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Generalized Detector under Nonorthogonal Multipulse Modulation in Remote Sensing Systems

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Abstract: - We develop the minimum mean-squared-error (MMSE) multiuser generalized detector for nonorthogonal multipulse modulation in wireless sensor networks employed by remote sensing systems over the noncoherent additive white Gaussian noise channel. The generalized detector (GD) is constructed based on the generalized approach to signal processing in the presence of noise. We analyze the asymptotic performance of the generalized detector and show that, unlike the case of linear modulation, the MMSE generalized detector does not generally approach the generalized detector discussed previously but outperforms the detector based on the generalized maximum-likelihood detection rule. However, it does approach a detector, which nulls out the multi-access interference. This detector is termed the multipulse generalized detector due to its similarity to the decorrelating detector. The probability of error for this generalized detector is derived and used to find the asymptotic multiuser efficiencies of both the multipulse generalized detector and the MMSE generalized detector. It is shown that for noncoherent binary signaling, in which the multipulse modulation is two-dimensional, the multipulse generalized detector is superior to the generalized maximum-likelihood detector. This result does not generalize to larger dimensionality signal set.

Key-Words: - Minimum mean-squared-error detection, multipulse modulation, multiuser detection, generalized detector, noncoherent detection.

1 Introduction

Orthogonal signaling is often employed in wireless sensor networks applied by remote sensing systems to communicate over noncoherent channels. However, when multiple sensor nodes and sinks share such a channel, the assignment of mutually orthogonal signal sets to each sensor node and sink requires a large bandwidth. Moreover, if each sensor node and sink employ a signal set, which is orthogonal but correlated with the other sensor nodes and sinks, then a low complexity receiver may produce an effective signal constellation, which is no longer orthogonal for each sensor node and sink. For these reasons, we consider the more general case of nonorthogonal multipulse modulation, in which each sensor node and sink are assigned a possibly correlated signal set, from which one signal is transmitted at each signaling period. There is also the possibility of bandwidth savings through the use of nonorthogonal multipulse modulation. Orthogonal signaling schemes require a bandwidth, which grows linearly with the number of signals employed, while nonorthogonal multipulse modulation can generally be made much more spectrally efficient. Zero-forcing or decorrelative detection of such signals has been studied recently in [1]–[4]. These detectors act to first remove the multiple-access interference through a perpendicular projection of the received data out of the span of the interfering sensor node's signals. This operation is followed by either the asymptotically optimal detector [4] if the sensor node's signal energies are available or by the generalized maximum-likelihood (GML) detector [1]–[4] in the absence of this information. Extension of the subspace tracking techniques of [5] is also discussed in [6] to develop a blind generalized maximum-likelihood detector, which employs subspace tracking to estimate the interfering sensor node's subspace. In this paper, we consider the use of the minimum mean-squared-error (MMSE) rule for nonorthogonal multipulse modulation in wireless sensor network employed by remote sensing systems under the generalized approach to signal processing in the presence of noise [7]–[11]. The MMSE detector in the universally adopted sense was previously derived in [6] and it was noted that for example therein, the GML detector appeared to outperform the MMSE detector asymptotically. We derive the asymptotic performance of the minimum mean-squared-error

(MMSE) GD and compare it to that of the GD derived in [7]–[11]. We show that MMSE GD does approach a detector, which completely nulls the multi-access interference. This detector is termed the multipulse generalized detector (MGD) due to its similarity by principles of functioning to the linear decorrelating detector of [12]. It is shown that the MGD and the MMSE GD outperform the GML detector for both high and low values of the signal-to-noise ratio (SNR). We prove that MGD and hence the MMSE GD are asymptotically superior to the GD for binary signaling [8] from a two-dimensional multipulse signal set. This result does not extend to higher dimension signaling. The performance comparison between the MGD and MMSE GD at low SNRs is still an open discussion.

2 Discrete Time Model

The discrete time model for the output of the noncoherent channel with nonorthogonal multipulse modulation can be presented in the following form [6]

$$\mathbf{y} = \mathcal{H}\mathcal{D}\,\tilde{\mathbf{b}} + \mathbf{n} \ . \quad (1)$$

The matrix

$$\mathcal{H} = [\mathbf{H}(1), \mathbf{H}(2), \ldots, \mathbf{H}(K)] \quad (2)$$

contains the signal vectors for each sensor node with

$$\mathbf{H}(k) = [\mathbf{h}_1(k), \mathbf{h}_2(k), \ldots, \mathbf{h}_M(k)] \quad (3)$$

and $\mathbf{h}_m(k)$ is the m-th signal corresponding to sensor node k. The vector

$$\tilde{\mathbf{b}} = [\mathbf{b}^T(1), \mathbf{b}^T(2), \ldots, \mathbf{b}^T(K)]^T \quad (4)$$

is a $MK \times 1$ vector with each $\mathbf{b}(k)$ a column of the $M \times M$ identity matrix, which selects the signal transmitted by sensor node k. That is,

$$\mathbf{H}(k)\mathbf{b}_m(k) = \mathbf{h}_m(k) \ . \quad (5)$$

The $MK \times MK$ matrix

$$\mathcal{D} = diag\{\sqrt{E_1}e^{j\theta_1(1)}, \sqrt{E_1}e^{j\theta_2(1)}, \ldots, \sqrt{E_1}e^{j\theta_M(1)}, \ldots, \sqrt{E_K}e^{j\theta_1(K)}, \ldots, \sqrt{E_K}e^{j\theta_1(M)}\} \quad (6)$$

contains the sensor node energies and phase terms. The individual gain parameters, $\sqrt{E_k}e^{j\theta_m(1)}$, are modeled as having an amplitude, $\sqrt{E_k}$, which is independent of the transmitted symbol, but a phase, $\theta_m(k)$, which may be hypothesis dependent. The additive noise, **n**, is modeled as zero-mean complex Gaussian with correlation matrix

$$M[\mathbf{n}\mathbf{n}^*] = \sigma^2\mathbf{I} \ . \quad (7)$$

Assuming that the phase terms are independent zero-mean random variables, the measurement **y** has first and second order statistics

$$\mathbf{m} = M[\mathbf{y}] = 0 \ , \ \mathbf{R} = M[\mathbf{y}\mathbf{y}^*] = \frac{1}{M}\mathcal{H}\,\mathbf{F}\,\mathcal{H}^* + \sigma^2\mathbf{I} \quad (8)$$

where

$$\mathbf{F} = diag\{E_1\mathbf{I}, \ldots, E_K\mathbf{I}\} \ . \quad (9)$$

We may expand this model when the k-th sensor node is of interest and has the transmitted signal $\mathbf{h}_m(k)$

$$\mathbf{y} = \sqrt{E_k}e^{j\theta_m(k)}\mathbf{h}_m(k) + \mathbf{S}(k)\beta + \mathbf{n} \ , \quad (10)$$

where the matrix

$$\mathbf{S}(k) \in \mathcal{C}^{N \times M(K-1)} \quad (11)$$

is formed from the matrices $\mathbf{H}(l)$ for $l \neq k$ and

$$\beta \in \mathcal{C}^{M(K-1)} \quad (12)$$

is formed by stacking vectors $\sqrt{E_l}e^{j\theta_{ml}}\mathbf{b}(k)$. We assume the sensor nodes communicate independently.

3 MMSE Generalized Detector

The MMSE GD estimator of the vector $\mathcal{D}\tilde{\mathbf{b}}$ according to the generalized approach to signal processing in the presence of noise [7]–[11] is given by

$$\hat{\mathcal{K}} = \hat{\mathcal{D}}\hat{\tilde{\mathbf{b}}} = 2\mathbf{F}\mathcal{H}^*\mathbf{R}^{-1}\mathbf{y} - \mathbf{y}^*\mathbf{R}^{-1}\mathbf{y} + \mathbf{n}_1^*\mathbf{R}^{-1}\mathbf{n}_1 , \quad (13)$$

where $\mathbf{n}_1$ is a vector of an additional noise source with the same statistical characteristics as the noise vector **n** included in the measurement **y**; $\mathbf{F}, \mathcal{H}$, and **R** are defined in the previous section. If we consider the k-th block of $\hat{\mathcal{K}} = \hat{\mathcal{D}}\hat{\tilde{\mathbf{b}}}$, $\hat{\mathbf{D}}(k)\,\hat{\mathbf{b}}(k)$, we obtain a decision rule for the MMSE GD in the case of the noncoherent channel

$$\hat{m}_{MMSE\,GD}(k) = \arg\max_m |\{\hat{\mathbf{D}}(k)\hat{\mathbf{b}}(k)\}_m|^2$$
$$= \arg\max_m \{|\,2\mathbf{h}_m^*(k)\mathbf{R}^{-1}\mathbf{y} - \mathbf{y}^*\mathbf{R}^{-1}\mathbf{y} + \mathbf{n}_1^*\mathbf{R}^{-1}\mathbf{n}_1\,|^2\} \ . \quad (14)$$

This is motivated by the fact that the true vector $\mathbf{D}(k)\,\mathbf{b}(k)$ has the form

$$\mathbf{D}(k)\mathbf{b}(k) = [0 \ \cdots \ 0 \ \mu_m(k) \ 0 \ \cdots \ 0]^T \ , \quad (15)$$

i.e., it is nonzero only in the m-th position when symbol m is transmitted by sensor node k. Geometrically, we see that the noncoherent generalized detector seeks the signal vector, $\mathbf{R}^{-0.5}\mathbf{h}_m(k)$, which is closest to measurement, $\mathbf{R}^{-0.5}\mathbf{y}$, in terms of the magnitude squared inner product. This MMSE GD chooses the maximum of a bank of M noncoherent generalized detectors.

4 Multipulse Generalized Detector

For nonorthogonal multipulse modulation the generalized detector (GD) is given by [13]

$$\hat{m}_{GD}(k) = \arg\max_m \frac{|2\mathbf{h}_m^*(k)\mathbf{P}_{\mathbf{S}(k)}^{\perp}\mathbf{y} - \mathbf{y}^*\mathbf{P}_{\mathbf{S}(k)}^{\perp}\mathbf{y} + \mathbf{n}_1^*\mathbf{P}_{\mathbf{S}(k)}^{\perp}\mathbf{n}_1|^2}{\mathbf{h}_m^*(k)\mathbf{P}_{\mathbf{S}(k)}^{\perp}\mathbf{h}_m(k)}, \quad (16)$$

where $\mathbf{P}_{\mathbf{S}(k)}^{\perp}$ is the orthogonal projection matrix with null space $\langle \mathbf{S}(k)\rangle$, the so-called multiple-access interference. The notation $\langle \mathbf{S}\rangle$ denotes the subspace spanned by the columns of the matrix **S**. For the case of linear coherent modulation, the MMSE GD is known to approach the GD [13]. However, we can show that the MMSE GD is not generally asymptotically equivalent to the GD for nonorthogonal multipulse modulation, although they are both zero-forcing, resulting in complete multiple-access interference removal

We can show that the MMSE GD can be asymptotically given by

$$\hat{m}_{MGD}(k) = \arg\max_m |\{2[\mathbf{H}^*(k)\mathbf{P}_{\mathbf{S}(k)}^{\perp}\mathbf{H}(k)]^{+}\mathbf{H}^*(k)\mathbf{P}_{\mathbf{S}(k)}^{\perp}\mathbf{y} - \mathbf{y}^*\mathbf{P}_{\mathbf{S}(k)}^{\perp}\mathbf{y} + \mathbf{n}_1^*\mathbf{P}_{\mathbf{S}(k)}^{\perp}\mathbf{n}_1\}_m|^2, \quad (17)$$

where the superscript "+"denotes the pseudoinverse, so long as $\text{Rank}\{\mathbf{H}(k)\} = \text{Rank}\{\mathbf{P}_{\mathbf{S}(k)}^{\perp}\mathbf{H}(k)\}$. This condition implies that $\mathbf{H}(k)$ and $\mathbf{S}(k)$ are linearly independent, i.e., that if $\text{Rank}\{\mathbf{H}(k)\} = r$ and $\text{Rank}\{\mathbf{S}(k)\} = p$, then $\text{Rank}\{\mathbf{H}(k)\mathbf{S}(k)\} = p + r$. This may be assumed without loss of generality since otherwise sensor node *k* is wasting power by communicating along a coordinate vector lying completely in the span of the interference. This condition does not require the matrix $\mathbf{H}(k)$ to be full rank (we can have $r < M$). Notice that the MMSE GD approaches the GD only when the interference-nulled correlation matrix, $\mathbf{H}^*(k)\mathbf{P}_{\mathbf{S}(k)}^{\perp}\mathbf{H}(k)$, is a scalar multiple of the identity matrix. We can show a correctness of (17) based on the following statements. Rewrite the matrix **R** given by (8) in the following form:

$$\mathbf{R} = \frac{E_k}{M}\mathbf{H}\mathbf{H}^* + \mathbf{B}, \quad (18)$$

with

$$\mathbf{B} = \frac{1}{M}\sum_{l \neq k} E_l \mathbf{H}(l)\mathbf{H}^*(l) + \sigma^2\mathbf{I} \quad (19)$$

and

$$E_k = M[|\mu(k)|^2]. \quad (20)$$

Application of the Woodbury identity yields

$$\mathbf{H}^*\mathbf{R}^{-1}\mathbf{y} = \frac{M}{E_k}[\frac{M}{E_k}\mathbf{I} + \mathbf{H}^*\mathbf{B}^{-1}\mathbf{H}]^{-1}\mathbf{H}^*\mathbf{B}^{-1}\mathbf{y}. \quad (21)$$

In [14] is shown that

$$\frac{E_k}{M}\mathbf{H}^*\mathbf{R}^{-1} \to [\mathbf{H}^*\mathbf{P}_{\mathbf{S}(k)}^{\perp}\mathbf{H}]^{+}\mathbf{H}^*\mathbf{P}_{\mathbf{S}(k)}^{\perp}, \quad (22)$$

where we have used the standard construction of the pseudoinverse of $\mathbf{P}_{\mathbf{S}(k)}^{\perp}\mathbf{H}$ in terms of its singular value decomposition. The term $\frac{E_k}{M}$ is hypothesis independent and may be dropped. The detector in (17) appears to be original and we call it the multipulse generalized detector (MGD). Using the results of [8] and [13], we find that the MGD may be derived by maximizing the likelihood function

$$f(\mathbf{y}) = \frac{1}{(\pi\sigma^2)^N}\exp\{-\frac{1}{\sigma^2}\|\mathbf{y} - \mathbf{H}(k)\mathbf{D}(k)\mathbf{b}(k) - \mathbf{S}(k)\beta\|^2\} \quad (23)$$

jointly over $\mathbf{D}(k)\mathbf{b}(k)$ and β, and choosing *m* as the entry of $\hat{\mathbf{D}}(k)\hat{\mathbf{b}}(k)$ of largest magnitude. It is, perhaps worth noting that MGD estimates the signal $\mathbf{H}(k)\mathbf{D}(k)\mathbf{b}(k)$ before imposing the *a priori* knowledge that this term is of the form $\sqrt{E_k}e^{j\theta_m(k)}\,\mathbf{h}_m(k)$, whereas the GD imposes this constraint from the outset.

5 Performance Analysis

In this section, we analyze the performance of the MGD. As this performance characterizes the MMSE GD asymptotically, it is useful in the investigation of both detectors. We can build asymptotically tight bounds of the probability of error, P_{MGD}^{er}, for the MGD via

$$\max_{l \neq m} - P_{MGD}^{er}(m,l) \le P_{MGD}^{er} \le \frac{1}{M}\sum_{m=1}^{M}\sum_{\substack{l=1 \\ l \neq m}}^{M} P_{MGD}^{er}(m,l), \quad (24)$$

where $P_{MGD}^{er}(m,l)$ is the probability that the *l*-th statistic in (17) is greater then the *m*-th statistic when signal *m* is transmitted. The upper and lower bounds are asymptotically coincident on the additive white Gaussian noise channel and so we will concentrate on the lower bound. Each term, $P_{MGD}^{er}(m,l)$, is found from

$$P_{MGD}^{er}(m,l) = \Pr\left[|\{(\mathbf{E}^*(k)\mathbf{E}(k))^{+}\mathbf{E}_m^*(k)\mathbf{y}\}_m|^2 < |\{(\mathbf{E}^*(k)\mathbf{E}(k))^{+}\mathbf{E}^*(k)\mathbf{y}\}_l|^2 \,\middle|\, H_m\right] = \Pr\left[\mathbf{y}^*\mathbf{E}(\mathbf{E}^*\mathbf{E})^{+}(\mathbf{e}_m\mathbf{e}_m^T - \mathbf{e}_l\mathbf{e}_l^T)(\mathbf{E}^*\mathbf{E})^{+}\mathbf{E}^*\mathbf{y} < 0 \,\middle|\, H_m\right] \quad (25)$$

where H_m is the hypothesis that signal m is transmitted, $\mathbf{e}_m$ is the m-th column of the identity matrix, $\mathbf{E} = \mathbf{P}_{\mathbf{S}(k)}^{\perp}\mathbf{H}(k)$, and we have dropped the dependency on k. We can form the eigendecomposition $\mathbf{E}^*\mathbf{E} = \mathbf{L}\boldsymbol{\Gamma}\mathbf{L}^*$, where $\boldsymbol{\Gamma} \in \mathfrak{F}^{r\times r}$ and $r = \mathrm{Rank}\{\mathbf{E}\}$. Introducing the following definition

$$\mathbf{u} = \frac{e^{-j\theta_m}}{\sigma}\mathbf{L}^*(\mathbf{E}^*\mathbf{E})^{+/2}\mathbf{E}^*\mathbf{y} \tag{26}$$

and

$$\mathbf{V} = \mathbf{L}^*(\mathbf{E}^*\mathbf{E})^{+/2}(\mathbf{e}_m\mathbf{e}_m^T - \mathbf{e}_l\mathbf{e}_l^T)(\mathbf{E}^*\mathbf{E})^{+/2}\mathbf{L}$$
$$= \begin{vmatrix}\mathbf{v}_1 & \mathbf{v}_2\end{vmatrix}\begin{vmatrix}\lambda_1 & 0 \\ 0 & \lambda_2\end{vmatrix}\begin{vmatrix}\mathbf{v}_1^* \\ \mathbf{v}_2^*\end{vmatrix}, \tag{27}$$

we obtain

$$P_{MGD}^{er}(m,l) = \Pr\left[\mathbf{u}^*\mathbf{V}\mathbf{u} < 0 \,\middle|\, H_m\right]. \tag{28}$$

We require that the matrix $\mathbf{V}$ has rank 2, if this condition is not satisfied then either $P_{MGD}^{er}(m,l) = 1$ or $P_{MGD}^{er}(l,m) = 1$, resulting in a probability of error, which approaches a constant as the signal-to-noise ratio, grows. The vector $\mathbf{u}$ is complex normal with correlation matrix

$$R[\mathbf{u}^*\mathbf{u}] = \mathbf{I} \tag{29}$$

and mean

$$M[\mathbf{u}] = \frac{1}{\sigma}\mathbf{L}^*(\mathbf{E}^*\mathbf{E})^{0.5}\mathbf{e}_m \tag{30}$$

under the hypothesis H_m. We may use results of [14] –[16] to define the probability of error in the following form

$$P_{MGD}^{er} \cong \exp\Big\{\min_{l\neq m}\big\{-\{\sqrt{4\sigma^4}[(\mathbf{H}^*\mathbf{P}_{\mathbf{S}}^{\perp}\mathbf{H})_{m,m}^{+} + (\mathbf{H}^*\mathbf{P}_{\mathbf{S}}^{\perp}\mathbf{H})_{l,l}^{+} + 2\,|\,(\mathbf{H}^*\mathbf{P}_{\mathbf{S}}^{\perp}\mathbf{H})_{m,m}^{+}\,|\,]\}^{-1}\big\}\Big\}. \tag{31}$$

Using our asymptotic expression for the probability of error, we can derive the asymptotic multiuser efficiency and the near-far resistance of the MMSE GD and MGD. According to [12], the asymptotic multiuser efficiency of the k-th sensor node is determined by the following form

$$\eta(k) = \sup\Big\{0 \le r \le 1,\ \lim_{\sqrt{4\sigma^4}\to 0}\frac{P_{\phi}(\sqrt{4\sigma^4})}{P_{SU}(\frac{\sqrt{4\sigma^4}}{r})} < \infty\Big\}, \tag{32}$$

where $P_{\phi}(\sqrt{4\sigma^4})$ is the probability of error for the k-th sensor node transmitting information signal to sink employing the MMSE (or multipulse) generalized detector ϕ with noise power $\sqrt{4\sigma^4}$ at the detector output, and $P_{SU}(\frac{\sqrt{4\sigma^4}}{r})$ is the probability of error for the MMSE GD (or MGD) in the absence of interfering sensor nodes and sinks, i.e., $\mathbf{S} = 0$ with the effective noise power $\frac{\sqrt{4\sigma^4}}{r}$ at the detector output.

Using the asymptotically tight expressions for $P_{\phi}(\sqrt{4\sigma^4})$, $P_{SU}(\frac{\sqrt{4\sigma^4}}{r})$ given in (31) (in the latter case we simply set $\mathbf{P}_{\mathbf{S}}^{\perp} = \mathbf{I}$), we can define the asymptotic multiuser efficiency in the following form $\eta(k)$

$$= \frac{\min_{l\neq m}\{(\mathbf{H}^*\mathbf{H})_{l,l}^{+} + (\mathbf{H}^*\mathbf{H})_{m,m}^{+}\}}{\min_{n\neq p}\{(\mathbf{H}^*\mathbf{P}_{\mathbf{S}}^{\perp}\mathbf{H})_{n,n}^{+} + (\mathbf{H}^*\mathbf{P}_{\mathbf{S}}^{\perp}\mathbf{H})_{p,p}^{+} + 2\,|\,(\mathbf{H}^*\mathbf{P}_{\mathbf{S}}^{\perp}\mathbf{H})_{n,p}^{+}\,|\}} \tag{33}$$

The near-far resistance of the detector is defined as the infimum of the asymptotic multiuser efficiency over the possible realizations of the interfering sensor node's and sink's powers. As the MMSE GD acts asymptotically to null the multiple-access interference, we find that the near-far resistance is simply the asymptotic multiuser efficiency given in (33).

6 Comparison with GD

It is interesting to compare the probability of error for MMSE GD and MGD given in (31) with the asymptotic expression for the probability of error of the GD. This latter quantity is known to be [10] and [13]

$$P_{GD}^{er} \cong \exp\Big\{-\frac{E_k}{\sqrt{8\sigma^4}}\min_{m\neq l}\left\|\mathbf{P}_{\mathbf{S}}^{\perp}\mathbf{h}_m\right\|^2\Big[1 - \frac{|\,\mathbf{h}_m^*\mathbf{P}_{\mathbf{S}}^{\perp}\mathbf{h}_l\,|}{\left\|\mathbf{P}_{\mathbf{S}}^{\perp}\mathbf{h}_m\right\|\left\|\mathbf{P}_{\mathbf{S}}^{\perp}\mathbf{h}_l\right\|}\Big]\Big\}. \tag{34}$$

It is clear that the expression in (31) and (34) are not generally equal. They are equal for the case of orthogonal signal with respect to $\mathbf{P}_{\mathbf{S}}^{\perp}$, i.e.,

$$\mathbf{H}^*\mathbf{P}_{\mathbf{S}}^{\perp}\mathbf{H} = \nu\,\mathbf{I}, \tag{35}$$

which is clear from the definitions of two tests. In general, there is no clear reason to choose the MMSE GD (or MGD) over the GD, at least in terms of asymptotic performance. For the case of binary signaling, i.e., $M = 2$, however, we can show below that asymptotically the MMSE GD (or MGD) is superior to the GD. For binary signaling, we let

$$\mathbf{E} = \mathbf{P}_{\mathbf{S}}^{\perp}\mathbf{H} = [\mathbf{e}_1, \mathbf{e}_2]. \tag{36}$$

Then the MMSE GD (or MGD) has the asymptotic probability of error

$$P^{er}_{MGD} \cong \exp\{-\frac{E_k \alpha_M}{8\sigma^4}\} \quad (37)$$

with

$$\alpha_M = \frac{2}{(\mathbf{E}^*\mathbf{E})^{-1}_{1,1} + (\mathbf{E}^*\mathbf{E})^{-1}_{2,2} + 2|(\mathbf{E}^*\mathbf{E})^{-1}_{1,2}|}$$

$$= \frac{2}{\frac{1}{\mathbf{e}_1^*\mathbf{P}^\perp_{\mathbf{e}_2}\mathbf{e}_1} + \frac{1}{\mathbf{e}_2^*\mathbf{P}^\perp_{\mathbf{e}_1}\mathbf{e}_2} + 2\frac{|\mathbf{e}_1^*\mathbf{P}^\perp_{\mathbf{e}_2}\mathbf{P}^\perp_{\mathbf{e}_1}\mathbf{e}_2|}{(\mathbf{e}_1^*\mathbf{P}^\perp_{\mathbf{e}_2}\mathbf{e}_1)(\mathbf{e}_2^*\mathbf{P}^\perp_{\mathbf{e}_1}\mathbf{e}_2)}}$$

$$= \frac{2(AB - |\beta|^2)}{A + B + 2|\beta|}, \quad (38)$$

where

$$A = \|\mathbf{e}_1\|^2, \; B = \|\mathbf{e}_2\|^2, \; \beta = \mathbf{e}_1^*\mathbf{e}_2. \quad (39)$$

We have assumed that **E** is invertible, since otherwise both MGD and GD test fail. In light of (34) we see that asymptotically, the GD has probability of error

$$P^{er}_{GD} \cong \exp\{-\frac{E_k \alpha_G}{8\sigma^4}\}. \quad (40)$$

The exponential parameter α_G is given by

$$\alpha_G = B\left[1 - \frac{|\beta|}{\sqrt{AB}}\right] \quad (41)$$

assuming without loss of generality that $A \geq B$. If $B > A$, we simply switch A and B. We are interested in the ratio $\frac{\alpha_M}{\alpha_G}$, noticing that if this ratio is greater then one, we have the MMSE GD (or MGD) outperforming the GD (asymptotically). Assuming that $A \geq B$, we have

$$\frac{\alpha_M}{\alpha_G} = \frac{2A + 2|\beta|\sqrt{\frac{A}{B}}}{A + B + 2|\beta|} \geq \frac{2A + 2|\beta|\sqrt{\frac{A}{B}}}{2A + 2|\beta|}$$

$$(\text{since } A + B \leq 2A) \quad \geq 1 \; (\text{since } \frac{A}{B} \geq 1. \quad (42)$$

The same argument works for the case of $B \geq A$ and we conclude that the MMSE GD (or MGD) is superior to the GD for large signal-to-noise ratio. Notice that equality is achieved when $A = B$. This appears to be most general statement that can be made about the asymptotic performance difference between the two detectors. For every value of M greater then two that we have considered, we have found signal sets for which $\alpha_G > \alpha_M$.

7 Simulation

We consider a noncoherent channel with $K = 3$ sensor sinks, each employing $M = 3$ signals transmitted by sensor nodes. The signals were taken to be length $N = 31$ codes (sensor node one used codes 5-7, sensor node two used codes 8-10, etc.), normalized to have unit form. The sensor node energies were chosen to be $E_1 = 1$, $E_2 = 5$, and $E_3 = 5$. The probability of error for the MMSE GD was estimated by Monte-Carlo simulation. The results are shown in Fig.1 along with the union upper bound on the MGD of (24). Notice that fit between the MMSE GD performance and the bound on the MGD appears quite good for this problem, even at relatively low signal-to-noise ratio. For comparison, we have plotted the union bound for the GD. It is shown that for noncoherent binary signaling, in which the multipulse modulation is two-dimensional, the MGD is superior to the GD. For this example, $\alpha_M = 0.34$ and $\alpha_G = 0.27$, and the MMSE GD (or MGD) outperforms GD, as expected. We have also plotted the union bound for the generalized maximum-likelihood (GML) detection rule [14] with the purpose to show a superiority of the use of the generalized approach to signal processing in the presence of noise with respect to classical and modern signal processing approaches. The exponential bound of (31) is plotted against the union bound of (24) in Fig.2. We see that the two bounds are asymptotically coincident, as predicted.

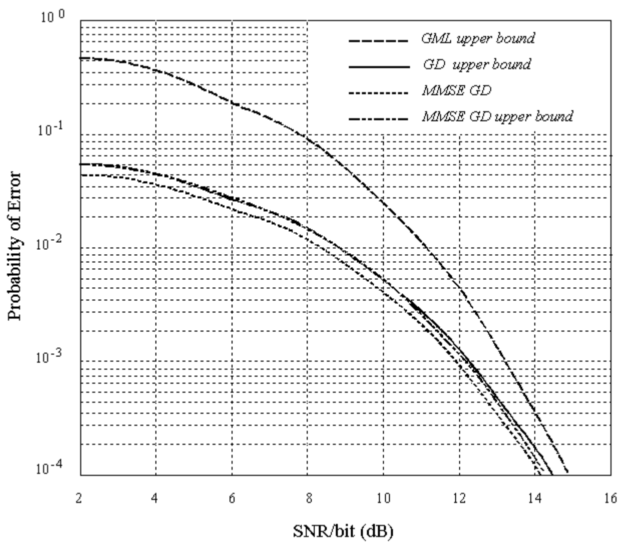

Figure1. The probability of symbol error for MMSE GD along with the asymptotic union bound of (24), the union bound of GD, and the union bound of the GML detector.

8 Conclusions

In this paper, we have investigated the MMSE GD, MGD, and GD for noncoherent nonorthogonal multipulse modulation and have compared with GML detection rule. It was observed that the former two detectors are asymptotically superior to the GD for binary signaling, a result which does not appear to generalize to larger cardinality signal constellations. We have also shown a great superiority of the use of the generalized approach to signal processing in the presence of noise under nonorthogonal multipulse modulation in wireless sensor networks over classical and modern approaches in signal processing. The asymptotic multiuser efficiency and near-far resistance were derived for the MMSE GD and MGD through a large signal-to-noise ratio approximation to the probability of error of the detectors. The MMSE GD requires knowledge of the sensor nodes' energy levels and the interfering sensor nodes' signal vectors. These requirements can be lifted by replacing the measurement correlation matrix by an estimate based on several observations.

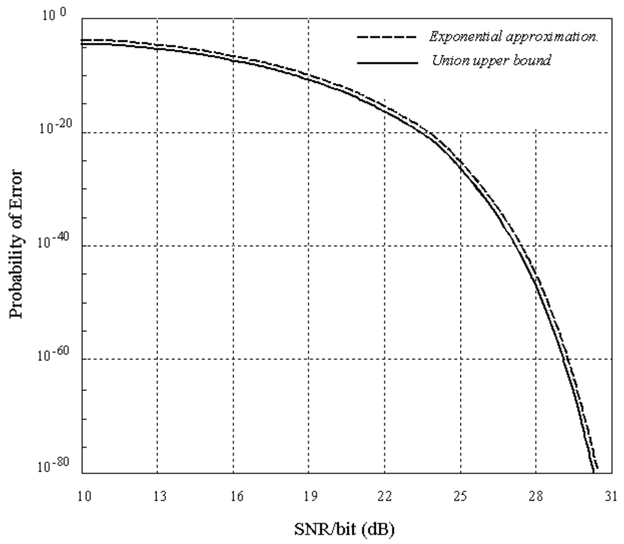

Figure 2. Comparison of the union upper bound given by (24) with the exponential approximation of (31).

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References:

[1] M.L. McCloud, L.L. Scharf, and L.T. McWhorter "Subspace coherence for detection in multiuser additive noise channels", in *Proc. SPAWC'97,* Paris, France, April 1997, pp. 225–228.

[2] M.K. Varanasi and A. Russ, "Noncoherent decorrelative multiuser detection for nonlinear nonorthogonal modulation", in *Proc. I.C.C.'97*, Montreal, Canada, June 1997.

[3] M.L. McCloud and L.L. Scharf, "Generalized likelihood detection on multiple access channels", in *Proc. Asilomar'97*, Monterey, CA, USA, November 1997.

[4] M.K. Varanasi and A. Russ, "Noncoherent décorrelative detection for nonorthogonal multipulse modulation over the multiuser Gaussian channel" *IEEE Trans. Commun*., Vol. COM-46, December 1998, pp.1675–1684.

[5] X. Wang and H.V. Poor, "Blind multiuser detection: A subspace approach", *IEEE Trans. Inform. Theory*, Vol. IT-44, March 1998, pp.677–689.

[6] M.L. McCloud and L.L. Scharf, "Interference estimation with applications to blind multiple access communication over fading channels", *IEEE Trans. Inform. Theory*, Vol.IT-46, May 2000, pp. 947–961.

[7] V. Tuzlukov, *Signal Processing in Noise: A New Methodology*, Minsk: IEC, 1998.

[8] V. Tuzlukov, "A new approach to signal detection", *Digital Signal Processing: A Review Journal*, Vol.8, No.3, 1998, pp.166–184.

[9] V. Tuzlukov, *Signal Detection Theory*,New York: Springer-Verlag, 2001.

[10] V. Tuzlukov, *Signal Processing Noise*, Boca Raton, London, New York, Washington D.C.: CRC Press, 2002.

[11] V. Tuzlukov, *Signal and Image Processing in Navigational Systems,* Boca Raton, London,New York, Washington D.C.: CRC Press, 2004.

[12] R. Lupas and S. Verdu, "Linear multiuser detectors for synchronous code-division multiple-access channels", *IEEE Trans. Inform. Theory*, Vol. IT-35, January 1989, pp.123–136.

[13] V. Tuzlukov, W.S.Yoon, and Y.D. Kim, "Wireless sensor networks based on the generalized approach to signal processing with fading channels and receive antenna array", *WSEAS Transactions on Circuits and Systems*, Issue 10, Vol. 3, December 2004, pp. 2149–2155.

[14] M.L. McCloud and L. Scharf, "Asymptotic analysis of the MMSE multiuser detector for nonorthogonal multipulse modulation", *IEEE Trans. Comm*., Vol.COM-49, No. 1, January 2001, pp. 24–30.

[15] M. Schwartz, W. Bennet, and S. Stein, *Communication Systems and Techniques*, New York, NY: IEEE Press, 1996.

[16] J.G. Proakis, *Digital Communications*, New York: McGraw-Hill, 1995.