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Analytical Modelling of Parameter Fluctuations with Applications to Analogue VLSI Circuits and Systems

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Abstract - Analogue circuits are sensitive to parameter fluctuations. Such sensitivity may cause an analogue circuit to have low operating performance. In this paper, an estimation method of parameter fluctuations that integrates design and test is proposed. It is based on analytical modelling equations to estimate the sensitivity of analogue circuits to parameter fluctuations as well as the uncertainty bound for robust operation, independent of simulation.

Key-Words: - Analogue, sensitivity, uncertainty, test

1 Introduction

The functions of an analogue circuit are complex and cannot be adequately described in simple terms. The great variety of possible input and output signals, and the number of possible circuit configurations in which an analogue circuit may be used make it extremely difficult to determine which of the many circuit parameters are important. Furthermore, accurate analogue simulation has to be performed at the device level and this is very demanding of CPU time and memory. Consequently it can only practically be applied to small analogue designs; otherwise the analysis effort and time requirements would be prohibitive. Techniques that are either aimed at testing or at identifying faults that have no effect on analogue circuits' behaviour [1], or aimed at synthesis-for-testability [2][3] generally have high computational cost even for relatively small circuits because of the number of variables and parameters involved. In an attempt to overcome some of these problems, the method presented in this paper is to replace the simulation with an analytical process. In contrast with simulation this process needs to be done only once, and therefore test-interface and time requirement for stimuli generation and application can significantly be reduced.

The faults of analogue circuits are classified as parametric faults [4][5][6] and hard faults [1][7]. Parametric faults are caused by slight variations in parameter values, whereas hard faults are caused by catastrophic variations in parameter values. As device dimensions continue to scale down, circuit performance becomes increasingly sensitive to deviations in the device fabrication process. Hence, parametric faults have become more important.

Techniques dealing with the testing of analogue circuits can be classified into two main groups. One group considers specification testing [6], and the other makes use of measurements to generate a test for a circuit [7]. The drawback associated with measurement-based techniques is the extensive number of simulation trials required and the determination of the input stimuli. With reference to specification testing, (i) it is extremely difficult to emulate the operational environment and (ii) the design aim is to make a specified performance robust to parameter fluctuations. This indicates that in certain cases changes in parameters cannot be detected as the circuit under test performs satisfactory in the presence of such changes. Furthermore, the dynamic behaviour of a circuit is the product of a number of transient responses that correspond to poles in the complex plane [10]. In reference [1] it was shown that abnormality in one transient response due to parameter changes may be masked by other transients, consequently such abnormality may not be detected at the functional level. Therefore, to accurately synthesise the degree of effect of parameter fluctuations on circuit behaviour, the fluctuations have to be defined within the dynamic behaviour. Parameter fluctuations may cause changes in the location of poles, which in turn result in changes in their corresponding transients and subsequently the overall behaviour. If for a maximum bound of fluctuations which drift a circuit closer to its marginal stability, optimal component values can be chosen. This allows the integration of design and test by coupling dynamic circuit behaviour to parameter fluctuations. This paper presents an analytical model to: (i) estimate the sensitivity of a circuit to parameter changes in terms of the positions of the poles; (ii) estimate the maximum bound of parameter fluctuations that maintains a normal functional behaviour.

2 Mathematical Modelling

For an undriven system the state-space representation of an analogue circuit is given:

$$x' = Ax(t) \tag{1}$$

where x(t): state vector (n elements), A: system-interconnection matrix (n.n).
with the pole polynomial

$$p(s) = \det(sI - A) \tag{2}$$

where s: complex variable, I: unit matrix.
The diagonal matrix of A (i.e. A_d) has its main diagonal representing the distinct poles of the circuit, which can be real or complex (λ_i; i: 1, 2, .., n).

$$A_d = (\lambda_{ij}) \tag{3}$$

where $\lambda_{ij} = 0$ when $i \neq j$.

In reference [11], it was shown that faults caused by parametric fluctuations introduce a change in the elements $\lambda_{ij}\}_{i=j}$ of A_d - thus a change in their corresponding transient responses $e^{\lambda_{ij} t}\Big|_{i=j}$

Fluctuations in parameters such as component values, temperature, non-linearity of an amplifier and variation in power supply may cause an analogue circuit to deviate from its desired response. If all parameters that affect circuit behaviour are lumped together and represented by Δ_p then the transfer function 'F' of an analogue circuit - under parameter fluctuations - can be defined as [12]:

$$F = F_n + \Delta_p \tag{4}$$

where F_n: nominal transfer function.

The relationship between F_n and Δ_p is shown in Fig.1.

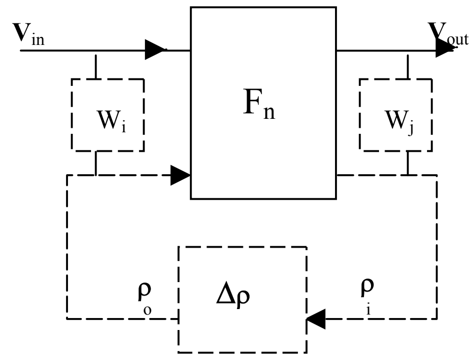

Fig.1 The relationship between F_n and Δ_p.

Consider that the change in parameters is relative magnitude:

$$F - F_n = F_n \Delta_\rho \tag{5}$$

or

$$F = F_n + F_n \Delta_\rho \tag{6}$$

with

$$W_i \rho_o F_n = W_j \rho_i \tag{7}$$

Re-arrange

$$\frac{\rho_i}{\rho_o} = W_i W_j^{-1} F_n = \Delta_\rho \tag{8}$$

Consider

$$V_{out} = W_j P_i \tag{9}$$

and

$$V_{in} = W_i P_o \tag{10}$$

where

$$\rho_o = \Delta V_{in} F_n W_j \tag{11}$$

The output V_{out}, under parameter fluctuations, is given by:

$$V_{out} = V_{in} F_n + W_i^{-1} \rho_o F \tag{12}$$

or

$$V_{out} = V_{in} F_n + W_i^{-1} F \Delta \rho_i \tag{13}$$

In a matrix form:

$$\begin{bmatrix} V_{out} \\ \\ \rho_o \end{bmatrix} = M \begin{bmatrix} V_{inp} \\ \\ \rho_i \end{bmatrix} \tag{14}$$

with the transfer matrix is given by

$$M = \begin{bmatrix} M_{11} & M_{12} \\ M_{21} & M_{22} \end{bmatrix} \tag{15}$$

with

$M_{11} = F$, $M_{12} = W_i^{-1} F^{-1}$, $M_{21} = W_i^{-1}$ and
$M_{22} = W_j W_i^{-1} F_n^{-1}$

One way of approaching testing an analogue circuit is via assessing its stability under parameter fluctuations. Considering that the transfer function F_n represents a stable circuit behaviour, then the circuit is said to be faulty-free or robust to parametric faults iff (if and only if) F_n remains stable for all allowable $\tilde{\Delta}_p$

That is

$$\left|\tilde{\Delta}_p\right| < \gamma \tag{16}$$

where γ: maximum bound to ensure stability and desired performance.

Hence, an analogue circuit may exhibit unstable behaviour only if $\left|\hat{\Delta}\right| > \gamma$ with

$$\lambda(M_{22}\hat{\Delta}) > 0 \tag{17}$$

where λ:a set of eignvalues of F, $\hat{\Delta}$: :change in Δ_ρ.

The determinant in a state-space form is given by:

$$\det(sI - A) = 0 \tag{18}$$

For a small variation in parameters: $|a_i| < \gamma$

where a_i is a coefficient of matrix A.

For small changes, the sensitivity 'S' of F_n with respect to a_i is given by:

$$S_{a_i}^{T_n} = (a_i / F_n) \cdot (\partial F_n / \partial a_i) \tag{19}$$

with reference to (19), the change in parameters represents a range of values,

$\gamma_{min} < a_i < \gamma_{max}$

The coefficient a_i now represents an uncertain range of values. It is important to know how close a circuit is to instability. This can be achieved by modifying (17) as follows:

$\lambda_s(M_{22}\hat{\Delta}) \le \kappa$; κ: integer

so that

$$\lambda_s(M_{22}\hat{\Delta}) = \lambda(M_{22}\hat{\Delta})\Big|_{\kappa \to 0}$$

and

$$\lambda = \lambda_s - |\kappa| \tag{20}$$

The value of κ, in effect, represents a maximum allowable fluctuation in parameters that retains a satisfactory functional behaviour.

3 Examples and Simulation

Depending on their realisation characteristics, analogue circuits may respond differently under certain parameter fluctuations. Different parameters can result in different effects on circuit behaviour. However, it is not always the case that parameter fluctuations yield faulty behaviour. For instance, some parameters may counteract the effect of each other with no noticeable overall change in circuit behaviour – thus the circuit is said to be robust to parameter changes.

Example I: consider the transfer function of a variable notch filter, as shown in (21).

$$F_n = (s^2 + \omega_o^2) \Big/ (s^2 + \frac{\omega_o}{Q} s + \omega_o^2) \tag{21}$$

with $a_o = \omega_o^2$, $a_1 = (\omega_o / Q)$

$$s^2 + \frac{\omega_o}{Q} s + \omega_o^2 = 0 \tag{22}$$

Assume that the components of the filter have been chosen to produce a notch at approximately 10Hz. Hence, $\omega_o = 62$, and $Q = 5$.

with the two poles of the filter are located at:

$s_1 = -6.2 + j62$ ($62 \angle 95.7^o$), $s_2 = -6.2 - j62$($62 \angle -95.7^o$)

Applying (20)

$$\begin{aligned} s_1 &= -5.5 - j61.68 \\ s_2 &= -5.5 + j61.68 \end{aligned}\Bigg|_{\kappa=0.7} \quad \begin{aligned} s_1 &= -5.2 - j61.68 \\ s_2 &= -5.2 + j61.68 \end{aligned}\Bigg|_{\kappa=1}$$

$$\begin{aligned} s_1 &= -3.2 - j61.68 \\ s_2 &= -3.2 + j61.68 \end{aligned}\Bigg|_{\kappa=3} \quad \begin{aligned} s_1 &= -1.2 - j61.69 \\ s_2 &= -1.2 + j61.69 \end{aligned}\Bigg|_{\kappa=5}$$

The effect of changing κ is shown in Fig.2, [12].

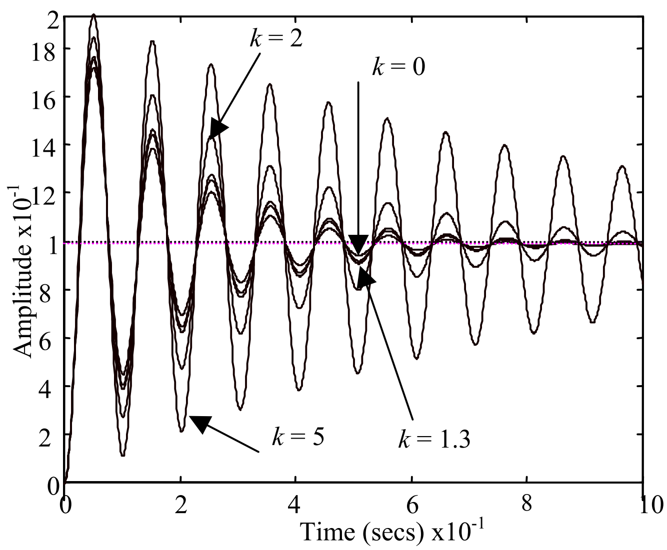

Fig.2 The effect on step-response of increasing *k*.

Fig.2 shows that for k = 1.3 the oscillatory behaviour of the circuit is very close to the nominal one - thus may not be detected by the conventional test techniques. However, such a vanishing change in the behaviour has about 9% increase in the phase compared to the nominal value.

From Fig.2 and for a robust operation $k < 1.3$. If k is chosen to be 1, and since $\lambda_{mag.} = 62$, then for a robust operation $\Delta\lambda/\lambda$ can be calculated using (20):$\Delta\lambda/\lambda = k/\lambda \Rightarrow < 0.016$;

For any chosen value of k, it is possible to predict a circuit's component values using equations (18) and (20). It was found that a change of 1% in either ω_o or Q has no effect on the phase and magntidue of λ. Hence, for a robust operation

$|\gamma| < 0.01$

The frequency response of the filter to parameter fluctuations is shown in Fig.3 (a) and (b).

The value of κ obtained above is consistent with 'γ' and also with the simulation results shown in Fig.3.

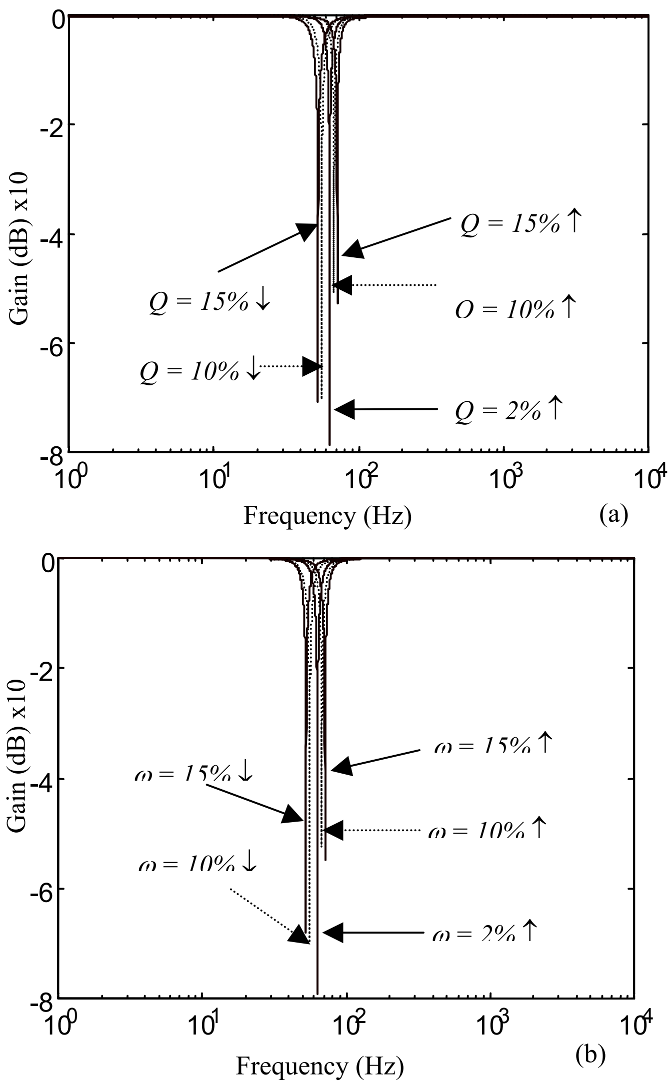

Fig.3. Frequency response for the notch filter.

<u>Example II</u>**:** There is a possibility that simultaneous or overlapped changes in parameters may counteract each other over a period of time. Consider the following transfer function of a second order state-variable:

$$T_{hp} = \frac{s^2}{s^2 + (\omega_o / Q)s + \omega_o^2} \qquad (23)$$

$$T_{bp} = \frac{\omega_o s}{s^2 + (\omega_o / Q)s + \omega_o^2} \qquad (24)$$

$$T_{lp} = \frac{\omega^2}{s^2 + (\omega_o / Q)s + \omega_o^2} \qquad (25)$$

The sensitivity equations are as follows:

$$S_{\omega_o}^{\Re_{notch}} \Rightarrow 0\Big|_{s\to 0}\ ; \quad S_{Q}^{\Re_{notch}} \Rightarrow 0\Big|_{s\to 0} \qquad (26)$$

and

$$S_{\omega_o}^{\Re_{state-variable,lp}} \Rightarrow 0\Big|_{s\to 0}\ ; \quad S_{Q}^{T_{state-variable,lp}} \Rightarrow 0\Big|_{s\to 0} \qquad (27)$$

$$S_{\omega_o}^{\Re_{state-variable,bp}} \Rightarrow -1\Big|_{s\to 0}\ ; \quad S_{Q}^{T_{state-variable,bp}} \Rightarrow +1\Big|_{s\to 0} \qquad (28)$$

$$S_{\omega_o}^{T_{state-variable,hp}} \Rightarrow -2\Big|_{s\to 0}\ ; \quad S_{Q}^{T_{state-variable,hp}} \Rightarrow 0\Big|_{s\to 0} \qquad (29)$$

Equation (26) shows that the notch filter is less sensitive to parameter changes at low frequencies. Equations (27), (28) and (29) show that the state-variable filter, when it is reconfigured as a low-pass or high-pass filter, is less sensitive to parameter variations at low frequencies. However, if this filter is reconfigured as a band-pass filter, its sensitivity increases as Q changes and decreases as ω_o changes. In this particular case, fluctuations in parameters may counteract each other and, hence, the circuit is considered to be robust to parameter fluctuations. For instance, under the same parameter fluctuations the state-variable filter exhibits tolerance to parameter fluctuations when it is configured as a bandpass filter, but the filter exhibits a noticeable deviation (doted line in Fig.4) from the normal response (straight line) if it is reconfigured as a high-pass filter.

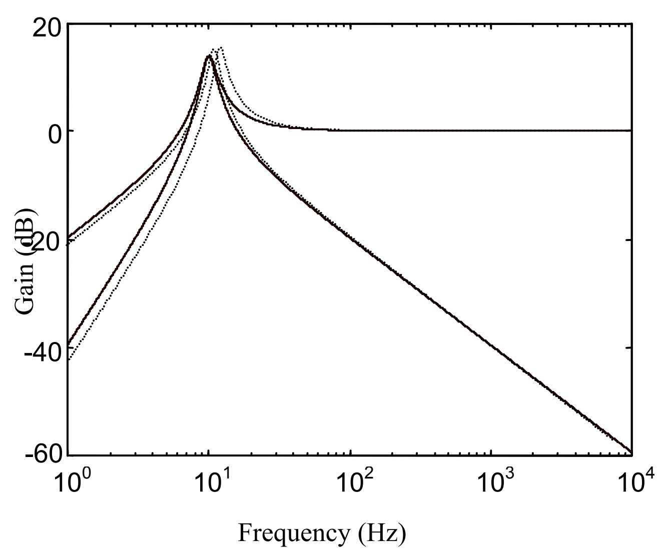

Fig.4 Frequency responses for the state-variable filter.

3.1 Estimation of component value

It is not the aim of this paper to show a systematic way of evaluation of circuit's components, but rather it illustrates how the method proposed can aid in estimation of components value. Consider the characteristic equation of the state-variable filter.

$$s^2 + \frac{\omega}{Q} + \omega^2 = 0$$

with $Q = (R/R_q)$, $\omega_o = (1/R_1 C)$, and

Assume that the filter is designed such that its two poles are located at

$s_1, s_2 = -1 \pm j10$,

then $Q = 5$ and $\omega = 10$

With $\kappa < 1.3$ and $|\gamma| < 0.01$, the new value for Q and ω is given as follows: $Q = 5.7$ and $\omega = 10.3$. The values chosen for R, R_q, R_1 and C should satisfy Q and ω, in order to satisfy a stable behavioural response. If the value chosen for the circuit's components provides a nominal value for Q and ω, then tolerances in, for instance, a component's value should maintain the value of Q or ω within that obtained in relation to κ. Otherwise, an unwanted oscillatory behaviour may occur.

4 CONCLUSION

An analytical method for estimation of sensitivity of analogue circuits to parameter fluctuations is presented. The method also allows for estimation of uncertainty bound for robust operation, independent of simulation. When integrated as part of a design process, component values for robust operation can be estimated.

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