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PSPICE modeling of Buck Converter by means of GTFs

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Abstract: The paper describes a way how to model the DC-DC buck converter by means of the so-called generalized transfer functions, utilizing both the Laplace and the *z*-transform operators. The resulting line-to-output and control-to-output transfer functions and the corresponding frequency responses can model some system behavior more truly than the classical *s*-domain averaged models, including special effects above the Nyquist's frequency. As an example, a concrete SPICE subcircuit is shown together with the results of AC analysis.

Keywords: - DC-DC converter, buck converter, GTF

1 Introduction

The averaging approach is commonly used for fast and economical analysis of switched DC-DC converters [1]. A drawback of these methods consists in the frequency limitation to the Nyquist's frequency $f_{switch}/2$. Near to and above this limit, the simulation ceases corresponding to reality.

For a more complex description of the dynamic features and for monitoring the stability, special approximate methods were developed [2] which model the influence of switching processes on the loop gain, particularly in the frequency region in the vicinity of Nyquist's frequency. However, these methods are based on continuous-time modeling by means of the s-domain transfer functions. That is why the processes above $f_{switch}/2$ are not modeled properly.

In this paper, modeling on the basis of mixed *s*-*z* description will be described, which overcame the above difficulties. The switched DC-DC converter is regarded as an analog linear time-varying system, where the externally controlled switches are modeled by time-varying resistances. To model such systems, the so-called generalized transfer functions (GTFs) [3] are applied. GTFs are functions of two well-known s and *z* operators, simultaneously describing both the continuous-time and the discrete-time behavior of switched converter.

This study includes a demonstration of the GTF approach for line-to-output transfer functions of buck switched converter operating in the continuous current mode (CCM). However, it is also valid for other types of switched DC-DC converters. The first description of this method including its programming in MATLAB was given in [4]. Here some extensions are made, which result in compiling the SPICE macromodel.

2 GTF of switched DC-DC converters

The well-known topology of buck converter and the switching diagram in Fig. 1 represent a two-phase switched circuit, when the active switch S causes by its ON/OFF operation inverse states of the passive switch D, i.e. OFF/ON. The relative duration of the ON state of the active switch with respect to the switching period is called duty ratio (D). This quantity is usually controlled in dependence on the output voltage, which is then stabilized via pulse width modulation.

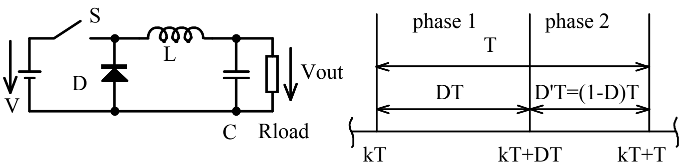

Fig. 1: Buck converter and its switching diagram.

After neglecting the nonlinear operation of switching devices, the converter can be described in each of its switching phases by a linear model with a dominant couple of state variables – the voltage across the capacitor and the inductor current.

Let us consider the following assumptions: linear model in each switching phase, continuous conducting mode, and constant duty ratio. Input voltage V will have a DC component V_0 and a superposed AC component $\hat{v}(t)$. The aim of the analysis is to obtain the frequency dependence of the AC component of the output voltage and the other observed signals without limitation of the bandwidth of signal $\hat{v}(t)$.

Due to the switching processes, the circuit variables of switched converters are not smooth time-domain functions. According to the theory of generalized transfer functions [3], these signals can be represented by smooth, so-called equivalent signals, which fulfill the following two conditions: (1) the original and equivalent signals have identical values at the sampling instances we are interested in (mostly on the boundaries of switching phases), (2) the bandwidth of the equivalent signals coincides with the bandwidth of the input signal. The first condition states that the equivalent signals form abstract envelopes of real signals. In the case of negligible influence of switching processes, there is a good agreement between the real signals and their envelopes. The second condition enables comparing the equivalent signals and the input signal in the frequency domain by means of linear theory. In other words, it enables defining the transfer functions.

Let us observe the converter state by means of vector **X**, which consists – in the simplest case – of a couple of state variables V_C and I_L. All the remaining circuit currents or voltages can be obtained by a linear combination of these state variables and the input voltage. In the following, we will assume a continuity of state variables both in the frame of switching phases and at transition instances between them.

Let us introduce the following notation:

$\mathbf{G}_1$, $\mathbf{G}_2$… matrices of natural responses of the converter within phase 1 and phase 2, respectively, defined on the assumption of zero-input signal v:

$$\mathbf{X}(kT+DT)=\mathbf{G}_1\mathbf{X}(kT)\,,$$
$$\mathbf{X}(kT+T)=\mathbf{G}_2\mathbf{X}(kT+DT)\,, \tag{1}$$

$\mathbf{g}_1$, $\mathbf{g}_2$…matrices of converter impulse responses within phase 1 and phase 2, respectively, defined on the assumption of input signal v as the Dirac impulse, operating at the beginning of switching phase,

$\mathbf{H}_1$, $\mathbf{H}_2$… vectors of forced responses of the converter to constant input signal V_0 within phase 1 and phase 2, respectively, considering zero initial state:

$$\mathbf{X}(kT+DT)=\int_0^{DT}\mathbf{g}_1(\xi)d\xi\, V_0=\mathbf{H}_1V_0\,,$$
$$\mathbf{X}(kT+T)=\int_0^{D''T}\mathbf{g}_2(\xi)d\xi\, V_0=\mathbf{H}_2V_0 \tag{2}$$

Taking into account the above notations and assumptions, the switched converter can be described by the following equations:

End of switching phase 1 – $t = kT+DT$:

$$\begin{aligned}&\mathbf{X}(kT+DT)=\\&=\mathbf{G}_1\mathbf{X}(kT)+\int_0^{DT}\mathbf{g}_1(DT-\xi)v(kT+\xi)d\xi\,.\end{aligned} \tag{3}$$

End of switching phase 2 – $t = kT+T$:

$$\begin{aligned}&\mathbf{X}(kT+T)=\mathbf{G}_2\mathbf{X}(kT+DT)+\\&+\int_0^{D'T}\mathbf{g}_2(D'T-\xi)v(kT+DT+\xi)d\xi\,.\end{aligned} \tag{4}$$

Now consider the input voltage as a sum of DC and AC components:

$$v=V_0+\hat{V}e^{st}\,, s=j\omega\,. \tag{5}$$

As a consequence of circuit linearity, the steady-state vector **X** will be also composed of DC and AC terms:

$$\mathbf{X}=\mathbf{X}_0+\hat{\mathbf{X}}e^{st}\,. \tag{6}$$

Taking into account equations (5) and (6), the following results can be derived from (3) and (4):

$$\begin{aligned}\mathbf{X}_0&=(\mathbf{I}-\mathbf{G}_1\mathbf{G}_2)^{-1}[\mathbf{G}_1\mathbf{H}_2+\mathbf{H}_1]\,V_0=\\&=(\mathbf{I}-\mathbf{G}_2\mathbf{G}_1)^{-1}[\mathbf{G}_2\mathbf{H}_1+\mathbf{H}_2]\,V_0\end{aligned} \tag{7}$$

$$\hat{\mathbf{X}}_1=(\mathbf{I}-\mathbf{G}_1\mathbf{G}_2z^{-1})^{-1}[\mathbf{G}_1\hat{\mathbf{H}}_2(s)z^{-D}+\hat{\mathbf{H}}_1(s)]\hat{V} \tag{8}$$

$$\hat{\mathbf{X}}_2=(\mathbf{I}-\mathbf{G}_2\mathbf{G}_1z^{-1})^{-1}[\mathbf{G}_2\hat{\mathbf{H}}_1(s)z^{-D'}+\hat{\mathbf{H}}_2(s)]\hat{V} \tag{9}$$

Equations (7) describe two equivalent ways of computing the state vector in DC steady state, whereas equations (8) and (9) represent state vectors of equivalent state signals of the converter, which correspond to the state variables after their sampling at the end of phases 1 ($\hat{\mathbf{X}}_1$) and phases 2 ($\hat{\mathbf{X}}_2$), respectively. In addition to the operator of the z-transform $z=e^{sT}$, the following quantities figure in the equations:

$$\hat{\mathbf{H}}_1(s)=\int_0^{DT}\mathbf{g}_1(t)e^{-st}dt\,,\ \hat{\mathbf{H}}_2(s)=\int_0^{D'T}\mathbf{g}_2(t)e^{-st}dt \tag{10}$$

and are the running Laplace transforms of converter impulse responses within phases 1 and 2, respectively. These functions generate s-domain poles of converter frequency responses.

Note that equations (7) for DC steady state directly result from equations (8), (9), (10) and (2) for $s = 0$.

Equations (8) a (9) describe generalized transfer functions of switched converter. Applying the double substitution $s = j\omega$, $z = e^{j\omega T}$ yields line-to-output frequency responses for an arbitrary bandwidth, i.e. without limitation to one half of the switching frequency, as is usual for the procedures published so far.

3 Modeling of buck converter

In the case of the buck converter in Fig. 1, the above modeling can be simplified. During switching phase 2, there is no connection between the input signal and the state variables. That is why matrix $\mathbf{g}_2$ and two vectors $\mathbf{H}_2$ and $\hat{\mathbf{H}}_2$ are zero.

To compile a PSPICE macromodel of buck converter, all components of Eqs. (8) and (9) were derived symbolically with the aid of the SNAP program [5], and then they were written in the SPICE language. The resulting subcircuit is given in Fig. 2. It models the GTF defined by Eq. (8). The modeling was based on the following specification of the converter in Fig. 1:

R_{on} of both the active and the passive switches are identical.

ESR resistances are considered both for the capacitor (R_C) and for the inductor (R_L).

To compute the transfer from the input V to the output V_{out}, the above equations must be completed with the output equation for buck converter:

$$V_{out} = (V_L + R_C I_L)/(1 + R_C / R_{load}) \tag{11}$$

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*subcircuit for modeling line-to-output transfer function of buck converter based on GTF
*
.subckt buckLTO_GTF in out params: fs=100k duty=0.543 Ron=10m RL=50m Rc=0.1 Rload=3 L=50u C=500u

.param duty2={1-duty} R1={Ron+RL} om0={sqrt((R1+Rload)/L/C/(Rc+Rload))}
+ B={Rc*Rload/(Rc+Rload)/L+R1/L+1/C/(Rc+Rload)} A={B/2} be={sqrt(om0^2-A^2)}
+ Kc={Rload/L/C/(Rc+Rload)} KL={1/L} omz={1/C/(Rc+Rload)} ce={(omz-A)/be} si {sin(be*duty/fs)}
+ si2 {sin(be*duty2/fs)} co {cos(be*duty/fs)} co2 {cos(be*duty2/fs)} ex {exp(-A*duty/fs)} ex2 {exp(-A*duty2/fs)}
* coordinates of matrix G1
+ gcc1 {(co-A*si/be)*ex+ex*si*(Rload*Rc+R1*Rc+R1*Rload)/L/be/(Rc+Rload)} gcl1 {ex*si*Rload/be/C/(Rc+Rload)}
+ glc1 {-gcl1*C/L} gll1 {gcl1/Rload+(co-A*si/be)*ex}
* coordinates of matrix G2
+ gcc2 {(co2-A*si2/be)*ex2+ex2*si2*(Rload*Rc+R1*Rc+R1*Rload)/L/be/(Rc+Rload)}
+ gcl2 {ex2*si2*Rload/be/C/(Rc+Rload)}
+ glc2 {-gcl2*C/L}
+ gll2 {gcl2/Rload+(co2-A*si2/be)*ex2}
**product G=G1*G2
+ g11 {gcc1*gcc2+gcl1*glc2} g12 {gcc1*gcl2+gcl1*gll2} g21 {glc1*gcc2+gll1*glc2} g22 {glc1*gcl2+gll1*gll2}
*s-domain transfer function in phase 1, output Vc
Econtc contc 0 LAPLACE {V(in)} {Kc/((s+A)^2+be^2)}
*s-domain transfer function in phase 1, output IL
Econtl contl 0 LAPLACE {V(in)} {Kl*(s+omz)/((s+A)^2+be^2)}
*first term of vector H1
Egc gc 0 LAPLACE {V(contc)} {1-exp(-s*duty/fs)*ex/be*((s+A)*si+be*co)};
*second term of vector H1
Egl gl 0 LAPLACE {V(contl)} {1-exp(-s*duty/fs)*ex*(s*(co+ce*si)+(A+be*ce)*co-(be-A*ce)*si)/(s+omz)}
*final programming of Eqs. (8) and (11)
Eout1 out1 0 LAPLACE {V(gc)}
+{(1-(g22-Rc*g21)*exp(-s/fs))/(1+Rc/Rz)/(1-(g11+g22)*exp(-s/fs)+(g11*g22-g12*g21)*exp(-2*s/fs))}
Eout2 out out1 LAPLACE {V(gl)}
+{(Rc-(Rc*g11-g12)*exp(-s/fs))/(1+Rc/Rz)/(1-(g11+g22)*exp(-s/fs)+(g11*g22-g12*g21)*exp(-2*s/fs))}

.ends; of buckLTO_GTF


*subcircuit for modeling line-to-output transfer function of buck converter based on Vorperian averaging model
*
.subckt buckLTO_VOR in out params:  duty=0.543 Ron=10m RL=50m Rc=0.1m Rload=3 L=50u C=500u

.param duty2={1-duty} R1={Ron+RL} re={Rc*Rload/(Rc+Rload)}
Evor out 0 LAPLACE {V(in)} {(duty*Rload+s*duty*C*Rc*Rload)/(duty*duty2*re+Rload+R1+
+s*(duty*duty2*re*C*(Rload+Rc)+C*(Rc*Rload+R1*Rc+R1*Rload)+L)+s*s*L*C*(Rload+Rc))}

.ends; of buckLTO_VOR
```

Fig. 2: PSPICE subcircuits for modeling line-to-output transfer functions of buck converter.

4 Simulation results

An example of PSPICE simulation of the line-to-output frequency response of buck converter is given in [1]. The circuit parameters are as follows (see also Fig. 1): L=50µH, C=500µF, $Rload$=3Ω.

An input voltage of 28V is converted to an output voltage of 15.2V, and the corresponding duty ratio is D = 0.543. The switching frequency is 100kHz. The converter operates in the CCM. The modeling in [1] does not include ESRs of inductor and capacitor. The switches were considered as ideal.

Results of PSPICE simulations based on the subcircuits in Fig. 2 are given in Fig. 3. The second subcircuit contains the transfer function of buck converter with the averaged Vorpérian model of PWM switch [6]. This equation has been obtained from the symbolic program SNAP. The corresponding frequency response does not reflect the switching processes and its validity is limited to the frequency region below one half of the switching frequency.

The second frequency response is a full representative of generalized transfer functions of the converter. It models comprehensively the crosstalk of wideband input signal to the output, without the $f_{switch}/2$ limitation.

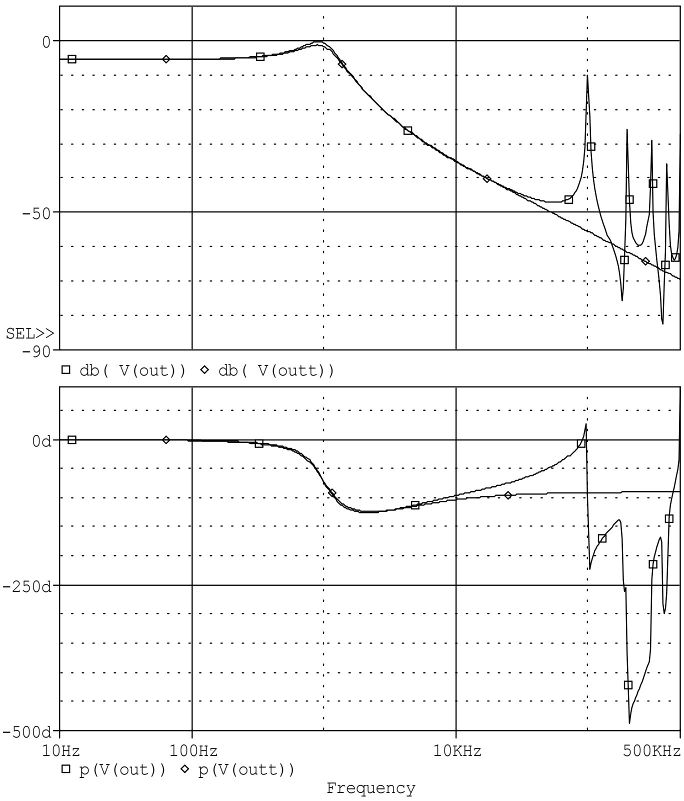

Fig. 3: PSPICE simulation of line-to-output frequency response of buck converter, ◇ averaged Vorpérian model, □ GTF-based model.

5 Conclusions

A novel method of AC analysis of switched DC-DC converters is described. This method utilizes the so-called generalized transfer functions. In comparison with the classical methods based on averaged modeling, the advantages consist in the modeling of converter behavior being more credible, particularly in the frequency range around $f_{switch}/2$, as well as in the ability to model correctly the transfers above this border frequency. A drawback consists in more complicated mathematical models and thus in their more difficult implementation in current simulation programs.

Modeling of the line-to-output transfer functions is described in the paper. The above approach can be also extended to a similar modeling of control-to-output frequency responses.

Acknowledgments

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