# Benes Networks with High Contrast Ratio Holographic Optical Switching Elements 

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#### Abstract

The Benes network has some advantages such as the lowest system insertion loss, the fewest switching elements, the fewest drivers, and fewer crossovers. However, a lower signal-to-noise ratio is its major disadvantage. In this presentation, the Benes network is proposed to obtain a higher SNR with high contrast ratio holographic optical switching elements.


Key-words: Benes network, Optical interconnection network, Contrast ratio, Signal-to-noise ratio, Holographic optical switching element, Polarization-selective element.

## 1 Introduction

To satisfy the requirements of optical communication systems, several optical interconnection networks have been proposed. Compared with other structures, the Benes network [1] has some advantages such as the lowest system insertion loss, the fewest switching elements, the fewest drivers, and fewer crossovers. However, it has a major disadvantage: lower signal-to- noise ratio (SNR). To improve the SNR of Benes networks, two kinds of technologies have been proposed previously: the first is to modify the network structure such as, dilated Benes network, modified dilated Benes network, and generally modified dilated Benes (GMDB) network [2, 3], and the second is to dilate the optical switching elements such as modified Benes network [4]. The SNR and the number of switching elements of these two network structures are similar, but the number of drivers of a modified Benes network is less than one third of the number of drivers of a GMDB network. However, the system insertion loss of the modified Benes network is large than the GMDB network's.

Holographic optical switching elements (HOSEs) have polarization-dependent characteristics. By well designs, highly
polarization-selective holographic elements can be designed and fabricated [5-12]. The HOSEs are three-dimensional devices, and have the advantages of flexibility and compactness. Utilizing these features, the sizes of the HOSEs can be adjusted, which may eliminate the necessity of interconnection lines between switching elements [12-16]. A high contrast ratio optical switching element with HOSEs has been proposed. Its contrast ratio can be higher than 58 dB [10]. The contrast ratio improvement of this device is the same as a dilated optical switching element. In this presentation, Benes network with high contrast ratio holographic optical switching elements is proposed to obtain a higher SNR, keep the fewest drivers, and reduce the system insertion loss.

## 2 Holographic Optical Switching Elements

When two optical signals pass through a $2 \times 2$ optical switching element, each desired optical signal in the output has a little crosstalk from the other signal. The crosstalk is the noise for the desired optical signal. The contrast ratio $(C R)$ is the optical power ratio between the desired and undesired output channels.

The crosstalk of a basic $2 \times 2$ HOSE is shown in Fig. 1. Fig. 1(a) shows the "straight" state of
a basic $2 \times 2$ HOSE, in which point A connects to point C and point B connects to point D . At point $C$, the signal and noise powers come from point $A$ and $B$, respectively. The signal power and noise power from point A to point C and D are $\alpha_{\text {EOHWP }}{ }^{2} \tau_{\mathrm{s}}^{2}$ and $\alpha_{\text {EOHWP }}{ }^{2} \eta_{\mathrm{s}}{ }^{2}$, respectively, where $\alpha_{\text {Eонwр }}$ is the efficiency of the electro-optic halfwave plate and its value is 0.85 [17]. Therefore, the crosstalk ( $\varepsilon$ ) can be obtained as

$$
\begin{equation*}
\varepsilon=\frac{\alpha_{\text {EOHWP }}{ }^{2} \eta_{s}^{2}}{\alpha_{\text {EOHWP }}{ }^{2} \tau_{s}{ }^{2}} \tag{1}
\end{equation*}
$$

and the contrast ratio ( $C R$ ) can be calculated as

$$
\begin{equation*}
C R=\frac{1}{\varepsilon}=\frac{\tau_{s}^{2}}{\eta_{s}^{2}} \tag{2}
\end{equation*}
$$

In this example, $\tau_{\mathrm{s}}$ is greater than $90 \%$ and $\eta_{\mathrm{s}}$ is less than 3\% [5]. Therefore, its contrast ratio $(C R)$ is greater than 900 and SNR is greater than 29 dB . Because the transmission efficiency is $\alpha_{\text {EOHwP }}{ }^{2} \tau_{\mathrm{s}}^{2}$, the insertion loss ( $L$ ) can be calculated as

$$
\begin{equation*}
L=10 \log _{10} \frac{1}{\alpha_{\mathrm{EOHWP}}{ }^{2} \tau_{s}^{2}}(\mathrm{~dB}) \tag{3}
\end{equation*}
$$

Therefore, its insertion loss is 2.3 dB .
Fig. 1(b) shows the "swap" state of a basic $2 \times 2$ HOSE, in which point A connects to point D and point B connects to point C. At point C, the signal and noise powers come from point $B$ and A, respectively. The signal power and noise power from point A to point D and C are $\alpha_{\text {EOHWP }}{ }^{2} \eta_{\mathrm{p}}^{2} \quad$ and $\quad \alpha_{\text {EOHWP }}{ }^{2} \tau_{\mathrm{p}}^{2}$, respectively. Therefore, the crosstalk ( $\varepsilon$ ) can be derived as

$$
\begin{equation*}
\varepsilon=\frac{\alpha_{\mathrm{EOHWP}}{ }^{2} \tau_{p}^{2}}{\alpha_{\mathrm{EOHWP}}{ }^{2} \eta_{p}^{2}} \tag{4}
\end{equation*}
$$

and the contrast ratio $(C R)$ can be calculated as

$$
\begin{equation*}
C R=\frac{1}{\varepsilon}=\frac{\eta_{p}{ }^{2}}{\tau_{p}{ }^{2}} . \tag{5}
\end{equation*}
$$

In this example, $\tau_{p}$ is less than $3 \%$ and $\eta_{p}$ is greater than $90 \%$ [5]. Therefore, its contrast ratio $(C R)$ is greater than 900 and SNR is greater than 29 dB , too. Because the transmission efficiency is $\alpha_{\text {EOHWP }}{ }^{2} \eta_{\mathrm{p}}^{2}$, the insertion loss ( $L$ ) can be calculated as

$$
\begin{equation*}
L=10 \log _{10} \frac{1}{\alpha_{\text {EOHWP }}^{2} \eta_{p}^{2}}(\mathrm{~dB}) \tag{6}
\end{equation*}
$$

Therefore, its insertion loss is 2.3 dB , too.


Fig. 1 The crosstalks in a holographic optical switching element: (a) Straight state; (b) Swap state, where HG and EOHWP are holographic grating and electro-optic halfwave plate, respectively.

Fig. 2 shows a $2 \times 2$ HOSE, which provides a higher contrast ratio. Fig. 2 (a) shows the "straight" state of a higher contrast ratio $2 \times 2$ HOSE, in which point $A^{\prime}$ connects to point $C^{\prime}$ and point $\mathrm{B}^{\prime}$ connects to point $\mathrm{D}^{\prime}$. At point $\mathrm{C}^{\prime}$, the signal and noise powers come from point $\mathrm{A}^{\prime}$ and $\mathrm{B}^{\prime}$, respectively. The signal power and noise power from point $\mathrm{A}^{\prime}$ to point $\mathrm{C}^{\prime}$ and $\mathrm{D}^{\prime}$ are $\alpha_{\text {EOHWP }}{ }^{2} \tau_{\mathrm{s}}^{4} \quad$ and $\quad \alpha_{\text {EOHWP }}{ }^{2} \eta_{\mathrm{s}}{ }^{4}$, respectively. Therefore, the crosstalk ( $\varepsilon_{\mathrm{HCR}}$ ) can be derived as

$$
\begin{equation*}
\varepsilon_{\text {HCR }}=\frac{\alpha_{\text {EOHWP }}{ }^{2} \eta_{s}^{4}}{\alpha_{\text {ЕонwP }}{ }^{2} \tau_{s}^{4}}, \tag{7}
\end{equation*}
$$

and the contrast ratio $\left(C R_{\mathrm{HCR}}\right)$ can be calculated as

$$
\begin{equation*}
C R_{\mathrm{HCR}}=\frac{1}{\varepsilon_{\mathrm{HCR}}}=\frac{\tau_{s}^{4}}{\eta_{s}{ }^{4}} . \tag{8}
\end{equation*}
$$

With the same values of $\tau_{s}$ and $\eta_{s}$, its contrast ratio ( $C R_{\mathrm{HCR}}$ ) is greater than 810000 . Because the transmission efficiency is $\alpha_{\text {EOHWP }}{ }^{2} \tau_{\mathrm{s}}{ }^{4}$, the insertion loss ( $L_{\mathrm{HCR}}$ ) can be calculated as

$$
\begin{equation*}
L_{\mathrm{HCR}}=10 \log _{10} \frac{1}{\alpha_{\mathrm{EOHWP}}{ }^{2} \tau_{s}^{4}} \tag{dB}
\end{equation*}
$$

Therefore, its insertion loss is 3.2 dB .
Fig. 2(b) shows the "swap" state of a higher contrast ratio $2 \times 2 \mathrm{HOSE}$, in which point $\mathrm{A}^{\prime}$ connects to point $\mathrm{D}^{\prime}$ and point $\mathrm{B}^{\prime}$ connects to point $\mathrm{C}^{\prime}$. At point $\mathrm{C}^{\prime}$, the signal and noise powers come from point $\mathrm{B}^{\prime}$ and $\mathrm{A}^{\prime}$, respectively. The signal power and noise power from point $\mathrm{A}^{\prime}$ to point $\mathrm{D}^{\prime}$ and $\mathrm{C}^{\prime}$ are $\alpha_{\text {EOHWP }}{ }^{2} \eta_{\mathrm{p}}^{4}$ and $\alpha_{\text {EOHWP }}{ }^{2} \tau_{\mathrm{p}}^{4}$, respectively. Therefore, the crosstalk ( $\varepsilon_{\mathrm{HCR}}$ ) can be derived as

$$
\begin{equation*}
\varepsilon_{\mathrm{HCR}}=\frac{\alpha_{\mathrm{EOHWP}}{ }^{2} \tau_{p}^{4}}{\alpha_{\mathrm{EOHWP}}{ }^{2} \eta_{p}{ }^{4}}, \tag{10}
\end{equation*}
$$

and the contrast ratio ( $C R_{\mathrm{HCR}}$ ) can be calculated as

$$
\begin{equation*}
\mathrm{C} R_{\mathrm{HCR}}=\frac{1}{\mathcal{E}_{\mathrm{HCR}}}=\frac{\eta_{p}^{4}}{\tau_{p}^{4}} . \tag{11}
\end{equation*}
$$

With the same values of $\tau_{p}$ and $\eta_{p}$, its contrast ratio $\left(C R_{\mathrm{HCR}}\right)$ is greater than 810000 , too. Because the transmission efficiency is $\alpha_{\text {EOHWP }}{ }^{2} \eta_{\mathrm{p}}{ }^{4}$, the insertion loss $\left(L_{\mathrm{HCR}}\right)$ can be calculated as

(a)

(b)

Fig. 2 The crosstalk in high contrast ratio holographic optical switching element: (a) Straight state; (b) Swap state.

$$
\begin{equation*}
L_{\mathrm{HCR}}=10 \log _{10} \frac{1}{\alpha_{\mathrm{EOHWP}}{ }^{2} \eta_{p}{ }^{4}}(\mathrm{~dB}) \tag{12}
\end{equation*}
$$

Therefore, its insertion loss is 3.2 dB , too.
In this higher contrast ratio $2 \times 2$ HOSE, the contrast ratio ( $C R_{\mathrm{HCR}}$ ) is increased from 900 to 810000, which results in that the SNR can be risen from 29 dB to 58 dB . This signal-to-noise ratio improvement of this device is the same as a dilated switching element.

## 3 Benes Network with Holographic Optical Switching Elements

The basic structure of an $N \times N$ Benes network is shown in Fig. 3. This network is a recursive structure consisting of three stages: the left stage, the inner stage, and the right stage. Both of the left and the right stages are constructed by $N / 22$ $\times 2$ switching elements. The inner stage has two $N / 2 \times N / 2$ subnetworks, and the basic one is a 2 $\times 2$ switching elements. In the left and right stages, the upper channels of switching elements connect to the upper subnetwork and the lower channels connect to the lower subnetwork. When the number of inputs and outputs are doubled, two stages of $2 \times 2$ switching elements have to be added.


Fig. 3. The basic structure of an $N \times N$ Benes networks.

In general, the insertion loss (IL) and the SNR of a system are related to the number of switching elements in an optical path. Because the basic $N / 2 \times N / 2$ subnetwork is a $2 \times 2$ switching element, the number of switching
elements in a optical path of an $N \times N$ Benes network is $2 \log _{2} N-1$. Therefore, the SNR of an $N \times N$ Benes network is

$$
\begin{equation*}
\mathrm{SNR}=|X|-10 \log _{10}(2 k-1)(\mathrm{dB}) \tag{13}
\end{equation*}
$$

and the system insertion loss is

$$
\begin{equation*}
\mathrm{IL}=(2 k-1) L(\mathrm{~dB}) \tag{14}
\end{equation*}
$$

where $k$ is $\log _{2} N,|X|$ and $L$ are the contrast ratio and insertion loss of switching elements in dB , respectively.

When an $N \times N$ Benes network is built with basic $2 \times 2$ HOSEs, its SNR and system insertion loss are $29-10 \log _{10}(2 k-1) \mathrm{dB}$ and $2.3(2 k-1) \mathrm{dB}$, respectively. To improve the SNR, a Benes network is constructed with higher contrast ratio $2 \times 2$ HOSEs, its SNR and system insertion loss are $58-10 \log _{10}(2 k-1) \mathrm{dB}$ and $3.2(2 k-1) \mathrm{dB}$, respectively. The SNR increased 29 dB .

## 4 Other Technologies

To improve the SNR of Benes networks, two kinds of technologies have been proposed previously: the first is to modify the network structure such as, dilated Benes network, modified dilated Benes network, and generally modified dilated Benes (GMDB) network [3], and the second is to dilate the optical switching elements such as modified Benes network [4].

The $4 \times 4$ GMDB network is shown in Fig. 4, which have been presented by Wojciech Kabacinski [3]. A GMDB network is constructed with basic $4 \times 4$ subnetworks. There are three stages of basic switching elements in this subnetwork. Each basic $4 \times 4$ subnetwork in a

- GMDB network consists of eight basic
- switching elements. To obtain a higher SNR
- only one optical signal pass through each basic switching element, and only two optical signals pass one basic $4 \times 4$ subnetwork.

The number of basic switching elements that a given optical path passes through decides the power attenuation value. In a GMDB network, there are $k$ stages of basic $4 \times 4$ subnetworks and there are three stages of basic switching elements in this $4 \times 4$ subnetwork. Therefore, the system insertion loss is

$$
\begin{equation*}
\mathrm{IL}=3 k L(\mathrm{~dB}) \tag{15}
\end{equation*}
$$



Fig. 4. $4 \times 4$ generally modified dilated Benes (GMDB) network.

In the worst case, each stage of a GMDB network except the first stage has a second-order crosstalk. Therefore, the total noise power at output point is $(k-1) \times \varepsilon^{2} \times P_{\text {in }}$ and the signal power is approximately $P_{\mathrm{in}}$. Hence, the SNR of a GMDB is

$$
\begin{equation*}
\mathrm{SNR}=2|X|-10 \log _{10}(k-1)(\mathrm{dB}) \tag{16}
\end{equation*}
$$

When an $N \times N$ GMDB network is consisted of basic $2 \times 2$ HOSEs, its SNR and system insertion loss are $58-10 \log _{10}(k-1) \mathrm{dB}$ and $6.9 k \mathrm{~dB}$, respectively.

The structure of a modified Benes network is the same as the Benes network as shown in Fig. 5. The difference between Benes and modified Benes networks is the structure of the switching elements. In a Benes network, each switching element is a basic switching element. In a modified Benes networks, each switching element is a dilated switching element that consists of four basic switching elements. Because the number of stages of the basic switching elements in an $N \times N$ Benes network is $2 k-1$, there are $2 k-1$ stages of dilated switching elements in an $N \times N$ modified Benes network. Therefore, the system insertion loss is

$$
\begin{equation*}
\mathrm{IL}=2(2 k-1) L(\mathrm{~dB}) \tag{17}
\end{equation*}
$$

and SNR of an $N \times N$ modified Benes network is

$$
\begin{equation*}
\mathrm{SNR}=2|X|-10 \log _{10}(2 k-1)(\mathrm{dB}) \tag{16}
\end{equation*}
$$

When an $N \times N$ modified Benes network is constructed with basic $2 \times 2$ HOSEs, its SNR and system insertion loss are $58-10 \log _{10}(2 k-1) \mathrm{dB}$ and $4.6(2 k-1) \mathrm{dB}$, respectively.


Fig. 5. The basic structure of an $N \times N$ modified Benes networks, all basic switching elements have been dilated.

Using the basic $2 \times 2$ HOSEs to build the Benes, modified Benes, and GMDB networks, their SNRs are $29-10 \log _{10}(2 k-1)$, $58-10 \log _{10}(2 k-1)$, and $58-10 \log _{10}(k-1)$, respectively, and their system insertion losses are $2.3(2 k-1) \mathrm{dB}, 4.6(2 k-1) \mathrm{dB}$, and $6.9 k \mathrm{~dB}$, respectively. Using the high contrast ratio HOSEs to construct a Benes network, its SNR and system insertion loss are $58-10 \log _{10}(2 k-1)$ dB and $3.2(2 k-1) \mathrm{dB}$, respectively.

Compared with a Benes network, the SNR improvement of a Benes network with high contrast ratio HOSEs, modified Benes network, and GMDB network are very close. All of them increased more than 29 dB . The system insertion losses of a Benes network with high contrast ratio HOSEs, modified Benes network, and GMDB network are $3.2(2 k-1) \mathrm{dB}, 4.6(2 k-1) \mathrm{dB}$, and $6.9 k \mathrm{~dB}$, respectively. Hence, the Benes network with high contrast ratio HOSE has the lowest system loss.

## 5 Conclusions

A Benes network with high contrast ratio $2 \times 2$ holographic optical switching elements has been proposed to improve the signal-to-noise ratio. Compared with a Benes network with basic $2 \times 2$ holographic optical switching elements, the SNR has been increased by 29 dB . Compared with a Benes network with basic $2 \times 2$ holographic
optical switching elements, the SNR has been increased by 29 dB . The signal-to-noise ratio improvement of this technology is the same as the modified Benes network and generally modified dilated Benes network. Compared with other two technologies, this technology has the lowest system insertion loss. Therefore, the performance of a Benes network with high contrast ratio $2 \times 2$ switching elements is better than other technologies when using holographic optical switching elements to construct a Benes network.

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## References:

[1] V. E. Benes, Mathematical Theory of Connecting Networks and Telephone Traffic. New York: Academic, 1965.
[2] Padmanabhan K. and Netravali A., Dilated networks for photonic switching, IEEE Transaction on Communications, vol. 35, 1987, pp. 1357-1367.
[3] Kabacinski W., Modified dilated Benes network for photonic switching, IEEE Transaction on Communications, vol. 47, 1999, pp. 1087-1091.
[4] J.-S. Deng and Y.-T. Huang, Modified Benes Networks for Photonic Switching, Proc. of SPIE, vol. 4595, 2001, pp. 74-82.
[5] Y.-T. Huang and Y.-H. Chen, PolarizationSelective Element with a Substrate-Mode Grating Pair Structure, Optics Letters, vol. 18, 1993, pp. 921-923.
[6] Y.-T. Huang, Polarization-selective volume holograms: general design, Applied Optics, vol. 33, 1994, pp. 2115-2120.
[7] J.-T. Chang, D.-C. Su, and Y.-T. Huang, Substrate-mode holographic polarizationdivision multi/demultiplexer for optical communications, Applied Optics, vol. 33, 1994, pp. 8143-8145.
[8] J.-T. Chang, D.-C. Su, and Y.-T. Huang, A
four channel polarization and wavelength separation element using substrate-mode stacked holograms, Applied Physics Letters, vol. 68, 1996, pp. 3537-3539.
[9] Y.-T. Huang and Y.-H. Chen, Optical switches with a substrate-mode grating structure, Optik, vol. 98, 1994, pp. 41-44.
[10] J.-S. Deng, M.-F. Lu, C.-P. Lee, and Y.-T. Huang, A High Contrast Ratio Optical Switch with Holographic Optical Switching Elements, WSEAS Transactions on Electronics, vol. 2, 2005, pp. 33-38.
[11] Y.-T. Huang, Polarization-Independent Optical Switch Composed of Holographic Optical elements, Optics Letters, vol. 20, 1995, pp. 1198-1200.
[12] Y.-T. Huang, J.-S. Deng, D.-C. Su, and J.-T. Chang, Holographic Polarization-Selective and Wavelength-Selective Elements in Optical Network Applications, Optical Memory and Neural Networks, vol. 6, 1997, pp. 249-260.
[13] J.-S. Deng, M.-F. Lu, and Y.-T. Huang, Double-Layer Networks with Holographic Optical Switches, Applied Optics, vol. 43, 2004, pp. 1342-1348.
[14] J.-S. Deng and Y.-T. Huang, Multistage Type 1 Network with Holographic Optical Switches, Proceeding of SPIE, vol. 2400, 1995, pp. 235-243.
[15] J.-S. Deng and Y.-T. Huang, New Network Structure: Cyclic Crossbar Network, Proceeding of SPIE, vol. 2692, 1996, pp. 222-232.
[16] M. Kato, H. Ito, T. Yamamoto, F. Yamagishi, and T. Nakagami, Multichannel optical switch that uses holograms, Optics Letters, vol. 17, 1992, pp. 769-771.
[17] Displaytech Asia-Pacific, http://www.displaytech.com/products/photo nics/index.html.

