluation the changes in social welfare seems very important. It is connected with relatively high economic growth what implicates increasing income inequality. The ranking procedures presented in the paper, applied to the theoretical income distributions well fitted to the data, can be a reliable source of information on economic situation of different social groups in Poland.

2 Ranking procedures

In order to rank income distributions from a point of view of social welfare the procedures for dominance relations based on Lorenz curves, generalized Lorenz curves or quantile functions can be very useful. They enable us to order distributions without dividing the process into two stages, first comparing the degree of inequality within each distribution and then introducing information on mean income.

Let us suppose that a social ordering of income distributions can be represented by the following welfare function:

$$W = \int_0^\infty U(y) f(y) dy$$

where: $f(y)$ denotes a density function of income and $U(y)$ represents an utility function of income, usually assumed to be increasing and concave.

The partial ordering of income distributions can be based on the following theorem (Atkinson [1]).

Theorem 1 Let $f_A(y)$ and $f_B(y)$ denote the density functions of income distributions *A* and *B*, $L_A(p)$ and $L_B(p)$ their corresponding Lorenz curves, μ_A and μ_B their mean incomes. For any strictly concave utility function $U(y)$:

If $\mu_A = \mu_B$ then

$$\int_0^\infty U(y) f_A(y) dy \geq \int_0^\infty U(y) f_B(y) dy \Leftrightarrow L_A(p) \geq L_B(p)$$

for all $p \in \langle 0,1 \rangle$.

The distribution *A* dominates *B* if and only if the Lorenz curve for *A* lies above the Lorenz curve for *B*. When the curves intersect it is impossible to make decision without further assumptions on a utility function.

Better ordering tools can be based on generalized Lorenz curves obtained by scaling up the ordinary Lorenz curve by the mean income (Shorrocks [6]). They enable us to compare distributions with different means, taking into account the efficiency preference. The generalized Lorenz dominance criterion is equivalent to the second-order stochastic dominance.

Theorem 2 Let $GL_A(p)$ and $GL_B(p)$ denote generalized Lorenz curves corresponding to the density functions $f_A(y)$ and $f_B(p)$. For any increasing and strictly concave utility function:

$$\int_0^\infty U(y)\, f_A(y)dy \geq \int_0^\infty U(y)\, f_B(y)dy \Leftrightarrow GL_A(p) \geq GL_B(p)$$

for all $p \in \langle 0,1 \rangle$.

The theorem 2 provides the ordering of income distributions with different means on condition that generalized Lorenz curves do not intersect. For complete ordering, a cardinal social welfare function that provides numerical values to all possible social states could be useful.

More basic and less restrictive dominance principle, based on strong Pareto law, was proposed by Saposnik [5]. It is called rank dominance and is equivalent to the first – order stochastic dominance.

Theorem 3 Let $Y_A(p)$ and $Y_B(p)$ denote the quantile functions of income distributions *A* and *B*. For any increasing and anonymous welfare function *W*:

$$\int_0^\infty U(y)\, f_A(y)dy \geq \int_0^\infty U(y)\, f_B(y)dy \Leftrightarrow Y_A(p) \geq Y_B(p)$$

for all $p \in \langle 0,1 \rangle$.

Bishop, Formby and Thistle [2] showed that much of power contained in generalized Lorenz dominance criterion is contained in comparisons of quantile functions.

3 Income distribution model

In many situations it seems reasonable to use theoretical income distributions, which show high consistency with the empirical ones. First, such an approach enables for the flattening of irregularities in empirical distributions, coming from the method of gathering information. Second, the use of the theoretical model simplifies and accelerates the analysis because all distribution characteristics can be expressed by the same parameters. Moreover, the maximum likelihood and ordinary least squares estimates of inequality measures can be provided easily, given the mathematical form of a density function or a cumulative distribution function.

A variety of probability functions has been suggested as suitable in describing the distributions of income by size. The lognormal distribution has been widely used in wage and income distribution analysis for many years. The advantage of this distribution is its simplicity; a disadvantage, however, is its poor fitting to the data, especially in the tails.

Unlike the lognormal, the Dagum model was based on empirical observations of income distributions made in many countries. Dagum [3] and Dagum and Lemmi [4] noted that the function describing income elasticity of a cumulative distribution function of income is convex, decreasing and bounded. It can be described by the following differential equation:

$$\varepsilon(y, F(y)) = \frac{d \ln F(y)}{d \ln y} = \beta_1 \left[1 - [\, f(y)^{\beta_2}\,]\right], \qquad (1)$$

for $y \geq 0, \beta_1, \beta_2 > 0$.

The cumulative distribution function of the Dagum model is the solution of the equation given by formula (1). It can be written as follows:

$$F(y) = (1 + \lambda y^{-\delta})^{-\delta}, \; y > 0 \qquad (2)$$

for $\beta, \lambda, \delta > 0$,

where: $\beta = 1/\beta_1$, $\delta = \beta_1 \beta_2$, $\lambda = \exp c$,

c – a constant of integration resulting from the solution of equation (2),

Parameters β and δ are inequality parameters of the Dagum distribution while λ is a parameter of scale.

The moments of order *r* about the origin corresponding to the model (2) known as Dagum type I distribution, are specified by the equation:

$$\mu_r = \beta \lambda^{r/\delta} B(1 - r/\delta, \beta + r/\delta) \text{ for } r < \delta \qquad (3)$$

where: $B(1 - r/\delta, \beta + r/\delta)$ - the beta function with parameters $(1 - r/\delta, \beta + r/\delta)$.

It follows from equation (3) that the moments of order *r* exist only for $r < \delta$. Hence, the moments of orders $r \geq \delta$ are infinite.

The Lorenz curve corresponding to the cumulative distribution function (2) can be written as follows:

$$L(p) = B^*\left[p^{1/\beta}; \beta + 1/\delta, 1 - 1/\delta\right] \quad (4)$$

for $\delta > 1, 0 \le p \le 1$,

where: $B^*\left[p^{1/\beta}; \beta + 1/\delta, 1 - 1/\delta\right]$ - the incomplete beta function.

The Gini concentration coefficient obtained on the basis of equation (4) has the form:

$$G = -1 + B(\beta, \beta) / B(\beta, \beta + 1/\delta) \quad (5)$$

where: $B(.,.)$- the beta function,

4 Application of the method

The methods mentioned above were applied to the analysis of per capita family income in Poland by socio-economic groups in the years 1999 and 2003. The basis for the calculations was continuous data obtained from the Household Budgets Survey, conducted quarterly by Polish Central Statistical Office. Household groups representing the basic socio-economic groups of the population were established on the basis of the exclusive or primary source of maintenance. The parameters of the Dagum model were estimated by means of the maximum likelihood method. To find the maximum of the logarithm of the likelihood function an individual numerical procedure has been applied. The results of the estimation are presented in table 1. The table contains estimated values of the Dagum model parameters as well as the statistical characteristics – mean income and Gini concentration ratio. These characteristics, being the functions of the Dagum model parameters, can also be treated as maximum likelihood (ML) estimates of the corresponding population values. It can be easily noticed that for almost all the distributions under consideration the higher mean in 2003 was connected with higher inequality. Only for the households of employees-farmers the inequality diminished. The estimates of λ, β and δ were then used to calculate the Lorenz, generalized Lorenz and quantile function ordinates for different socio-economic groups. Figures 1-3 show $L(p), GL(p)$ and $F^{-1}(p)$ for the households of self-employed in the years 1999 and 2003. The distributions cannot be ranked on the basis of the Lorenz criterion (fig. 1) because the mean incomes differ significantly and the greater mean is accompanied by greater inequality. Figures 2 and 3 show that the income distribution in 2003 dominates the distribution in 1999 according to the both generalized Lorenz and rank dominance criteria. Similar results were obtained for the remaining socio-economic groups. It drives to the conclusion that the differences between inequality levels in the years 1999 and 2003 were relatively small compared with variations in mean incomes.

Table 1. ML Estimates for the Dagum Model

Household type		Dagum model parameters			Statistical characteristics	
		λ	β	δ	Mean income (in 100 PLN)	Gini ratio
Employees	1999	0,1419	1,3405	2,8840	7,116	0,3277
	2003	0,2857	1,3716	2,5954	9,306	0,3641
Employees farmers	1999	0,1000	0,7918	3,4568	5,381	0,3071
	2003	0,1023	1,2345	3,2674	6,355	0,2926
Farmers	1999	0,1080	1,4483	2,0125	6,438	0,4715
	2003	0,1000	1,7449	1,9368	6,929	0,4810
Self-employed	1999	0,2799	1,2409	2,6655	8,773	0,3602
	2003	0,4121	1,3949	2,1613	11,171	0,4396
Retirees	1999	0,2401	0,8667	4,7133	7,619	0,2209
	2003	0,5718	0,9649	4,2458	9,506	0,2377
Pensioners	1999	0,1467	0,6339	4,1080	5,856	0,2787
	2003	0,3945	0,6132	3,9833	7,315	0,2902
Maintained from non-earned sources	1999	0,1000	0,6077	2,7273	4,204	0,4135
	2003	0,2105	0,7363	2,5522	6,038	0,4188

Source: Authors' calculations.

Fig.1. Lorenz curves for per capita income in the households of self-employed

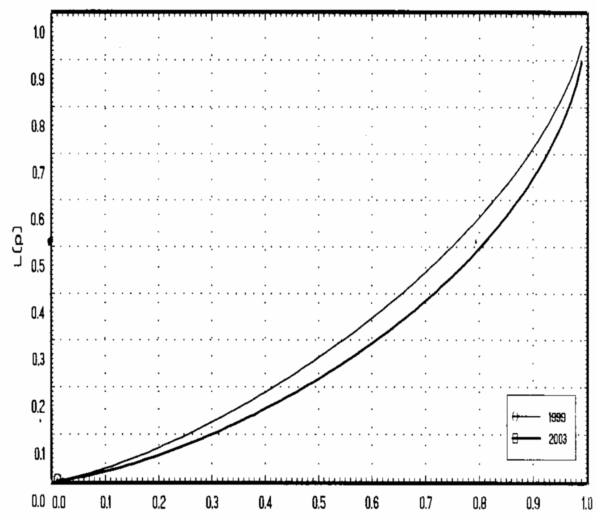


Fig.2. Quantile functions for per capita income in the households of self-employed

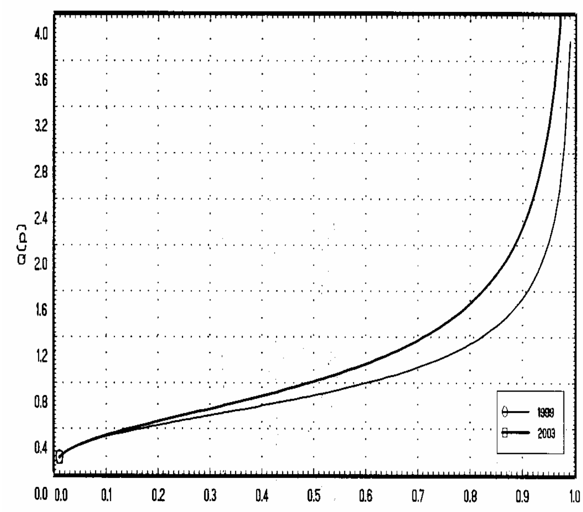
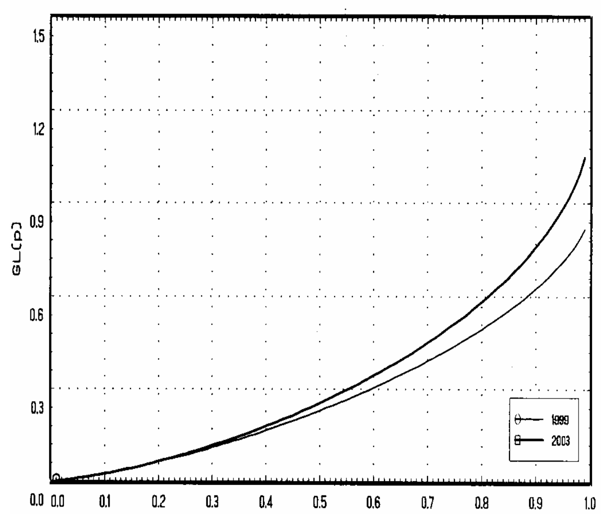


Fig.3. Generalized Lorenz curves for per capita income in the households of self-employed



5 Conclusion

Ranking of income distributions based on social welfare functions can be very useful in the analysis of wage an income distributions in Poland. The period of economic transformation (since 1990) was characterized by a series of fundamental changes in economy. The changes influenced, among other things, income and earnings distribution by size. It was connected, on one hand, with greater possibilities of economic activity of different social groups, and on the other hand, with a growing polarization of personal income. Assuming more or less general form of social welfare function it is possible to find compromise between efficiency and equity preference of a population of income receivers. It is worth mentioning that it would be advisable to investigate also non-income factors of social welfare (better education, health, standard of living) to complete the analysis.

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