

# The Quasi Optimal Design Strategy for Electronic Circuits

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*Abstract:* - This paper address the problem of diminishing the total computer time needed to complete the design process of electronic circuits. Using a new formulation, based on the optimal control theory, a general methodology is introduced and the idea of the quasi-optimal design strategy's existence is developed. Some numerical results are presented in this paper to support this affirmation and to show the great perspective that this methodology has in design's theory.

*Key-Words:* - General design strategy, optimization, optimal control.

## 1 Introduction

Since early approaches to the optimal control theory, the application to practical problems became obvious specially in physic systems, those that have the particularity to be a controllable dynamic system. As examples of such applications are: a lunar soft landing [1]; the landing of a jet aircraft in an aircraft carrier [2]; a micro-satellite's stabilization [3]; and so on. Other interesting applications have include nuclear research, electricity supply, chemical process industries; and the military field like the design of multilayer antireflection coatings, useful in evading radar systems [4]. For the first time optimal control has been applied to the system design theory [5]-[6].

As a result, a new and more general theory for design process of systems, the General Design Strategy (GDS) [5]-[6] for systems which are described by means of non-linear algebraic equations, has been obtained. On the bases of this theory it is possible to compare the different types of systems and circuits design strategies. The GDS produce a set of  $2^M$  different basic design strategies, where traditional methodology is contained, and where  $M$  is the nodal equations number. On the other hand, a more general approach generates an infinite number of different design strategies. It is clear that among these strategies there is one optimal strategy. The quasi-optimal design strategy is defined as the strategy that achieves the optimum of the design objective function of the design process for the minimal computer time, which is equivalent to the minimal-time problem of the optimal control theory.

In section 2, a new formulation for designing based on control theory is introduced. How to reproduce all the basic design strategies generated by the new formulation is explained in section 2. The optimal strategy that jumps trough basic strategies is explained in section 4. A numerical example is given in section 5 to show the great perspectives of the new proposal in diminishing the computer time needed for the total design process.

## 2 General design formulation

It is possible to generalize the circuits and systems design process by introducing an optimal control vector in it, defined as

$$U = (u_1, u_2, \dots, u_M) \quad (1)$$

where  $u_j \in \Omega$ ,  $\Omega = \{0;1\}$  and  $M$  is the total number of nodal equations of the circuit. The main system equations can be defined then by

$$\frac{dx_i}{dt} = f_i(X, U), \quad i = 1, 2, \dots, M \quad (2)$$

where  $X$  is the total variables vector that is divided in two parts, one for independent variables ( $X'$ ), and another one for dependent variables ( $X''$ ), to obtain

$$X = \{X', X''\}. \quad (3)$$

The convenience of equation (3) becomes clear in next section. In order to meet with the design requirements an objective function is defined and then minimized, in this case

$$F(X, U) = C(X) + \varphi(X, U) \quad (4)$$

where  $C(X)$  is the simple objective function that contains the actual requirements while  $\varphi(X, U)$  term are the so call penalty functions that simulates the system according to control variables vector selection and is defined as

$$\varphi(X, U) = \sum_{j=1}^M u_j g_j^2(X). \quad (5)$$

Here the terms  $g_j^2(X)$  are taken from the main system equations

$$g_j(X) = 0 \quad (6)$$

where the squared form avoid contribution eliminations by opposite signs. The equation (4) is subject to

$$(1 - u_j) g_j(X). \quad (7)$$

System equations (7) is a modified formulation of equation (6) that gives the possibility to decide which equation is part of the system and which one is not, by means of the control variables.

This new methodology is the General Design Strategy (GDS) for circuits and systems that has great perspectives of diminishing the total computer time required for the design process.

### 3 GDS generation

In Fig. 1 the flow diagram for the general methodology GDS is shown. It begins with the necessary initializations such as the reading of values for the  $X$  total variables and the vector  $U$ . The vector  $U$  is defined by equation (1), and  $M$  is the total number of nodal equations, thus there exist  $2^M$  possible combinations and so possible design strategies. After variables initialization there is the GDS module, that will be useful in next section, it contains the two main parts of design process. Within Part I the system  $(1 - u)g_j(X)$  with  $j = 1, 2, \dots, M$ , is solved, while Part II is divide in two subparts, one for evaluation of penalty functions given by equation

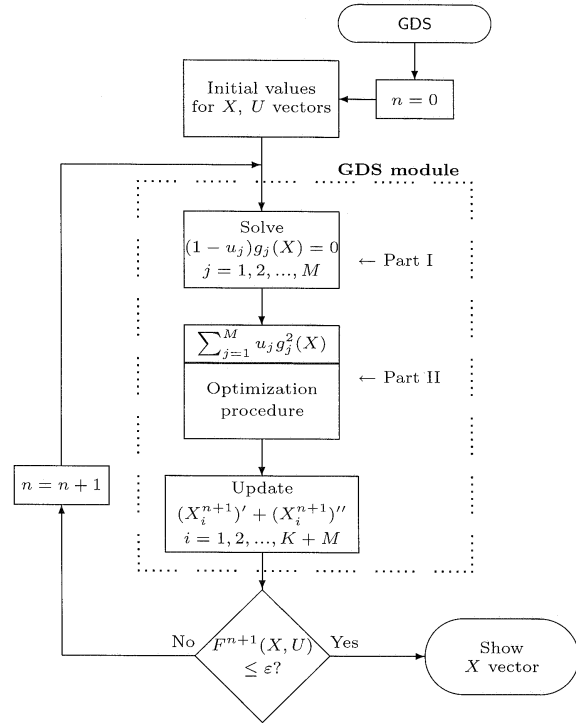


Fig. 1. Flow diagram simplified for General Design Strategy (GDS).

(5) and contained in the objective function (4); and other one for optimization procedure that is applied over  $F(X)$ . Notice at this point that  $u_j = 0$  means presence of equation  $j$  in the system, and absence of penalty function  $j$ , on the contrary  $u_j = 1$  means absence of the equation  $j$  in the system and presence in penalty function  $j$  in Part II. GDS module finish with an update of the total variables vector  $X^{n+1}$  defined by equation (3), and affected by control variables vector  $U$  as follows:

$$(X^{n+1})' = (x_1^{n+1}, x_2^{n+1}, \dots, u_1 x_{K+1}^{n+1}, \dots, u_M x_M^{n+1})$$

and

$$(X^{n+1})'' = (\bar{u}_1 x_{K+1}^{n+1}, \bar{u}_2 x_{K+2}^{n+1}, \dots, \bar{u}_M x_M^{n+1})$$

where  $\bar{u}_j = (1 - u_j)$ . Independent variables  $x_1^{n+1}, x_2^{n+1}, \dots, x_K^{n+1}$  are always independent but dependent variables  $x_{K+1}^{n+1}, x_{K+2}^{n+1}, \dots, x_M^{n+1}$  can be seen as dependent or independent variables by means of control variables  $u_j$  which values decides whether a

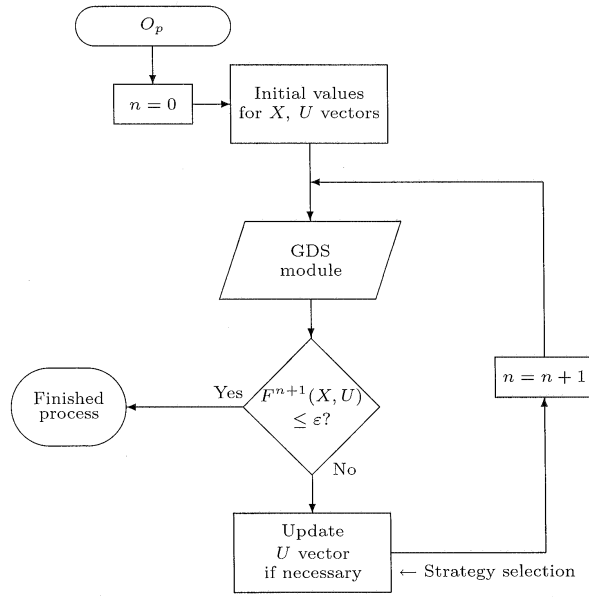


Fig. 2. Flow diagram for the quasi-optimal design strategy.

dependent variable will be considered as independent one or keep his dependent nature. Here  $u_j = 0$  means that  $x_j^{n+1} \in (X^{n+1})''$  and  $u_j = 1$  means that  $x_j^{n+1} \in (X^{n+1})'$ . Clearly the cases where  $x_j^{n+1}$  belongs to both  $(X^{n+1})'$  and  $(X^{n+1})''$  or where neither belongs to  $(X^{n+1})'$  nor  $(X^{n+1})''$  are not allowed. The flow diagram for GDS finish with the evaluation of  $F^{n+1}(X, U)$  and asking if design requirements have been reached, if yes, actual value of X vector is shown and the design process is over; else another step take place and the process is continued.

We conclude that the total control variables vector  $U$  used in the GDS, give us the possibility to generate not only the traditional methodology but other set of design strategies that never have been consider before. This new design strategies arise owing to all possible value combinations ( $2^M$ ) of control variables  $u_j, j = 1, 2, \dots, M$  under the assumption that  $U$  vector is fix along the entire design process

#### 4 The quasi optimal design strategy

In the past section we consider  $U$  fix within GDS, however there exist the possibility to change values of control vector at any numerical step of the design

process, this would means a switch of strategy. Many switches can be applied along the complete design process, defining each switch, a new design strategy. We name basic strategy when  $U$  is fixed along the design process. The utility of switching strategies is the approach to the optimal trajectory that we name quasi-optimal trajectory ( $O_p$ ). This idea is justified by the maximal principle of the optimal control [7].

The simplified flow diagram for  $O_p$  is shown in Fig 2. It begins with the necessary initializations, specially for  $X$  and  $U$  vectors, and thus an initial design strategy is selected. Using the GDS Module of Fig. 1, an optimization procedure is applied to minimize  $F(X, U)$  that is defined by equation (4), subject to the system equations (7). After this, we need to now if design conditions has been meet, it is considered so when  $F(X, U)$  has a value equal or less than a certain  $\epsilon$ , if design conditions are not completed yet, the module of strategy selection is used to decide whether a new design strategy is needed or not. The process continues until desire conditions or maximum iterations' number, have been satisfied.

The quasi-optimal strategy gives the possibility to reduce the CPU time needed to complete the total design process. This affirmation will be illustrated with a numerical example in the next section.

#### 5 Numerical Results

As an example of the new proposal a two stages amplifier circuit is analyzed in this section, see Fig 3.

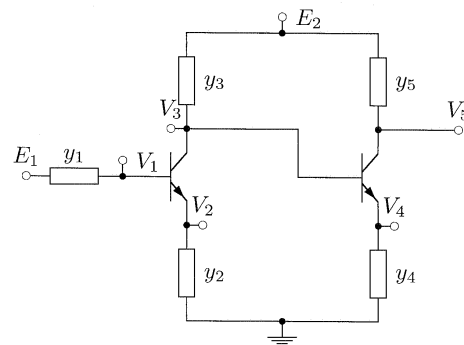


Fig. 3. Two stages amplifier circuit.

This circuit has five nodal voltages  $V_1, V_2, \dots, V_5$  that are considered dependent variables under traditional methodology and five admittances  $y_1, y_2, \dots, y_5$  which actually are independent variables. Therefore, the total variables vector has ten elements,

$X = (x_1, x_2, \dots, x_{10})$ . On the other hand, the simple objective function is given as

$$C(X) = (x_{10} - k_v)^2 + (V_{BE1} - \rho_1)^2 + (V_{BC1} - \rho_2)^2 + (V_{BE2} - \rho_3)^2 + (V_{BC2} - \rho_4)^2 \quad (8)$$

where it is suitable to define  $k_v = 9.0V$ ,  $\rho_1 = \rho_3 = 0.28V$  and  $\rho_2 = \rho_4 = -2.0V$ . The total objective function is given by equation (4), where  $M = 5$  for this example. The model used for transistors of Fig. 3 is the BJT Spice2 [8] and its characteristics are selected the same,  $I_{S1} = I_{S2} = 20 \times 10^{-9} A$ ,  $\beta_{F1} = \beta_{F2} = 100$  and  $\beta_{R1} = \beta_{R2} = 1$ . The result analysis of some design strategies for gradient and Davidon-Fletcher-Powell (DFP) optimization methods using initial vector

$$X^0 = (2,2,2,2,2,5,3,5,6,4,6,1,6,7) \quad (9)$$

is shown in Table 1, the control variables  $U$  is fixed along of design process in this case. For both gradient and DFP methods, the best strategy is that generated by control vector  $U = (1,1,1,0,1)$  with 1.82 s. and 0.55 s. respectively. The traditional methodology corresponds to control vector  $U = (0,0,0,0,0)$  with a CPU consumption time of 236.34 s. and 62.64 s. for gradient and DFP methods respectively. Clearly the traditional methodology is not the optimal one for this example but this situation is valid for all circuits analyzed.

Table 1. Some design strategies for the amplifier of Fig. 3.

U	Gradient		DFP	
	No. of Iterations	CPU Time(s)	No. of Iterations	CPU Time(s)
00000	105395	256.34	14076	62.64
00101	14860	25.54	2625	8.35
00111	23517	31.58	2676	6.71
01101	4529	6.21	791	1.98
10101	2471	2.21	395	0.66
10110	88742	122.98	12874	32.84
10111	7624	5.33	799	1.11
11100	80723	111.55	15835	39.71
11101	2652	1.82	392	0.55
11110	102014	104.08	13564	24.93
11111	15938	2.69	1586	0.67

The result of analysis for the circuit of Fig. 3 when control vector  $U$  is variable along the design process is shown in Table 2. The quasi-optimal design

strategy has CPU time consumptions of 0.504 s. and 0.147 s. for the gradient and DFP optimization methods respectively. This means that the gain in time of  $Op$  against the traditional methodology is 509 and 427 for the gradient and DFP methods respectively.

Table 2. Data of quasi-optimal design strategy  $Op$  for circuit of Fig. 3.

Method	$U$	No. of Iterations	CPU time (s)	Gain
Gradient	(11111, 00000, 11111)	105 2 321	0.504	509
	DFP	(11111, 00000, 11111)		

On the other hand, it must be said that in all circuits analyzed the traditional methodology has been not the optimal one.

## 6 Conclusion

The general design formulation for electronic circuits design process gives the possibility to increase the total number of basic design strategy and actually permits an infinite number of possible design trajectories. According to the optimal control theory among this possible trajectories there exist one that is optimal in the sense of time. The approach to this optimal trajectory, the quasi-optimal strategy, shows a great perspective in decreasing the total computer time needed to complete the design process.

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