

# Speed Sensorless Rotor Flux Estimation in Vector Controlled Induction Motor Drive

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*Abstract:* - This paper presents a speed sensorless rotor flux estimation algorithm in a vector controlled induction motor drive. The proposed method is based on observing a newly defined state which replaces the unknown terms containing rotor flux and speed on right hand side of the state equation of the motor. A new mathematical model of the motor is derived after introducing the above mentioned state. Rotor flux estimation is achieved using a modified Blaschke equation obtained after introducing the new state into the Blaschke equation. Rotor speed is computed using a simple equation derived using the newly defined state.

*Key-words:* Induction motor, vector control, flux estimation, speed estimation, reduced order observer.

## 1 Introduction

Vector Control (VC) [1, 2] has revolutionised the use of induction motors in high performance drive applications. Conventional VC drive uses speed sensor such as a shaft encoder for speed control. However, a speed sensor can not be mounted in certain applications such as a motor drive in hostile environment or a high speed drive etc. It also requires careful cabling arrangement with proper attention to electrical noise. Moreover, it makes the drive system more bulky and expensive. Therefore, a lot of research are underway to develop good speed estimation methods. Recently several speed sensorless vector control schemes have been proposed [3-12].

Induction machines do not allow rotor flux to be easily measured, therefore, for vector control one has to resort to flux estimation. The current model (CM) and the voltage model (VM) are the traditional solutions, and their benefits and drawbacks are well known [13]. Various observers for flux estimation were analyzed in the pioneering work by Verghese and Sanders [14]. Over the years several other have been presented, many of which include speed estimation [6-12, 15].

In [6] extended kalman filter was used for estimating the rotor flux and speed using a full order model of the motor assuming that rotor speed is a constant. A speed adaptive flux observer was proposed in [7] for estimating rotor flux and speed.

In [8] Gopinath style reduced order observer was used for estimating the rotor flux while the speed was computed using an equation derived from the motor model. Tajima et al [9] proposed MRAS [3] with novel pole allocation method for speed estimation while rotor flux estimation was done using Gopinath's observer. Yan et al [10] proposed a flux and speed estimator based on the sliding-mode control methodology. Ohtani et al [11] used the voltage model for flux estimation overcoming the problem associated with integrator and low pass filter while speed was computed by subtracting slip speed from the synchronous speed. In [12] voltage model was used for rotor flux estimation [11] and speed estimation was achieved based on a reduced order observer which estimates a new quantity that is used along with rotor flux in computing the speed.

In this paper we present a speed sensorless rotor flux estimation algorithm in a vector controlled induction motor drive. The proposed method is based on observing a newly defined state [12] which is a function of rotor flux and speed. Availability of this state makes the speed sensorless rotor flux estimation possible. Rotor flux estimation proposed in this work is achieved using a modified Blaschke equation obtained after introduction of the quantity into the motor model. Rotor speed is computed using a simple equation which is derived using the newly defined state [12].

## 2 Induction Motor Model

The induction motor model in stationary stator reference frame  $\alpha$ - $\beta$  is given by:

$$\frac{d\boldsymbol{\Psi}_r}{dt} = \mathbf{A}_{11}\boldsymbol{\Psi}_r + \mathbf{A}_{12}\mathbf{i}_s \quad (1)$$

$$\frac{d\mathbf{i}_s}{dt} = \mathbf{A}_{21}\boldsymbol{\Psi}_r + \mathbf{A}_{22}\mathbf{i}_s + \mathbf{A}_{23}\mathbf{v}_s \quad (2)$$

Where

$$\mathbf{A}_{11} = -(R_r / L_r)\mathbf{I} + \omega\mathbf{J}, \mathbf{A}_{12} = (L_m R_r / L_r)\mathbf{I},$$

$$\mathbf{A}_{21} = L_m / (\sigma L_s L_r) \{ (R_r / L_r)\mathbf{I} - \omega\mathbf{J} \}$$

$$\mathbf{A}_{22} = -\left\{ R_s / (\sigma L_s) + R_r L_m^2 / (\sigma L_s L_r^2) \right\} \mathbf{I},$$

$$\mathbf{A}_{23} = 1 / (\sigma L_s) \mathbf{I}, \boldsymbol{\Psi}_r = [\Psi_{r\alpha} \quad \Psi_{r\beta}]^T,$$

$$\mathbf{i}_s = [i_{s\alpha} \quad i_{s\beta}]^T, \mathbf{I} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} \text{ and } \mathbf{J} = \begin{bmatrix} 0 & -1 \\ 1 & 0 \end{bmatrix}$$

## 3 Rotor Flux and Speed Estimation

We introduce a new state into the motor model which when introduced will make the right hand side of the (1) and (2) independent of the unknowns – the rotor flux and speed. Let's define the new state as:

$$\mathbf{Z} = -\mathbf{A}_{11}\boldsymbol{\Psi}_r \quad (3)$$

A new motor model is obtained after introduction of the new state as given below:

$$\frac{d\boldsymbol{\Psi}_r}{dt} = \mathbf{A}_{12}\mathbf{i}_s + \mathbf{A}_{14}\mathbf{Z} \quad (4)$$

$$\frac{d\mathbf{i}_s}{dt} = \mathbf{A}_{22}\mathbf{i}_s + \mathbf{A}_{23}\mathbf{v}_s + \mathbf{A}_{24}\mathbf{Z} \quad (5)$$

$$\frac{d\mathbf{Z}}{dt} = \mathbf{A}_{32}\mathbf{i}_s + \mathbf{A}_{34}\mathbf{Z} \quad (6)$$

where  $\mathbf{A}_{14} = -\mathbf{I}$ ,  $\mathbf{A}_{24} = \{L_m / (\sigma L_s L_r)\}\mathbf{I}$ ,

$$\mathbf{A}_{32} = (L_m R_r^2 / L_r^2)\mathbf{I} - \omega(L_m R_r / L_r)\mathbf{J} \quad \text{and}$$

$$\mathbf{A}_{34} = \mathbf{A}_{11}$$

It can be seen from (3) and (4) that speed and rotor flux can be estimated if  $\mathbf{Z}$  is known. A Gopinath's reduced order observer [16] is constructed using (5) and (6) for estimating  $\mathbf{Z}$ . The  $\mathbf{Z}$  observer equation is as given below [12]:

$$\frac{d\hat{\mathbf{Z}}}{dt} = \mathbf{A}_{32}\mathbf{i}_s + \mathbf{A}_{34}\hat{\mathbf{Z}} + \mathbf{G}\left(\frac{d\mathbf{i}_s}{dt} - \frac{d\hat{\mathbf{i}}_s}{dt}\right) \quad (7)$$

where  $\mathbf{G} = \begin{bmatrix} g_1 & g_2 \\ -g_2 & g_1 \end{bmatrix}$  is the observer gain.

Using equation (6) for  $\frac{d\hat{\mathbf{i}}_s}{dt}$  the observer equation becomes:

$$\begin{aligned} \frac{d\hat{\mathbf{Z}}}{dt} = & \mathbf{A}_{32}\mathbf{i}_s + \mathbf{A}_{34}\hat{\mathbf{Z}} \\ & + \mathbf{G}\left(\frac{d\mathbf{i}_s}{dt} - \mathbf{A}_{22}\mathbf{i}_s - \mathbf{A}_{23}\mathbf{v}_s - \mathbf{A}_{24}\hat{\mathbf{Z}}\right) \end{aligned} \quad (8)$$

The observer poles can be placed at the desired locations in the stable region of the complex plane by properly choosing the values of the elements of the G matrix. In order to avoid derivative of the stator current in the algorithm we introduce another new quantity:

$$\mathbf{F} = \hat{\mathbf{Z}} - \mathbf{G}\mathbf{i}_s \quad (9)$$

Finally, the observer is of the following form:

$$\begin{aligned} \frac{d}{dt}\mathbf{F} = & (\mathbf{A}_{32} + \mathbf{A}_{34}\mathbf{G} - \mathbf{G}\mathbf{A}_{22} - \mathbf{G}\mathbf{A}_{24}\mathbf{G})\mathbf{i}_s - \mathbf{G}\mathbf{A}_{23}\mathbf{v}_s \\ & + (\mathbf{A}_{34} - \mathbf{G}\mathbf{A}_{24})\mathbf{F} \end{aligned} \quad (10)$$

$$\hat{\mathbf{Z}} = \mathbf{F} + \mathbf{G}\mathbf{i}_s \quad (11)$$

Assuming no parameter variation and no speed error, the equation for error dynamics is given by:

$$\frac{d}{dt}\tilde{\mathbf{Z}} = \frac{d}{dt}(\mathbf{Z} - \hat{\mathbf{Z}}) = (\mathbf{A}_{34} - \mathbf{A}_{24}\mathbf{G})\tilde{\mathbf{Z}} \quad (12)$$

Eigenvalues of  $(\mathbf{A}_{34} - \mathbf{A}_{24}\mathbf{G})$  are the observer poles which are as given below:

$$P_{obs1,2} = -\left(\frac{R_r}{L_r} + \frac{L_m}{\sigma L_s L_r} g_1\right) \pm j\left(\omega - \frac{L_m}{\sigma L_s L_r} g_2\right) \quad (13)$$

It is to be noted here that the model of the motor used in implementing the observer algorithm has been developed assuming that the derivative of the rotor speed is zero. It is valid to make such an assumption since the dynamics of rotor speed is

much slower than that of electrical states. Moreover, such an assumption allows estimation without requiring the knowledge of mechanical quantities of the drive such as load torque, inertia etc.

### 3.1 Rotor Flux Estimation

Rotor flux may be obtained from the modified Blaschke equation (4) which is obtained after introduction of the new state  $\mathbf{Z}$ . However, rotor flux computation by pure integration suffers from dc offset and drift problems. To overcome the above problems a low pass filter is used instead of pure integrator and the phase error due to low pass filtering is approximately compensated by adding low pass filtered reference flux with the same time constant as used above [11]. The equation of the proposed rotor flux estimator is as given below:

$$\hat{\Psi}_r = \frac{\tau}{1 + \tau s} (A_{12} i_s + A_{14} \mathbf{Z}) + \frac{1}{1 + \tau s} \Psi_r^* \quad (14)$$

where  $\tau$  is the LPF time constant. The command rotor flux  $\Psi_r^*$  is obtained as follows:

$$\Psi_r^* = \begin{bmatrix} \Psi_{r\alpha}^* \\ \Psi_{r\beta}^* \end{bmatrix} = \begin{bmatrix} \Psi_r^* \cos \rho^* \\ \Psi_r^* \sin \rho^* \end{bmatrix} \quad (15)$$

where  $\Psi_r^* = L_m i_{sd}^*$  and  $\rho^*$  the command rotor flux angle is as given by:

$$\rho^* = \int \omega_s^* dt \quad (16)$$

$\omega_s^*$  the command rotor flux speed is computed as given below:

$$\omega_s^* = \omega_{sl}^* + \hat{\omega} \quad (17)$$

The command slip speed  $\omega_{sl}^*$  is given by:

$$\omega_{sl}^* = \frac{R_r i_{qs}^*}{L_r i_{ds}^*} \quad (18)$$

We know that the equation of the back emf is given by:

$$\mathbf{e} = \frac{L_m}{L_r} \frac{d\Psi_r}{dt} = \frac{L_m}{L_r} (A_{12} i_s + A_{14} \mathbf{Z}) \quad (19)$$

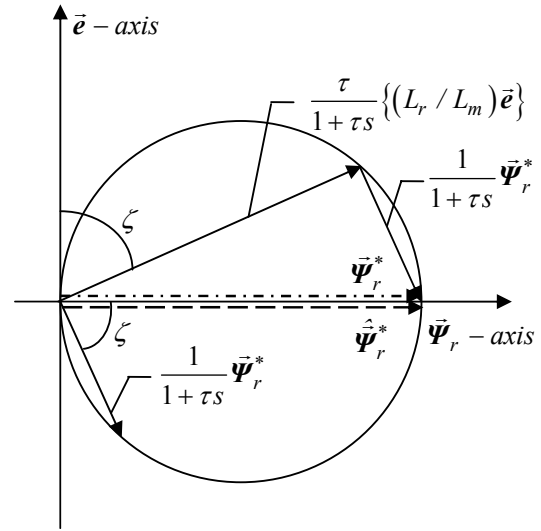


Fig. 1. Obtaining of estimated rotor flux

Now, equation (14) may also be written as:

$$\hat{\Psi}_r = \frac{\tau}{1 + \tau s} \left( \frac{L_r}{L_m} \mathbf{e} \right) + \frac{1}{1 + \tau s} \Psi_r^* \quad (20)$$

Fig. 1. explains how estimated flux is obtained using equation (20).

### 3.2 Speed Estimation

Performing matrix multiplication of  $\Psi_r^T \mathbf{J}$  with equation (3) we have:

$$Z_\alpha \Psi_{r\beta} - Z_\beta \Psi_{r\alpha} = (\Psi_{r\alpha}^2 + \Psi_{r\beta}^2) \omega \quad (21)$$

This simple equation is used for computing speed by replacing the actual values of  $\mathbf{Z}$  and flux by estimated ones as given below [12]:

$$\hat{\omega} = \frac{\hat{Z}_\alpha \hat{\Psi}_{r\beta} - \hat{Z}_\beta \hat{\Psi}_{r\alpha}}{\hat{\Psi}_{r\alpha}^2 + \hat{\Psi}_{r\beta}^2} \quad (22)$$

The block diagram of the flux and speed estimator is shown in Fig. 1.



First, acceleration and speed reversal at no load is performed. A speed command of 150 rad/s at 0.5 s is given to the drive system which was initially at rest, and then the speed is reversed at 3 s. The response of the drive is shown in Fig. 4. Fig. 4 (a) shows reference ( $\hat{\omega}^*$ ), actual ( $\omega$ ), estimated ( $\hat{\omega}$ )

speed, and speed estimation error ( $\omega - \hat{\omega}$ ). The module of the actual ( $|\Psi_r|$ ), estimated ( $|\hat{\Psi}_r|$ ) rotor flux, and rotor flux estimation error ( $|\Psi_r| - |\hat{\Psi}_r|$ ) are shown in Fig. 4 (b). Fig. 4 (c) and (d) shows respectively the locus of the actual and estimated rotor flux.

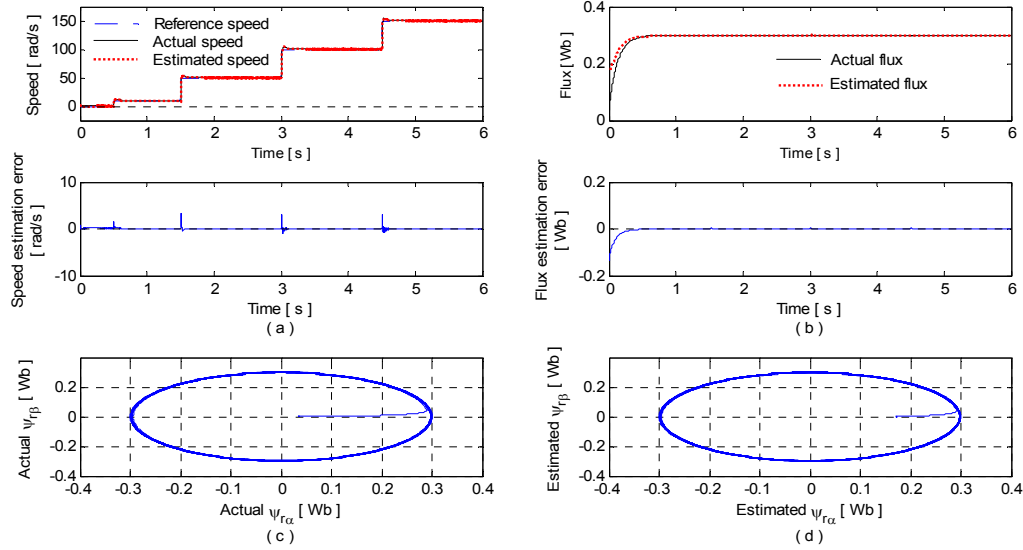


Fig. 5. No-load operation with step increase in speeds; (a) reference ( $\hat{\omega}^*$ ), actual ( $\omega$ ) and estimated ( $\hat{\omega}$ ) speed, and speed estimation error; (b) actual ( $|\Psi_r|$ ) and estimated ( $|\hat{\Psi}_r|$ ) rotor flux, and rotor flux estimation error ( $|\Psi_r| - |\hat{\Psi}_r|$ ); (c) locus of actual flux ( $\Psi_{r\beta}$  vs.  $\Psi_{r\alpha}$ ); (d) locus of estimated flux ( $\hat{\Psi}_{r\beta}$  vs.  $\hat{\Psi}_{r\alpha}$ ).

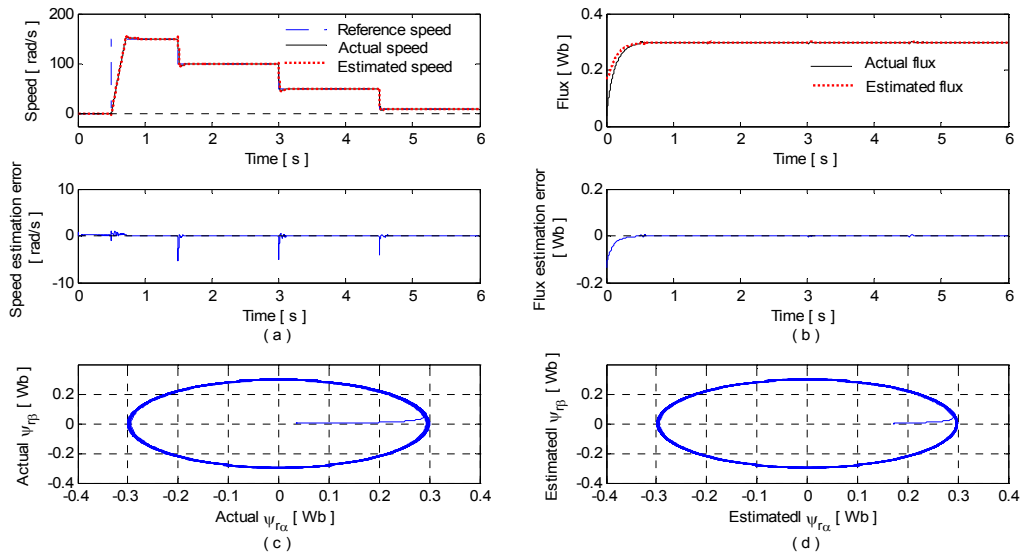


Fig. 6 Operation at full load at various speeds; (a) reference ( $\hat{\omega}^*$ ), actual ( $\omega$ ) and estimated ( $\hat{\omega}$ ) speed, and speed estimation error; (b) actual ( $|\Psi_r|$ ) and estimated ( $|\hat{\Psi}_r|$ ) rotor flux, and rotor flux estimation error ( $|\Psi_r| - |\hat{\Psi}_r|$ ); (c) locus of actual flux ( $\Psi_{r\beta}$  vs.  $\Psi_{r\alpha}$ ); (d) locus of estimated flux ( $\hat{\Psi}_{r\beta}$  vs.  $\hat{\Psi}_{r\alpha}$ ).

Then, the drive is subjected to step increase in speed under no load condition. It is accelerated from rest to 10 rad/s at 0.5 s, then accelerated further to 50 rad/s, 100 rad/s and 150 rad/s at 1.5 s, 3 s and 4.5 s respectively. Fig. 5 shows the estimation of rotor flux and speed, and the response of the sensorless drive system.

Further, the performance of the estimator is verified under loaded conditions at various operating speeds. The fully loaded drive is accelerated to 150 rad/s at 0.5 s and then decelerated in steps to 100 rad/s, 50 rad/s and 10 rad/s at 1.5 s, 3 s and 4.5 s respectively. Fig. 6 shows the estimation results and response of the loaded drive system.

## 5 Conclusion

A speed sensorless rotor flux estimation algorithm in vector controlled induction motor drive has been proposed. The rotor flux estimation was achieved using a modified Blaschke equation obtained after introducing a newly defined state. The rotor speed was computed using a simple equation obtained using the newly defined state. Accurate estimations of rotor flux and speed were achieved under both transient and steady state conditions, and the response of the sensorless vector controlled induction motor drive was found to be satisfactory.

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