# Shape analysis of left ventricle using spherical harmonics functions 

${ }^{1}$ A. Ben Abdallah, ${ }^{2}$ H. Essabbah, ${ }^{2}$ M. H. Bedoui<br>1Groupe de Recherche en Images et Formes de Tunisie (GRIFT), Ecole Nationale des Sciences de l'Informatique (ENSI) - 2010, La Mannouba-Tunisie<br>2Laboratoire de Biophysique - TIM, UR 08-27,<br>Faculté de Médecine de Monastir - 5019, MONASTIR<br>TUNISIE<br>asma.babdallah@cristal.rnu.tn


#### Abstract

This paper presents an approach for the parameterization of 3D triangular mesh surfaces of spherical topology objects. Our approach is decomposed in two steps. The first step consists in parameterizing the surface by defining a continuous one-to-one mapping from the surface of the object to the surface of a unit sphere [10]. The optimization of initial parameterization is formulated as a constrained optimization problem. The second step consists in expanding the surface into a series of spherical harmonic functions. The new approach is illustrated with modelling the left ventricle at stress and at rest using the sphercical harmonics model and myocardial scintigraphic data. Since we are interested in measuring and analysing left ventricle model deformation between stress and rest, normalization of the spherical harmonics descriptors set is accomplished to eliminate differences due to rotation, translation and magnification.


Key-Words - spherical harmonics model, 3D reconstruction, 3D shape descriptors, surface parameterization, left ventricle, myocardial scintigraphy.

## 1 Introduction

2
Reconstruction, shape modelling and object description are among the main problems which can be faced in the analysis and interpretation of medical images field. Often, it is necessary to model anatomical structures to obtain much closer fit and compact representation.
Several approaches have been used for 3D anatomical structures modelling, more precisely, the heart, using several medical imaging techniques. Thus, in [4], the author has used hierarchical Fourier descriptors for the modelling of various shape 3D objects. In [1]-[3]-[10], spherical harmonics have been used as a 3D model of the heart using angiographic data [14]. In [6], algebraical surfaces have been applied to model the ventricle using SPECT (Single Photon Emission Computed Tomography) images. Generalized algebraical surfaces have been used, as well, by [3], for the left ventricle reconstruction using angiographic data, and more precisely superquadrics.
Variant approaches (hyperquadric, hybrid hyperquadric) have been used in [7]-[8] for cardiac 3D images analysis. Let's also mention, studies in [9] which have used the parametric superquadric model associated to free-form deformations to characterise the heart motion over time using tomographic images and superquadric model tacked as a reference shape and combined with
displacement function.
In this paper, we propose to use the spherical harmonics model and data extracted from scintigraphic images to model the left ventricle. We use the normalized spherical harmonics descriptors to analyse left ventricle model deformation between stress and rest which permits us to conclude if ever an ischemia exists and to quantify it. The diagnosis of the pathology will allow to provide a patient with an adequate therapy. More precisely, in order to extract set of 3D points which corresponds to epicardium and endocardium wall respectively, we have to segment the original data. Then, we fit the surface which is decomposed into a spherical harmonic basis on this set of 3D points through an iterative reconstruction process which leads to determine the optimal harmonic number. Finally we use the spherical harmonics descriptors to estimate if ever pathology exists or not.

## 2 Spherical harmonic model (SH)

Harmonic surfaces are widely studied [11]-[13] and used in different applications (geodesy, biology ...). A spherical topology surface can be parameterised by spherical coordinates $(\varphi, \theta)$ [15]. The spherical harmonic basis functions can be defined as:

$$
\begin{gather*}
Y_{l}^{m}(\theta, \phi)=\sqrt{\frac{2 L+1(l-m)!}{4 \pi(l+m)!}} P_{l}^{m}(\cos \theta) e^{i m \phi} \\
\quad(1) \\
Y_{l}^{-m}(\theta, \phi)=(-1)^{m} Y_{l}^{m^{*}}(\theta, \phi) \tag{2}
\end{gather*}
$$

Where $P_{m}^{l}$ is the associated Legendre function.

## 3 Surface parameterization

The appropriate parameterization of the points of a surface description is a key problem. The generation of such a parameterization has been formulated and solved as a large constrained optimization problem [10]. The algorithm use as input a surface object represented by square faces mesh which separates the initial simply 6connected object and background voxels. To overcome this limitation, we propose a new approach to parameterize the triangular mesh surface of 3D objects.
First, a harmonic map is created to initially map the mesh vertices (object space) to unit sphere (parameter space). The main idea is to start with an initial parameterization [10].
To obtain a homogenous distribution of the parameter space over the surface, the initial parameterization is modified in a constrained optimization procedure considering two criteria:

1. Area preservation: Every object region must map to a region of proportional area in parameter space (eq.3).

$$
\begin{equation*}
\min _{f} \sum_{i \in \Delta}\left[\left|\frac{\text { aire }_{p, i}}{4 \pi}-\frac{\text { aire }_{o, i}}{\sum_{i \in \Delta} \text { aire }_{o, i}}\right|^{2}\right. \tag{3}
\end{equation*}
$$

Where $f$ is the mapping function from space object to parameter space that assigns parameter values to every vertex of the surface.
2. Minimal distortion: Every triangle should map to a spherical triangle in parameter space (eq.4).

$$
\begin{equation*}
\left(\sum_{j=0}^{2}\left|\cos \alpha_{i j p}-\cos \alpha_{i j o}\right|^{2}\right) \tag{4}
\end{equation*}
$$

Where $\operatorname{Cos} \alpha_{i j p}\left(\operatorname{Cos} \alpha_{i j o}\right.$ respectively) is the angle cosinus of the $i^{\text {ième }}$ triangle $j(j=0 . .2)$ of parameter space (object space respectively)

## 4 spherical harmonics descriptors

The spherical harmonic surface can be written as an expansion of spherical harmonic basis functions. A set of
coefficients is then used to express the surface in compact way:

$$
\begin{equation*}
S(\theta, \phi)=\sum_{l=0}^{L} \sum_{m=-1}^{l} c_{l}^{m} Y_{l}^{m}(\theta, \phi) \tag{5}
\end{equation*}
$$

Where $c_{l}^{m}=\left(c_{x l}^{m}, c_{y l}^{m}, c_{z l}^{m}\right)^{T}$ are 3-D vectors and they are obtained by solving a least-squares problem. The values of the basis functions are gathered in the matrix $B=B_{i, j(l, m)}$ with $B_{i, j(l, m)}=Y_{l}^{m}\left(\theta_{i}, \phi_{i}\right)$ where $j(l, m)$ is a function assigning an index to every pair ( $l, m$ ) and $i$ denotes the indices of $n_{\text {vert }}$ points to be approximated. The coordinates of thse points are ranged in $S=\left(S_{1}, S_{2}, \ldots, S_{n v e r t}\right)^{T}$ and all coefficients are gathered in the matrix $C=\left(C_{0}^{0}, C_{1}^{-1}, C_{1}^{0}, \ldots\right)^{T}$. The coefficients that best approximate the points in a leas-squares sense are obtained by:

$$
\begin{equation*}
C=\left(B^{T} B\right)^{-1} B^{T} S \tag{6}
\end{equation*}
$$

The spherical coefficients are determined and normalized with respect to rotation and translation using the first order ellipsoid.

Finally, to compare and discriminate two surfaces $S$ and S', the following distance $D$ can be defined:

$$
\begin{equation*}
D^{2}=\sum_{n=0}^{L} \sum_{m=0}^{l}\left(c_{l}^{m}-c_{l}^{\prime m}\right)^{2} \tag{7}
\end{equation*}
$$

## 5 Scintigraphic images

Myocardial sintigraphy is an imaging modality which provides functional information. We used myocardium SPECT images performed at stress and rest with ${ }^{201} \mathrm{Tl}$ (fig. 2a). The myocardial perfusion is estimated by comparing images at two different instants rest and stress and it is indicated when ischemia is suspected. The diagnosis is obtained by comparing the topology of myocardium blood flow at these tow different instants.

## 6 Results

We start by segmenting the 3D images. This segmentation has enabled us to obtain two 3D point sets representing the surface of anatomical structures of respectively both the epicardium and the endocardium wall. These 3D point set will be used as input for the reconstruction algorithm (fig.2).


Fig. $2 a$ : Scintigraphic image sequences at stress and at rest for one voluntary,


Fig. 2b : Rendered surface of left ventricle at stress and at rest respectively,
Fig. 2: Rendered surface of left ventricle at stress and at rest respectively after segmentation of scintigraphic image sequences of voluntary 1.

For the two examples (fig. 3, and fig. 4), we have analysed scintigraphic data of one healthy voluntary and a pathologic case respectively. Fig. 3 shows an example of rendered surface of the epicardium wall at stress and rest respectively for one healthy voluntary (fig. 3a, fig. 3c), epicardium modelisation with SH model at stress and rest respectively (fig. 3b, fig. 3d). For this case the distance D value calculated is 0.62 .
Fig. 4 shows an example of rendered surface of the epicardium wall at stress and rest respectively for one patient (fig. 4a, fig. 4c), epicardium modelisation with SH model at stress and rest respectively (fig. 4b, fig. 4d). For the pathologic case the distance D is 0.39 .


Fig. 3a : Rendered surface of the epicardium wall of voluntary 1, Fig. 3b : Epicardium modelisation with SH model at stress and at rest.


Fig. 3c : Rendered surface of the epicardium wall voluntary 1, Fig. 3d : Epicardium modelisation with SH model at rest.


Fig. 4 a : Rendered surface of the epicardium wall of patient 1 , Fig. 4b : Epicardium modelisation with SH model at stress.


Fig. 4c : Rendered surface of the epicardium wall of patient 1, Fig. 4d : Epicardium modelisation with SH model at rest

Finally, we have used the spherical harmonics descriptors to analyse and quantify left ventricle model deformation between stress and rest which permits us to conclude if ever an ischemia exists and to quantify it using a distance D value calculated (7) (table1, table2).

| Distance between epicardial wall at stress and at rest |  |  |
| :---: | :---: | :---: |
| State | Initial | Normalized |
| healthy | 17.70 | 0.62 |
| moderate | 12.00 | 0.39 |
| moderate | 13.8901 | 0.784365 |
| healthy | 0 | 0 |
| critical | 24.342 | 0.713892 |
| moderate | 2.25005 | 0.120104 |
| critical | 12.092 | 0.482052 |
| moderate | 16.4202 | 0.395356 |
| critical | 10.43 | 0.258 |
| moderate | 20.04 | 0.99 |
| healthy | 6.436 | 0.355 |
| moderate | 33.17 | 0.99 |

Table 1. Comparison of epicardial wall distance (respectively normalized distance) at stress and at rest for 12 patients.

| Distance between <br>  <br> endocardial wall at stress and at <br> rest |  |  |
| :---: | :---: | :---: |
| State | Initial | Normalized |
| healthy | 13.0433 | 1.08633 |
| moderate | 12.0951 | 0.39391 |
| moderate | 16.289 | 1.07207 |
| healthy | 20.0685 | 1.59268 |
| critical | 18.859 | 1.02522 |
| moderate | 3.13687 | 0.231625 |
| critical | 8.24444 | 0.501197 |
| moderate | 25.3268 | 0.802311 |
| critical | 10.51 | 0.44 |
| moderate | 2.507 | 0.168 |
| healthy | 3.738 | 0.414 |
| moderate | 22.07 | 1.78 |

Table 2. Comparison of endocardial wall distance (respectively normalized distance) at stress and at rest for 12 patients.


## 7 Conclusion

In this work, we have presented a new triangular mesh surface parameterization. The parameterization enables us to expand the object surface into a series of spherical harmonics functions. Invariant object-centered descriptors are obtained by rotating the parameter net and the object into standard positions. We have modelled the left ventricle at stress and at rest using the sphercical harmonics model and myocardial scintigraphic data. The normalized spherical harmonics descriptors obtained at stress and at rest respectively are used to analyse and quantify left ventricle model deformation between stress and rest. Precisely, a numerical distance is calculated. A significant difference is obtained between distances relating to the voluntary data and pathologic.

We have developed a tool which permits us to superpose the result of scintigraphic analyse on the
vascular bed to precise the location and the impact of the pathology. We aim at correlating data from scintigraphic (functional imagery) and coronography information (morphological imagery) in order to to estimate the impact of stenosis on myocardial tissue irrigation or for concluding on a dimension of the coronary artery obstruction using only a myocardial scintigraphy.

We defined a protocol for recruitment of typical patients with CAD in order to enrich our data base. The patients recurred are those having moderate lesion (between $50 \%$ and $70 \%$ of obstruction) and having undergone a coronary angiography and myocardiac perfusion SPECT.

Once this training is acquired, our tool will allow us to estimate, using only myocardial perfusion SPECT data as input, the potential evolution of the lesion and the necessity of perfoming a coronarography.

## References

[1] R. B. Schudy, and D. Ballard, "Towards an anatomical model of heart motion as seen in 4D cardiac ultrasound data", in 6 th Conf on Computer Applications in radiology ans Computer Aided Analysis of Radiological Images, pp. 366-376, 1979.
[2] A. H. Barr, "Superquadrics and angle preserving transformations", IEEE Computer Graphics and Applications, 1, 1981.
[3] Chang Wen Chen, and Thomas S. Huang, "Left Ventricle motion analysis by hierarchical descomposition", in ICASSP, pp. 273-376, 1992.
[4] L.H. Staib, and J.S. Duncan, "Deformable Fourier models for surface finding in 3D images", Proc. VBC'92 Conf., pp.90-194, 1992.
[5] R. Prokop, and A. Reeves, A survey of moment-based techniques for unoccluded object representation and recognition, Computer Vision, Graphics and Image Processing: Graphical Models, 54(5):438-460, 1992.
[6] J. Cauvin, J. Boire, M. Zanca, J. Bonny, J, "Maublant, and A. Veyre, 3D modeling in myocardial TL SPECT", Computerized Medical Imaging and Graphics, 17(4-5):345-350, 1993.
[7] K. B. S. Han, and D. B. Goldgof. "Using hyperquadrics for shape recovery from range data", in Proc. Of the $4^{\text {th }}$ ICCV, Berlin, Germany, pp. 492-496, 1993.
[8] I. Cohen, and L. Cohen. "A hybrid hyperquadric model for 2D and 3D data fitting", rapp. Tech. 2188, INRIA, Rocquencourt, 1994.
[9] E. Bardinet, N. Ayache, and L. Cohen. "Fitting of isosurfaces using superquadrics and free-form deformations", in Proceedings of the IEEE Workshp on Biomedical Image Analysis, Seattle, Washington, juin 1994.
[10] Ch. Brechbuhler, G. Gerig, and O. Kuhler, "Parametrization of closed surfaces for 3D shape description", Computer Vision and Image Understanding, Vol. 61, No. 2, pp. 154-170, 1995.
[11] G. Burel, and H. Henocq, "Determination of orientation of 3D objects using spherical harmonics". Graphical Models and Image Processing, 57, 1995.
[12] M. Nash, "Mechanics and Material Properties of the Heart using an Anatomically Accurate Mathematical Model". PhD thesis, University of Auckland, 1998.
[13] F. Ghorbel. "Toward an unified approach for invariant description", Special issue on image coding, Annales des Télécom, 1998.
[14] G. Gerig, M. Styner, G. Székely, "Statistical Shape model for segmentation and structural analysis", IEEE ISBI, 2002.
[15] A. Ben Abdallah, T. Bannour, F. Ghorbel, "3-D Shape Modelling of Left Ventricle Using Surface Parameterization Description". IEEE ICTTA’04. Damascus, Syria, Avril 2004.
[16] A. Ben Abdallah, M.H. Bedoui, H. Essabah, "Coronographic Results and Myocardiac Scintigraphy Data Rrelationship: Towards Coronary Artery Disease Prognosis". IEEE ICTTA’04. Damascus, Syria, Avril 2004.

