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Fault reconstruction in nonlinear dynamical systems using differential algebraic methods

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Abstract: In this paper we tackle the *diagnosis problem in nonlinear systems under failure* using *algebraic observability* and *differential transcendence degree* concepts. The proposed methodology is applied to a hydraulic system. Numerical simulations are presented to illustrate the effectiveness of the suggested approach.

Key-words: Diagnosis problem, algebraic observability, differential transcendence degree.

1 Introduction

Systems diagnosis has been studied for more than three decades, see for instance [1]. Early research was strongly oriented towards the design of algorithms which are capable of realizing the fault diagnostic task in linear systems. In [1] a review of the observer-based fault diagnosis approaches for deterministic nonlinear dynamic systems is given, also, some schemes extending the well-known diagnosis methods for linear systems to the nonlinear case. Besides a direct extension of the unknown input observer (UIO) results in linear systems to the nonlinear case was considered. Another appealing approach is the one based on differential geometric methods [2, 7]. Alternatively, some authors have proposed solutions to the Fault Detection and Identification Problem for a nonlinear systems class in an algebraic and differential setting [3, 4]. For instance, in [3] and [4] an approach has been considered to solve the diagnosis problem. It consists on translating the solvability of the problem in terms of the algebraic observability of the variable modelling the *fault*. The connection between diagnosability and observability of faults has not been studied previously. However, it is important to note mentions of quite close notions in earlier works [6] including input observability, fault detectability, distinguishability and fault isolability [1]. The framework in which this paper is conceived is essentially based in the language of differential algebra. In [3, 4], the methodologies employed for the observer design only include full order observers without considering uncertainty estimation.

In this paper, the fault dynamics is considered as an uncertainty. In the proposed procedure, the construction of a full order observer is not necessary, instead of this, a reduced order uncertainty observer is constructed using differential algebraic techniques applied to the fault estimation in the diagnosis problem.

The proposed methodology is applied to a hydraulic system, which was studied before in [7]. Here, a full estimation of the fault is obtained, meanwhile in [7] the work is limited to fault detection. The class of systems for which this methodology can be applied contains systems depending on the inputs and their time derivatives in a polynomial form.

The rest of this paper is organized as follows: In Section 2 some basic definitions on observability and systems diagnosability in a differential algebraic framework are introduced. The statement of the problem and the diagnosability condition are described in Section 3. In Section 4 the application of the proposed methodology to a hydraulic system is shown. In Section 5, the construction of a reduced order uncertainty observer is described. In section 6, numerical results are given. Finally, in Section 7 the paper is closed with some concluding remarks.

2 Basic Definitions

We start introducing, some basic differential algebra definitions given in [3, 4, 5], and references there in.

Definition 1 A differential field extension L/k is given by two differential fields k and L, such that: i) k is a subfield of L, ii) the derivation of k is the restriction to k of the derivation of L.

Example $\mathbb{Q}$,$\mathbb{R}$ and $\mathbb{C}$ are trivials differential field extensions, where $\mathbb{Q} \subset \mathbb{R} \subset \mathbb{C}$.

Definition 2 An element is said to be differentially algebraic with respect to the field k if it satisfies a differential algebraic equation with coefficients over k.

Example Let $\mathbb{R}\left\langle e^{at}\right\rangle/\mathbb{R}$ a differential field extension, where $\mathbb{R} \subset \mathbb{R}\left\langle e^{at}\right\rangle$, $x=e^{at}$ is a solution of $P(x)=\dot{x}-ax=0$ (a is a constant).

Definition 3 An element is said to be differentially transcendental over k, if and only if, it is not differentially algebraic over k.

Definition 4 Let L/k a differential field extension. A differential transcendental family, which is the greatest with respect to the inclusion, is called a differentially transcendental base of L/k. The cardinality of the base is called the differential transcendence degree of L/k:

$$difftrd^{\circ}(L/k), \quad (1)$$

for more details see [5].

Definition 5 The differential output rank ρ of a system is equal to the differential transcendence degree of the differential extension $k\langle y\rangle$ over the differential field k, i.e.:

$$\rho = difftrd^{\circ}k\langle y\rangle/k. \quad (2)$$

Property 1. *The differential output rank ρ, of a system is smaller or equal to min(m, p):*

$$\rho = difftrd^{\circ}k\langle y\rangle/k \leq min(m,p), \quad (3)$$

where m, p are the total number of inputs and outputs respectively.

Definition 6 A system is left-invertible, if and only if, the differential output rank is equal to the total number of inputs, i. e.

$$\rho = m$$

Property 2 (Left invertibility). *If a system is differentially left-invertible then the input u can be recovered from the output by means of a finite number of ordinary differential equations.*

Definition 7 A dynamics is a finitely generated differential algebraic extension $G/k\langle u\rangle$ $(k\langle u,\xi\rangle, \xi \in G)$. Any element of G satisfies an algebraic differential equation with coefficients being rational functions over k in the elements of u and a finite number of their time derivatives.

Example Let consider the input-output system $\ddot{y} + \omega^2 sen(y) = u$, equivalent to the system:

$$\Sigma_B \left\{ \begin{array}{rcl} \dot{x}_1 &=& x_2 \\ \dot{x}_2 &=& -\omega^2 sin(x_1) + u \\ y &=& x_1 \end{array} \right. \quad (4)$$

system (4) is a dynamics of the form $\mathbb{R}\langle u,y\rangle/\mathbb{R}\langle u\rangle$ where $G = \mathbb{R}\langle u,y\rangle$, $y \in G$ and $k = \mathbb{R}$. Any solution of (4) satisfies the following differential algebraic equation:

$$\left(y^{(3)} - \dot{u}\right)^2 + \left(\dot{y}\left(y^{(2)} - u\right)\right)^2 = \left(\omega^2\dot{y}\right)^2.$$

Definition 8 Let a subset $\{u, y\}$ of G in a dynamics $G/k\langle u\rangle$. An element in G is said to be algebraically observable with respect to $\{u, y\}$ if it is algebraic over $k\langle u,y\rangle$. Therefore, a state x is said to be algebraically observable, if and only if, it is algebraically observable with respect to $\{u, y\}$. A dynamics $G/k\langle u\rangle$, with output y in G is said to be algebraically observable if, and only if, the state has this property.

Example System Σ_B in (4) with output $y \in \mathbb{R}\langle u,y\rangle$ is algebraically observable, since x_1 and x_2 satisfy two differential algebraic polynomials with coefficients in $\mathbb{R}\langle u,y\rangle$, i.e.

$$\begin{array}{rcl} x_1 - y &=& 0 \\ x_2 - \dot{y} &=& 0. \end{array}$$

3 On the diagnosability condition

Let consider the class of nonlinear systems described by:

$$\begin{array}{l} \dot{x}(t) = A(x, \bar{u}) \\ y(t) = h(x, u), \end{array} \quad (5)$$

where $x = (x_1, ..., x_n)^T \in R^n$ is a state vector, $\bar{u} = (u, f) = (u_1,...,u_{m-\mu}, f_1,...,f_\mu) \in R^{m-\mu} \times R^\mu$ where u is a known input vector and f is an unknown fault vector, $y = (y_1, ..., y_p) \in R^p$ is the output. A and h are assumed to be analytical vector functions.

In the next paragraphs some concepts concerning to the diagnosability problem will be presented [3, 4].

Definition 9 (Algebraic observability) An element $f \in k\langle\bar{u}\rangle$ is said to be algebraically observable if f satisfies a differential algebraic equation with coefficients over $k\langle u,y\rangle$.

Definition 10 (Diagnosability) The class of nonlinear systems described by (5) is said to be diagnosable if it is possible to estimate the fault f from the system equations and the time histories of the data u and y, i. e., it is diagnosable if f is algebraically observable with respect to u and y.

In other words it is required that each fault component be able to be written as a solution of a polynomial equation in f_i and finitely many times derivatives of u and y with coefficients in k:

$$H_i(f_i, u, \dot{u}, ..., y, \dot{y}, ...) = 0.$$

Example Let consider the following linear system

$$\begin{array}{rcl} \dot{x}_1 &=& x_2 \\ \dot{x}_2 &=& -x_2 + f \\ y_1 &=& x_2 \end{array} \quad (6)$$

It is not hard to see that system (6) is diagnosable according to definition 8, that is to say, the fault f in the system is algebraically observable with respect to u and y and satisfies a differential algebraic polynomial with coefficients in $k\langle u,y\rangle$ as follows:

$$\dot{y} + y - f = 0$$

Rearranging terms the following differential algebraic polynomial is obtained

$$f = \dot{y} + y, \tag{7}$$

then, f can be estimated from $\dot{y}$ and y. Condition (7) is called the **diagnosability condition**.

It was already pointed out in [4] that a diagnosable system is not necessarily observable, and vice versa. Indeed, the following system

$$\left\{ \begin{array}{rcl} \dot{x}_1 & = & -x_1 + x_2, \\ \dot{x}_2 & = & x_2 + u + f, \\ y & = & x_2, \end{array} \right. \tag{8}$$

is diagnosable, i. e. $f = \dot{y} - y - u$, but it is not observable since x_1 is not observable with respect to u and y. ■

The following result relates the observability and diagnosability conditions

Theorem 1 [4] *If system* (5) *is observable then it is diagnosable iff f is observable with respect to u, y and x.* ■

Remark 1 This is an immediate consequence of the general transitivity property of the observability condition [4].

3.1 On the minimal number of measurements

The basic practical question is: How many measurements does one need to make a system diagnosable?. An answer to the question would be a valuable piece of information to the system expert who wants to optimize the number of sensors for fault detection purposes [4]. Before starting the main result, an useful Lemma concerning the Towers of Differential Fields Extensions and some definitions are given. This results will be used subsequently in the proof of Theorem 2.

Lemma 1 [8] Take *K, L, M* , differential fields, where $K \subseteq L \subseteq M$, then:

$$difftrd^{\circ} M/K = difftrd^{\circ} M/L + difftrd^{\circ} L/K. \quad ■ \tag{9}$$

Another key result is given in the following proposition:

Proposition 1 The differential transcendence degree of the differential field extension $k\left\langle \bar{u}, y\right\rangle / k\left\langle \bar{u}\right\rangle$ is equal to zero, that is to say

$$difftrd^{\circ} k\left\langle \bar{u}, y\right\rangle / k\left\langle \bar{u}\right\rangle = 0. \tag{10}$$

Proof From *Property 1* of the differential output rank, there exists only two cases: $\rho = m$ and $\rho < m$.

By applying Lemma 1 to the differential field extension $k\left\langle \bar{u}, y\right\rangle / k$, the following decomposition is obtained:

$$\begin{array}{rcl} k & \subseteq & k\left\langle y\right\rangle \subseteq k\left\langle \bar{u}, y\right\rangle \\ k & \subseteq & k\left\langle \bar{u}\right\rangle \subseteq k\left\langle \bar{u}, y\right\rangle \end{array}$$

then

$$\begin{array}{l} difftrd^{\circ} k\left\langle \bar{u}, y\right\rangle / k = difftrd^{\circ} k\left\langle \bar{u}, y\right\rangle / k\left\langle y\right\rangle \\ + difftrd^{\circ} k\left\langle y\right\rangle / k \end{array} \tag{11}$$

$$\begin{array}{l} difftrd^{\circ} k\left\langle \bar{u}, y\right\rangle / k = difftrd^{\circ} k\left\langle \bar{u}, y\right\rangle / k\left\langle \bar{u}\right\rangle \\ + difftrd^{\circ} k\left\langle \bar{u}\right\rangle / k. \end{array} \tag{12}$$

Case 1 Let suppose that $\rho = m$, then by *Property of left invertibility*

$$difftrd^{\circ} k\left\langle \bar{u}, y\right\rangle / k\left\langle y\right\rangle = 0$$

Replacing in (11) this implies that:

$$difftrd^{\circ} k\left\langle \bar{u}, y\right\rangle / k = m \tag{13}$$

From (12) and (13) it is not hard:

$$difftrd^{\circ} k\left\langle \bar{u}, y\right\rangle / k\left\langle \bar{u}\right\rangle + difftrd^{\circ} k\left\langle \bar{u}\right\rangle / k = m \tag{14}$$

and using the following relation which is given by definition:

$$difftrd^{\circ} k\left\langle \bar{u}\right\rangle / k = m,$$

then:

$$difftrd^{\circ} k\left\langle \bar{u}, y\right\rangle / k\left\langle \bar{u}\right\rangle = 0. \tag{15}$$

and the first part of the proof is concluded .

Case 2 Let consider $\rho < m$, then y_e and y_T can be defined, such that the output vector y_e is differentially transcendental over the differential field $k\left\langle y\right\rangle$ and $y_T = [y, y_e]$, which means that:

$$k\left\langle y_T\right\rangle = k\left\langle y, y_e\right\rangle,$$

then $k\left\langle y, y_T\right\rangle / k\left\langle y_T\right\rangle$ is a differential algebraic extension, that is to say:

$$difftrd^{\circ} k\left\langle y, y_T\right\rangle / k\left\langle y_T\right\rangle = 0 \tag{16}$$

without loss of generality, let suppose that

$$\rho_T = m$$

or in other way

$$\rho_T = \rho + \rho_e = m.$$

But in Case 1 was proved that if $\rho_T = m$, then

$$difftrd^{\circ} k\left\langle y_T, \bar{u}\right\rangle / k\left\langle \bar{u}\right\rangle = 0$$

and from (16) this implies that

$$difftrd^{\circ} k\left\langle y, \bar{u}\right\rangle / k\left\langle \bar{u}\right\rangle = 0$$

and the proof is concluded. ■

The following theorem corresponds to the main result of this paper.

Theorem 2 *System (*5*) is diagnosable iff* $difftrd^{\circ} k\langle u, y\rangle / k\langle u\rangle = \mu$*, where* μ *is the number of components of the fault* f*.*

Proof By applying Lemma 1 to differential field extension $k\langle\bar{u},y\rangle/k$, the following relation is obtained:

$$k\subseteq k\langle u\rangle\subseteq k\langle u,y\rangle\subseteq k\langle\bar{u},y\rangle$$

then

$$\begin{array}{l} difftrd^{\circ}k\langle\bar{u},y\rangle/k=difftrd^{\circ}k\langle\bar{u},y\rangle/k\langle u,y\rangle \\ +difftrd^{\circ}k\langle u,y\rangle/k\langle u\rangle+difftrd^{\circ}k\langle u\rangle/k. \end{array} \tag{17}$$

On the other hand

$$k\subseteq k\langle u\rangle\subseteq k\langle\bar{u}\rangle\subseteq k\langle\bar{u},y\rangle$$

$$\begin{array}{l} difftrd^{\circ}k\langle\bar{u},y\rangle/k=difftrd^{\circ}k\langle\bar{u},y\rangle/k\langle\bar{u}\rangle \\ +difftrd^{\circ}k\langle\bar{u}\rangle/k\langle u\rangle+difftrd^{\circ}k\langle u\rangle/k. \end{array} \tag{18}$$

From (17) and (18) the following is given

$$\begin{array}{l} difftrd^{\circ}k\langle u,y\rangle/k\langle u\rangle=difftrd^{\circ}k\langle\bar{u},y\rangle/k\langle\bar{u}\rangle \\ +difftrd^{\circ}k\langle\bar{u}\rangle/k\langle u\rangle-difftrd^{\circ}k\langle\bar{u},y\rangle/k\langle u,y\rangle \end{array} \tag{19}$$

Using *Proposition 1* in (19)

$$\begin{array}{l} difftrd^{\circ}k\langle u,y\rangle/k\langle u\rangle=difftrd^{\circ}k\langle\bar{u}\rangle/k\langle u\rangle \\ -difftrd^{\circ}k\langle\bar{u},y\rangle/k\langle u,y\rangle \end{array} \tag{20}$$

From (20), is now possible to demonstrate Theorem 2.

(Sufficiency)
Let suppose that the $difftrd^{\circ}k\langle u,y\rangle/k\langle u\rangle$ is equal to the number of fault components; since each fault component is differentially transcendental over $k\langle u\rangle$, then, $difftrd^{\circ}k\langle\bar{u}\rangle/k\langle u\rangle$ is also equal to the number of fault components which implies:

$$difftrd^{\circ}k\langle\bar{u},y\rangle/k\langle u,y\rangle=0,$$

that is to say, the fault f is algebraic over $k\langle u,y\rangle$ (or f is diagnosable), which concludes the first part of the proof.

(Necessity)
Let f to be diagnosable, this implies that f is algebraic over $k\langle u,y\rangle$ and the following equality is true:

$$difftrd^{\circ}k\langle u,y\rangle/k\langle u\rangle=difftrd^{\circ}k\langle\bar{u}\rangle/k\langle u\rangle, \tag{21}$$

Is also known that $difftrd^{\circ}k\langle\bar{u}\rangle/k\langle u\rangle=\mu$, since all the fault components are by definition differentially transcendental over the differential field $k\langle u\rangle$. Then, the equality (21) can be expressed as follows

$$difftrd^{\circ}k\langle u,y\rangle/k\langle u\rangle=\mu,$$

where μ is the number of components of the fault f. And the proof is concluded. ■

Remark 2 Theorem 2, provides a way to determine the diagnosability of a class of nonlinear systems described by (5). ■

4 Hydraulic System

The hydraulic system was presented in [7], which consists of a spool valve and a single rod piston acting on an inertial load (see Figure 1). The external force F_e controls the flow entering the head side chamber of the piston from a pressure supply P_a. The rod side chamber is always connected to the return pressure P_r.

Then, it is necessary to detect and estimate two faults in this system: a drop of the spool control force F_e, and an increase of the internal leakage of the piston which is normally assumed to be negligible.

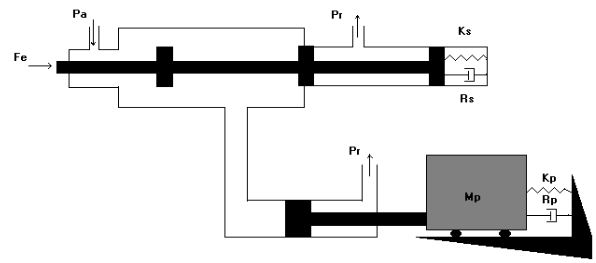

Fig. 1. Hydraulic system

The following notations will be used: x_1, displacement of the spool; x_2, velocity of the spool; x_3, displacement of the piston; x_4, velocity of the piston; x_5, pressure at the head side chamber; f_1, failure mode corresponding to the control force; f_2, failure mode corresponding to the internal leakage of the piston; A_p, area of the piston; D, diameter of the spool; B, bulk modulus; C_d, discharge coefficient; ρ, density of the fluid; K_s and R$_s$, respectively spring and damping coefficients associated to the spool; K_p and R_p respectively spring and damping coefficients associated to the load; M_s, and M_p respectively mass of the spool and mass of the piston together with the load.
Now, the model of the process is presented

$$\begin{array}{l} \dot{x}_1=x_2 \\ \dot{x}_2=-(K_sx_1+R_sx_2)/M_s+(F_e-F_F-f_1)/M_s \\ \dot{x}_3=x_4 \\ \dot{x}_4=(-K_px_3-R_px_4+A_px_5)/M_p \\ \dot{x}_5=\frac{B}{A_px_3}C_d\pi Dx_1\sqrt{\frac{2}{\rho}(P_a-x_5)}-B\frac{x_4}{x_3}-\frac{x_5x_4}{x_3} \\ -\frac{B}{A_p}\frac{f_2}{x_3} \end{array} \tag{22}$$

where $F_F=\frac{2C_d\pi D}{\rho}x_1(P_a-x_5)$ represents the full flow force acting on the spool. The available measurements are $y=\left[\begin{array}{cc} y_1 & y_2 \end{array}\right]^T=\left[\begin{array}{cc} x_1 & x_3 \end{array}\right]^T$.
All the state variables $x_i, i=1,...,5$ take values in closed intervals $[a_i,b_i]$, $i=1,...,5$. The position measurements are calibrated so that the lower bounds of the interval are positive, and thus the division by x_3 does not cause any problem.
Firstly, it is necessary to construct an algebraic equation to each component of the fault with coefficients in $k\langle u,y\rangle$.

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Obtaining a second time derivative of y_1:

$$\ddot{y}_1 = -\left(K_s y_1 + R_s \dot{y}_1\right)/M_s + \left(F_e - F_F - f_1\right)/M_s \tag{23}$$

Where:

$$\begin{aligned} F_F &= \tfrac{2C_d \pi D}{\rho} y_1 (P_a - x_5), \\ x_5 &= \left(M_p \ddot{y}_2 + K_p y_2 + R_p \dot{y}_2\right)/A_p \end{aligned} \tag{24}$$

and replacing (24) in (23), it is possible to obtain a differential algebraic polynomial for f_1 whose coefficients are in $k\langle u, y\rangle$

$$\begin{aligned} &0 = -\ddot{y}_1 - \left(K_s y_1 + R_s \dot{y}_1\right)/M_s + (F_e - \tfrac{2C_d \pi D}{\rho} y_1 \\ &(P_a - ((M_p \ddot{y}_2 + K_p y_2 + R_p \dot{y}_2/A_p)) - f_1)/M_s \end{aligned} \tag{25}$$

Also, replacing y_1 and y_2 in $\dot{x}_5$, the following equation is obtained:

$$\dot{x}_5 = \frac{B}{A_p y_2} C_d \pi D y_1 \sqrt{\frac{2}{\rho}(P_a - x_5)} - B\frac{\dot{y}_2}{y_2} - \frac{x_5 \dot{y}_2}{y_2} - \frac{B}{A_p}\frac{f_2}{y_2} \tag{26}$$

where, replacing $x_5 = \left(M_p \ddot{y}_2 + K_p y_2 + R_p \dot{y}_2\right)/A_p$, yields:

$$\begin{aligned} &0 = -\left(M_p \dddot{y}_2 + K_p \dot{y}_2 + R_p \ddot{y}_2\right)/A_p + \tfrac{B}{A_p y_2} C_d \pi D y_1 \\ &\left(\sqrt{\tfrac{2}{\rho}(P_a - \left(\left(M_p \ddot{y}_2 + K_p y_2 + R_p \dot{y}_2\right)/A_p\right))}\right) \\ &-B\tfrac{\dot{y}_2}{y_2} - \tfrac{((M_p \ddot{y}_2 + K_p y_2 + R_p \dot{y}_2)/A_p)\dot{y}_2}{y_2} - \tfrac{B}{A_p}\tfrac{f_2}{y_2} \end{aligned} \tag{27}$$

Equations (25) and (27) are the differential algebraic polynomials for f_1 and f_2 with coefficients in $k\langle u, y\rangle$. However, these polynomials depends on second and third derivatives of the output, which are unknown. So, it is not possible to construct a reduced order observer for the fault in a direct way. On the other hand, by Theorem 1, it is known that if a system is observable (i. e. the state *x* is algebraically observable) then it is diagnosable *iff* the fault *f* is observable with respect to *u, y* and *x*. It is not hard to see that this system is algebraically observable as follows:

$$\begin{aligned} x_1 - y_1 &= 0, \\ x_2 - \dot{y}_1 &= 0, \\ x_3 - y_2 &= 0, \\ x_4 - \dot{y}_2 &= 0, \\ A_p x_5 - \left(M_p \ddot{y}_2 + R_p \dot{y}_2 + K_p y_2\right) &= 0. \end{aligned}$$

So by Theorem 1, it is only necessary that each component of the fault be algebraically observable with respect to *u, y* and *x,* and it is not hard to see that this condition is satisfied.

Then, from (22) the following equations are obtained

$$\begin{aligned} &f_1 = -M_S \dot{x}_2 - (K_S y_1 + R_s x_2) + F_e - F_F \\ &f_2 = -\dot{x}_5 y_2 A_p/B + C_d \pi D y_1 \sqrt{\tfrac{2}{\rho}(P_a - x_5)} \\ &-A_p x_4 - \tfrac{A_p}{B} x_4 x_5 \end{aligned}$$

where it is clear that $difftrd^\circ R\langle u.y\rangle / R\langle u\rangle = 2$.

5 Reduced Order Uncertainty Observer

Let consider system (5). The fault vector *f* is unknown and it is assimilated as a state with uncertain dynamics. Then, in order to estimate it the state vector is extended to deal with the unknown fault vector. The new extended system is given by

$$\begin{aligned} \dot{x}(t) &= A(x, \bar{u}) \\ \dot{f} &= \Omega(x, \bar{u}) \\ y(t) &= h(x, u) \end{aligned} \tag{28}$$

where $\Omega(x, \bar{u})$ is assumed to be a bounded uncertain function

Note that a classic Luenberger observer can not be constructed because the term $\Omega(x, \bar{u})$ is unknown. The above problem is overcome using a reduced order uncertainty observer in order to estimate the failure variable *f.*

Hypotheses

H1: $\Omega(x, \bar{u})$ is bounded, i.e., $|\Omega(x, \bar{u})| \leq M$.

H2: $f(t)$ is algebraically observable over $k\langle u, y\rangle$.

H3: γ is a $\mathbb{C}^1$ real-valued function.

Where $\Omega(x, u)$ is a function which depends on the state vector and possibly the input. Next Lemma describes the construction of a proportional reduced-order observer for (28).

Lemma 2 [9] The system

$$\dot{\hat{f}} = K\left(f - \hat{f}\right) \tag{29}$$

is an asymptotic reduced order observer for system $\dot{f} = \Omega(x, \bar{u})$, where $\hat{f}$ denotes the estimate of fault f. Fault f is given by its algebraic equation with coefficients in $k\langle u, y\rangle$ and $K \in \mathbb{R}^+$ determines the desired convergence rate of the observer. ■

Corollary The dynamic system (29) along with

$$\dot{\gamma} = \psi(x, \bar{u}, \gamma), \text{ with } \gamma_0 = \gamma(0) \text{ and } \gamma \in \mathbb{C}^1 \tag{30}$$

constitute a proportional asymptotic reduced order fault observer for the system (28), where γ is a change of variable which depends on the estimated fault $\hat{f}$, and the states variables. ■

The performance of the reduced order observer estimator is shown by means of numerical simulations.

6 Numerical Results

Before to construct the reduced-order observer used for the fault estimation, the states x_2, x_4 and x_5, must be estimated, so it is proposed the following reduced-order observer:

For x_2 and x_4 it follows that

$$\begin{aligned} \hat{x}_2 &= \dot{\hat{y}}_1 = K_1(y_1 - \hat{y}_1) \\ \hat{x}_4 &= \dot{\hat{y}}_2 = K_2(y_2 - \hat{y}_2) \end{aligned}$$

In order to estimate x_5 the following system is constructed :

$$\begin{aligned} \hat{x}_5 &= \gamma_5 + \frac{K_5 M_p}{A_p}\hat{x}_4,\ \gamma_5 \in \mathbb{C}^1 \\ \dot{\gamma}_5 &= K_5\left[(K_p x_3 + R_p \hat{x}_4)/A_p - \gamma_5 - \frac{K_5 M_p}{A_p}\hat{x}_4\right] \end{aligned}$$

Then, it is now possible to construct the *diagnosability condition* for each one of the fault components as follows. In the case of f_1:

$$f_1 = -M_S \dot{\hat{x}}_2 - (K_S y_1 + R_s \hat{x}_2) + F_e - F_F \quad (31)$$

In order to eliminate the derivative $\dot{x}_2$, the following change of variable γ_{f_1} is proposed.
From (31), it is easy to obtain the following system, which is a reduced order observer for f_1:

$$\begin{aligned} &\hat{f}_1 = \gamma_{f_1} - K_{f_1} M_S \hat{x}_2,\ \gamma_{f_1} \in \mathbb{C}^1 \\ &\dot{\gamma}_{f_1} = K_{f_1}\left[-(K_S y_1 + R_S \hat{x}_2) + F_e - F_F - \right. \\ &\left. \gamma_{f_1} + K_{f_1} M_S \hat{x}_2\right] \end{aligned}$$

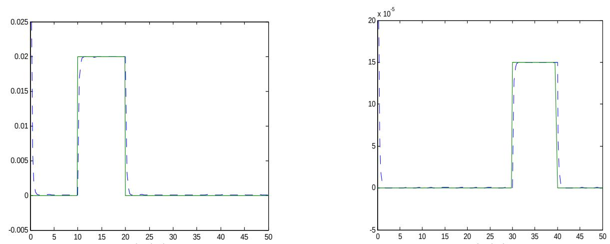
In a similar manner, for f_2

$$f_2 = -\dot{\hat{x}}_5 y_2 A_p/B + C_d \pi D y_1 \sqrt{\tfrac{2}{\rho}(P_a - \hat{x}_5)} - A_p \hat{x}_4 - \tfrac{A_p}{B}\hat{x}_4 \hat{x}_5$$

Then, the following system is a reduced order observer for f_2:

$$\begin{aligned} &\hat{f}_2 = \gamma_{f_2} - \tfrac{K_{f_2} A_p}{B} y_2 \hat{x}_5,\ \gamma_{f_2} \in \mathbb{C}^1 \\ &\dot{\gamma}_{f_2} = K_{f_2}\left[C_d \pi D y_1 \sqrt{\tfrac{2}{\rho}(P_a - \hat{x}_5)} - A_p \hat{x}_4 \right. \\ &\left. -\gamma_{f_2} + \tfrac{K_{f_2} A_p}{B} y_2 \hat{x}_5\right] \end{aligned}$$

Numerical simulations corresponding to this example are presented in figure 2.



a) b)

Fig 2. Hydraulic system. a) Fault f_1 (solid line) and estimate $\hat{f}_1$ (dotted line). b) Fault f_2 (solid line) and estimate $\hat{f}_2$ (dotted line).

We verify the performance of the proportional reduced-order fault estimator by means of numerical simulations. Next, the values used for the simulation of the hydraulic system are presented.

$M_S = 0.1kg$, $R_S = 2.1Ns/m$, $K_S = 10^3 N/m$, $D_p = 0.2m$ (Diameter of the piston), $D = 0.01m$, $\rho = 840kg/m^3$, $B = 10^9 N/m^2$, $C_d = 0.7kg/m^3$, $P_a = 220 \times 10^5 N/m^2$, $M_p = 5 \times 10^3 kg$, $R_p = 10^4 Ns/m$, $K_p = 5 \times 10^5 N/m$.

With $K_1 = 10$, $K_2 = 10$, $K_5 = 10$, $K_{f_1} = 2$, $K_{f_2} = 2$, and the following initials conditions:
$\gamma_5(0) = \gamma_{f_1}(0) = \gamma_{f_2}(0) = 0.001$.

Note that the estimate fault converges to the actual fault.

7 Concluding remarks

We have tackled the *diagnosis problem in nonlinear systems* under failure conditions using the *algebraic observability* and the *differential transcendence degree of a differential field extension* concepts. A reduced-order uncertainty observer is used to estimate the fault variable. Numerical simulations were presented to illustrate the effectiveness of the suggested approach.

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