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Robust Anti-Windup Controller Synthesis: A Mixed $\mathcal{H}_2/\mathcal{H}_\infty$ Setting

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Abstract: - In this paper an approach for robust *anti-windup* compensation design for multivariable controllers based on the characterization of $\mathcal{H}_2$-$\mathcal{H}_\infty$ norms as Linear Matrix Inequalities (LMI) is presented. The method consider a mix of $\mathcal{H}_2$-$\mathcal{H}_\infty$ performance indexes on the transfer functions of the disturbances to the controlled output, and of the actuator output signal to the controller output signal. The controller and the compensation gain are obtained by numerical solution of the LMIs system. The robustness is considered by assuring the closed loop performance, spite of unknown changes on the actuator saturation limits.

Key-Words: - Anti-windup compensation. Multiobjective control. $\mathcal{H}_2$-$\mathcal{H}_\infty$ Control. Linear Matrix Inequalities.

1 Introduction

In general, the controlled industrial processes present actuator saturation problems. In control theory that restriction is denominated the bounded control problem, which can be solved by substitution of the control policy.

On the other hand, the control systems can operate in multiple environments and with multiple objectives. Each specific situation defines the operation mode, which can require a controller commutation. The modes commutation is the substitution in the plant inputs, considering that the controller output is replaced by another.

As a result of substitutions and limitations, the plant inputs will be different to the controller's output. When this happens, the controller outputs don't drive the plant appropriately and the controller's states will be strongly updated, [7, 9]. This effect is called Wind-Up. In global terms, the wind-up is one inconsistency among the control input given to the process and the internal states of the controller. The adverse effect of the wind-up is a significant performance deterioration, overshoots and even instability, [9, 2].

The wind-up problem can be handled by means of compensation where, in a first stage, it is designed the control system without taking into account the restrictions; and in a second stage, some compensation scheme is found, with the purpose of minimizing the limitations and commutations effect. The last outlined focus has been denominated the *anti-windup bumpless transfer problem* (AWBT), [7].

The main advantage of this design methodology is that no restrictions are placed on the original linear controller design. The main disadvantage is that although the linear controller and anti-windup compensator both affect the closed-loop performance; so, the effect of the linear controller on the performance under saturation is completely ignored. In addition, the possible changes of the saturation limits is not considered, which can result very inconvenient.

For robustness in the AWBT compensation design, [11] present a general formulation of the problem of multivariable AWBT controller synthesis. The resulting synthesis method demonstrates graceful performance degradation whenever input nonlinearities are active through minimization of a weighted $\mathcal{L}_2$ gain. The AWBT controller synthesis, using static compensation, is cast as a convex optimization over linear matrix inequalities. [15] present a method based on $\mathcal{L}_2$ performance. An static anti-windup compensator is obtained and both the nominal performance problem and robust performance problem can be reduced to a generalized eigenvalue problem. [6] study linear anti-windup augmentation for linear control systems with saturated linear plants in the special case when the anti-windup compensator can only modify the input and the output of the windup-prone linear controller and the performance is measured in terms of the $\mathcal{L}_2$ gain from exogenous inputs

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to selected performance outputs. [12] present an approach for AWBT compensation design for PID controllers based on the characterization of $\mathcal{H}_2$ and $\mathcal{H}_\infty$ norms as Linear Matrix Inequalities. The robustness is considered by assuring the closed loop performance, spite of unknown changes on the actuator saturation limits.

In this paper a generalization of the approach given in [13] is presented. In this case, the multivariable controllers are considered, and the performance is studied from multi-objective criteria on the closed loop transfer function of the control system and on the transfer function of the actuator output respect to controller output.

2 Problem formulation

In order to introduce the robustness problem for the anti-windup compensation design; let us consider the following linear model:

$$\begin{aligned} \dot{x}(t) &= Ax(t) + B_1\omega(t) + B_2\sigma(u) \\ z(t) &= C_1x(t) + D_{11}\omega(t), \\ y(t) &= C_2x(t) + D_{21}\omega(t), \end{aligned} \tag{1}$$

where $x \in \Re^n$ are the states, $\omega \in \Re^r$ are the disturbances, $u \in \Re^p$ are the controls, $z \in \Re^m$ are the controlled outputs, and $y \in \Re^q$ are the measured outputs. A, B_1, B_2, C_1, C_2, D_{11}, D_{21} are known matrices, which have appropriate dimensions.

The non-linear function $\sigma(\circ)$ denotes the actuator saturation, which is defined by

$$\sigma(u_i) = \begin{cases} u_{i_{min}} & \text{if} \quad u(t) < u_{i_{min}} \\ u(t) & \text{if} \quad u_{i_{min}} \le u(t) \le u_{i_{max}} \\ u_{i_{max}} & \text{if} \quad u(t) > u_{i_{max}}, \quad i = 1, 2, \ldots, p \end{cases} \tag{2}$$

The AWBT compensation problem is formulated due to the limitations and/or substitutions, where a non linearity appears among the controller's output and the effective process input.

The effective control input $\sigma(u)$ is a non-linear function of the controller's output $u(t)$. In order to satisfy the control requirements, a dynamic controller with compensation is considered, which is given by

$$\begin{aligned} \dot{\zeta}(t) &= \mathbf{A}_c\zeta(t) + \mathbf{B}_c y(t) + \mathbf{E_c}[\sigma(u) - u] \\ u(t) &= \mathbf{C}_c\zeta(t) + \mathbf{D}_c y(t), \end{aligned} \tag{3}$$

where $\mathbf{A}_c$, $\mathbf{B}_c$, $\mathbf{C}_c$ $\mathbf{D}_c$ corresponds to the dynamical matrices of the controller, which are design parameters. $\mathbf{E_c}$ represents the compensation gain, which is also a design parameter.

When the controlled system is in saturation, the signal $\vartheta = \sigma(u) - u$ is not null, and performance deterioration, overshoots and/or instability are presents. For compensation, the controller dynamic matrices $\mathbf{A}_c$, $\mathbf{B}_c$, $\mathbf{C}_c$, and $\mathbf{D}_c$ are designed without considering the actuator saturation, while the effect of the saturation on the performance in closed loop is minimized by means of the selection of the gain $\mathbf{E_c}$. The compensation have effect when the signal $\vartheta = \sigma(u) - u$ is not null, and an additional feedback is incorporated in order to make that signal be null, again, through the upgrade of the control signal u, which will be inside of the actuator saturation limits.

Thus, the system in closed loop is given by

$$\begin{aligned} \dot{x} &= Ax + B_1\omega + B_2u + B_2\vartheta \\ \dot{\zeta} &= \mathbf{B}_cC_2x + \mathbf{A}_c\zeta + \mathbf{E_c}\vartheta \\ z &= C_1x + D_{11}\omega \\ u &= \mathbf{D}_cC_2x + \mathbf{C}_c\zeta(t) + \mathbf{D}_cD_{21}\omega, \end{aligned} \tag{4}$$

which is equivalent to

$$\begin{aligned} \dot{x} &= (A + B_2\mathbf{D}_cC_2)x + B_2\mathbf{C}_c\zeta + \\ &\quad (B_1 + B_2\mathbf{D}_cD_2)\omega + B_2\vartheta \\ \dot{\zeta} &= \mathbf{B}_cC_2x + \mathbf{A}_c\zeta + \mathbf{B}_cD_{21}\omega + \mathbf{E_c}\vartheta \\ z &= C_1x + D_{11}\omega, \\ u &= \mathbf{D}_cC_2x + \mathbf{C}_c\zeta + \mathbf{D}_cD_{21}\omega. \end{aligned} \tag{5}$$

The performance of the closed loop system without saturation is studied from the transfer function of the disturbance ω to the controlled output z, [10]. In this case, be $T_{z\omega}$ such transfer function

$$T_{z\omega}(s) = \left[\begin{array}{c|c} \mathbb{A} & \mathbb{B}_1 \\ \hline \mathbb{C}_1 & \mathbb{D}_1 \end{array}\right] = \mathbb{C}_1(s\mathbb{I} - \mathbb{A})^{-1}\mathbb{B}_1 + \mathbb{D}_1 \tag{6}$$

where

$$\mathbb{A} = \begin{pmatrix} A + B_2\mathbf{D}_cC_2 & B_2\mathbf{C}_c \\ \mathbf{B}_cC_2 & \mathbf{A}_c \end{pmatrix}, \quad \mathbb{B}_1 = \begin{pmatrix} B_1 + B_2\mathbf{D}_cD_2 \\ \mathbf{B}_cD_{21} \end{pmatrix},$$

$$\mathbb{C}_1 = \begin{pmatrix} C_1 & 0 \end{pmatrix}, \quad \mathbb{D}_1 = D_{11}.$$

Problem 1: By performance requirements, without saturation, is necessary to design a dynamic controller (3) such that

1. The closed loop system (5) be asymptotically stable.
2. $\|T_{z\omega}\|_2 < \mu$ or $\|T_{z\omega}\|_\infty < \gamma$, where the performance indexes $\mu > 0$, $\gamma > 0$.

Some approaches are well known in the literature on robust control, [1, 16, 5].

When the saturation is present is necessary to conserver the stability properties. It is guaranteed by means of the global stability of the bounded input system, that which, in case that this problem concerns, is guaranteed by means of the compensation gain design. This way, the synthesis problem consists in designing $\mathbf{E_c}$ that guarantees the effectiveness of the compensation under some robust stability condition in perturbed systems. The closed loop stability is satisfied, in first place, for the appropriate selection of the dynamic matrices of the controller. The second condition, lied to the bounded input systems stability, will allow the compensation under global stability. Some results can be found in the literature, [7, 8, 11].

On the other hand, it is necessary to consider aspects of robustness with relationship to the operation of the actuators. In this sense, we should design the compensation gain in order to guarantee the closed loop stability and to minimize the effect of the *disturbance* signal ϑ on the control signal u when the saturation becomes present. Therefore, the compensation gain design problem can be focused starting from the norms $\mathcal{H}_2$ and $\mathcal{H}_\infty$.

Let consider the transfer function of the control signal with respect to the signal ϑ

$$T_{u\vartheta}(s) = \left[\begin{array}{c|c} \mathbb{A} & \mathbb{B}_2 \\ \hline \mathbb{C}_2 & \mathbb{D}_2 \end{array}\right] = \mathbb{C}_2(s\mathbb{I}-\mathbb{A})^{-1}\mathbb{B}_2 + \mathbb{D}_2 \qquad (7)$$

where

$$\mathbb{B}_2 = \begin{pmatrix} B_2 \\ \mathbf{E_c} \end{pmatrix}, \qquad \mathbb{C}_1 = \begin{pmatrix} \mathbf{D}_c C_2 & \mathbf{C}_c \end{pmatrix}, \quad \mathbb{D}_2 = 0.$$

Problem 2: Given the dynamic system (1), to design the *compensation gain* $\mathbf{E_c}$ for the controller (3), such that:

1. The closed loop system (5) be asymptotically stable.

2. The effect of signal ϑ on the control signal u be minimum, in some sense.

Thus, robust performance indexes with respect to saturation limits changes can be obtained, which is a typical demand in an industrial processes control environment where the actuator elements: control valves, hydraulic actuators, etc., can be deteriorated for the intensive use, parts obsolescence, construction materials degradation, among other aspects. Therefore, it is necessary to design compensation mechanisms, in order to guarantee some robustness characteristics, which should consider changes in the actuator devices performance, [13, 15].

In summary, we have formulated two problem: a typical optimal control problem and a robust compensation synthesis problem. Both problems can be outlined as a multiobjective control problem:

Problem: Given the dynamic system (1), to design the *controller with compensation* (3) such that:

1. The closed loop system (5) be asymptotically stable.

2. $\|T_{z\omega}\|_2 < \mu$ or $\|T_{z\omega}\|_\infty < \gamma$, where the performance indexes $\mu > 0$, $\gamma > 0$; **subject to that**

3. The effect of signal ϑ on the control signal u be minimum, in some sense.

Under this formulation, the design objectives can be a mix of $\mathcal{H}_2$ performance, H_∞ performance, asymptotic disturbance rejection, which can derive a system of linear matrix inequalities, just as it has been established in the multiobjective $\mathcal{H}_2/\mathcal{H}_\infty$ controller synthesis, [14, 4].

In this context, it is possible to develop a systematic design technique that combines important aspects the feedback control synthesis with anti-windup compensation design taking advantage of the formulation of multiple objectives.

3 Control-Compensation Synthesis

In this point, we intend to satisfy the design constraints that imposes the anti-windup compensation, such as has been formulated in the problem 2. Thus, we want design the controller with compensation (3) such that the closed loop system (5) be asymptotically stable and $||T_{u\vartheta}||_2^2 < \mu$ or $||T_{u\vartheta}||_\infty < \gamma$.

3.1 $\mathcal{H}_2$ Formulation

In this case, we want design the controller (3) such that $||T_{u\vartheta}||_2^2 < \mu$, for all $\mu > 0$, which guarantees requirements of anti-windup compensation. The following lemma is a well known result, which completely characterizes the $\mathcal{H}_2$ norm constraint through LMI [1, 14].

Lemma 3.1 *The inequality $||T_{u\vartheta}||_2^2 < \mu$ holds if, and only if, $\mathbb{D} = 0$ and there exists symmetric matrices $\mathbb{X} > 0$, and $\mathbb{W}$ such that*

$$\begin{bmatrix} \mathbb{A}\mathbb{X} + \mathbb{X}\mathbb{A}^T & \mathbb{B} \\ (\circ)^T & -\mathbb{I} \end{bmatrix} < 0, \quad \begin{bmatrix} \mathbb{W} & \mathbb{C}\mathbb{X} \\ (\circ)^T & \mathbb{X} \end{bmatrix} > 0, \qquad (8)$$
$$tr(\mathbb{W}) < \mu.$$

is feasible.

Proposition 3.1 *Consider the system defined by (1) and the controller with compensation given by (3). The controlled system is asymptotically stable with robust compensation, because $||T_{u\vartheta}||_2^2 < \mu$, iff there exists symmetric matrices of n order $\mathbf{X} > 0$ and $\mathbf{Y} > 0$; matrix $\mathbf{Q} \in \Re^{n\times n}$; matrices $\mathbf{L} \in \Re^{p\times n}$, $\mathbf{F} \in \Re^{n\times q}$, $\mathbf{R} \in \Re^{p\times q}$, $\mathbf{M} \in \Re^{n\times p}$; and a symmetric matrix $\mathbb{W} \in \Re^{p\times p}$ satisfying the following LMIs:*

$$\begin{bmatrix} A\mathbf{X} + \mathbf{X}A^T + B_2\mathbf{L} + \mathbf{L}^T B_2^T & A + B_2\mathbf{R}C_2 + \mathbf{Q}^T & B_2 \\ (\circ)^T & \mathbf{Y}A + A^T\mathbf{Y} + \mathbf{F}C_2 + C_2^T\mathbf{F}^T & \mathbf{Y}B_2 + \mathbf{M} \\ (\circ)^T & (\circ)^T & -\mathbb{I} \end{bmatrix} < 0,$$

$$\begin{bmatrix} \mathbb{W} & \mathbf{L} & \mathbf{R}C_2 \\ (\circ)^T & \mathbf{X} & \mathbb{I} \\ (\circ)^T & (\circ)^T & \mathbf{Y} \end{bmatrix} > 0,$$

$$tr(\mathbb{W}) < \mu.$$

The controller is obtained from

$$\begin{pmatrix} \mathbf{A}_c & \mathbf{B}_c \\ \mathbf{C}_c & \mathbf{D}_c \end{pmatrix} = \begin{pmatrix} \mathbf{V}^{-1} & -\mathbf{V}^{-1}\mathbf{Y}B_2 \\ 0 & \mathbb{I} \end{pmatrix} \begin{pmatrix} \mathbf{Q} - \mathbf{Y}A\mathbf{X} & \mathbf{F} \\ \mathbf{L} & \mathbf{R} \end{pmatrix} \begin{pmatrix} \mathbf{U}^{-1} & 0 \\ -C_2\mathbf{X}\mathbf{U}^{-1} & \mathbb{I} \end{pmatrix}.$$

The compensation gain matrix $\mathbf{E_c}$ *is given by*

$$\mathbf{E_c} = \mathbf{V}^{-1}\mathbf{M};$$

where $\mathbf{V}$ *and* $\mathbf{U}$ *are non singular matrices satisfying* $\mathbf{YX} + \mathbf{VU} = \mathbb{I}$.

Proof

The proof is based on the typical linearization procedure of the matrix inequalities through the congruence transformation and variable changes. ■

This formulation guarantees the stability in the case of saturation with a minimum effect on the controller's output signal.

3.2 $\mathcal{H}_\infty$ Formulation

In this environment, we want to design the controller 3 such that $||T_{u\vartheta}||_\infty < \gamma$, for all $\gamma > 0$. It is well known that the $\mathcal{H}_\infty$ norm has a characterization as LMI constraints according to the Bounded Real Lemma [1, 14]:

Lemma 3.2 *The inequality* $||T_{u\vartheta}||_\infty < \gamma$ *holds if, and only if, there exist a symmetric matrix* $\mathbb{X}$*, such that*

$$\begin{bmatrix} \mathbb{A}^T\mathbb{X} + \mathbb{X}\mathbb{A} & \mathbb{X}\mathbb{B} & \mathbb{C}^T \\ (\circ)^T & -\gamma\mathbb{I} & \mathbb{D}^T \\ (\circ)^T & (\circ)^T & -\gamma\mathbb{I} \end{bmatrix} < 0. \qquad (9)$$

Proposition 3.2 *Consider the system defined by (1) and the controller with compensation given by (3). The controlled system is asymptotically stable with robust compensation, because* $||T_{u\vartheta}||_\infty < \gamma$*, iff there exists symmetric matrices of n order* $\mathbf{X} > 0$ *and* $\mathbf{Y} > 0$*; matrix* $\mathbf{Q} \in \Re^{n\times n}$*; matrices* $\mathbf{L} \in \Re^{p\times n}$, $\mathbf{F} \in \Re^{n\times q}$, $\mathbf{R} \in \Re^{p\times q}$, $\mathbf{M} \in \Re^{n\times p}$*; such that the following LMI is satisfied:*

$$\begin{bmatrix} A\mathbf{X} + \mathbf{X}A^T + B_2\mathbf{L} + \mathbf{L}^T B_2^T & A + B_2\mathbf{R}C_2 + \mathbf{Q}^T & B_2 & \mathbf{L}^T \\ (\circ)^T & \mathbf{Y}A + A^T\mathbf{Y} + \mathbf{F}C_2 + C_2^T\mathbf{F}^T & \mathbf{Y}B_2 + \mathbf{M} & C_2^T\mathbf{R}^T \\ (\circ)^T & (\circ)^T & -\gamma\mathbb{I} & 0 \\ (\circ)^T & (\circ)^T & (\circ)^T & -\gamma\mathbb{I} \end{bmatrix} < 0.$$

The controller is obtained from

$$\begin{pmatrix} \mathbf{A}_c & \mathbf{B}_c \\ \mathbf{C}_c & \mathbf{D}_c \end{pmatrix} = \begin{pmatrix} \mathbf{V}^{-1} & -\mathbf{V}^{-1}\mathbf{Y}B_2 \\ 0 & \mathbb{I} \end{pmatrix} \begin{pmatrix} \mathbf{Q} - \mathbf{Y}A\mathbf{X} & \mathbf{F} \\ \mathbf{L} & \mathbf{R} \end{pmatrix} \begin{pmatrix} \mathbf{U}^{-1} & 0 \\ -C_2\mathbf{X}\mathbf{U}^{-1} & \mathbb{I} \end{pmatrix}.$$

The compensation gain matrix $\mathbf{E_c}$ *is given by*

$$\mathbf{E_c} = \mathbf{V}^{-1}\mathbf{M};$$

where $\mathbf{V}$ *and* $\mathbf{U}$ *are non singular matrices satisfying* $\mathbf{YX} + \mathbf{VU} = \mathbb{I}$.

Proof

In a similar way that in the previous case, the demonstration is constructed from appropriate transformation and changes of variables on the LMIs. ■

The performance index based-on the $\mathcal{H}_\infty$ norm corresponds to the $\mathcal{L}_2$ gain of the controller output signal with respect to the actuator output signal.

3.3 Anti-Windup Controller via Mixed $\mathcal{H}_2/\mathcal{H}_\infty$

Starting from the previous results, it is possible to outline different design objectives considering the control performance and the compensation requirements. For example, the following objectives can be formulated:

1. $\|T_{z\omega}\|_\infty < \mu$ subject to $||T_{u\vartheta}||_2 < \gamma$.
2. $\|T_{z\omega}\|_2 < \mu$ subject to $||T_{u\vartheta}||_\infty < \gamma$.

In order to propitiate the study, we particularly show the $\mathcal{H}_\infty$-$\mathcal{H}_2$ multiobjective design problem. The $\mathcal{H}_2$-$\mathcal{H}_\infty$ case can be treated in the same way.

Corollary 3.1 *The system defined by (1) is stabilizable by the full order dynamic output feedback control with compensation (3) such that* $||T_{z\omega}||_\infty < \mu$ *and* $||T_{u\vartheta}||_2^2 < \gamma$*, iff there exists symmetric matrices of n order* $\mathbf{X} > 0$ *and* $\mathbf{Y} > 0$*; matrix* $\mathbf{Q} \in \Re^{n\times n}$*; matrices* $\mathbf{L} \in \Re^{p\times n}$, $\mathbf{F} \in \Re^{n\times q}$, $\mathbf{R} \in \Re^{p\times q}$, $\mathbf{M} \in \Re^{n\times p}$*; and a symmetric matrix* $\mathbb{W} \in \Re^{p\times p}$ *satisfying the following LMIs:*

$$\begin{bmatrix} A\mathbf{X} + \mathbf{X}A^T + B_2\mathbf{L} + \mathbf{L}^T B_2^T & A + B_2\mathbf{R}C_2 + \mathbf{Q}^T & B_1 + B_2\mathbf{R}D_{21} & \mathbf{X}C_1^T \\ (\circ)^T & \mathbf{Y}A + A^T\mathbf{Y} + \mathbf{F}C_2 + C_2^T\mathbf{F}^T & \mathbf{Y}B_1 + \mathbf{F}D_{21} & C_1^T \\ (\circ)^T & (\circ)^T & -\mu\mathbb{I} & D_{11}^T \\ (\circ)^T & (\circ)^T & (\circ)^T & -\mu\mathbb{I} \end{bmatrix} < 0.$$

$$\begin{bmatrix} A\mathbf{X} + \mathbf{X}A^T + B_2\mathbf{L} + \mathbf{L}^T B_2^T & A + B_2\mathbf{R}C_2 + \mathbf{Q}^T & B_2 \\ (\circ)^T & \mathbf{Y}A + A^T\mathbf{Y} + \mathbf{F}C_2 + C_2^T\mathbf{F}^T & \mathbf{Y}B_2 + \mathbf{M} \\ (\circ)^T & (\circ)^T & -\mathbb{I} \end{bmatrix} < 0,$$

$$\begin{bmatrix} \mathbb{W} & \mathbf{L} & \mathbf{R}C_2 \\ (\circ)^T & \mathbf{X} & \mathbb{I} \\ (\circ)^T & (\circ)^T & \mathbf{Y} \end{bmatrix} > 0,$$

$$tr(\mathbb{W}) < \gamma.$$

The controller is obtained from

$$\begin{pmatrix} \mathbf{A}_c & \mathbf{B}_c \\ \mathbf{C}_c & \mathbf{D}_c \end{pmatrix} = \begin{pmatrix} \mathbf{V}^{-1} & -\mathbf{V}^{-1}\mathbf{Y}B_2 \\ 0 & \mathbb{I} \end{pmatrix} \begin{pmatrix} \mathbf{Q} - \mathbf{Y}A\mathbf{X} & \mathbf{F} \\ \mathbf{L} & \mathbf{R} \end{pmatrix} \begin{pmatrix} \mathbf{U}^{-1} & 0 \\ -C_2\mathbf{X}\mathbf{U}^{-1} & \mathbb{I} \end{pmatrix}.$$

The compensation gain matrix $\mathbf{E_c}$ *is given by*

$$\mathbf{E_c} = \mathbf{V}^{-1}\mathbf{M}; \qquad (10)$$

where $\mathbf{V}$ *and* $\mathbf{U}$ *are non singular matrices satisfying* $\mathbf{YX} + \mathbf{VU} = \mathbb{I}$.

This result shows that one can easily find the appropriate LMI formulation for each particular specification, where $\mathcal{H}_2/\mathcal{H}_2$ and $\mathcal{H}_\infty/\mathcal{H}_\infty$ criterium can also be considered. For instance, the design problem can involve an constraint on the controlled signal, and a constraint on the control signal from the saturation signal. Then, it possible find the corresponding synthesis LMIs on the multiobjective problem, and solve the LMI system numerically to derive a solution.

4 Numerical Example

Let us consider the F-8 fighter aircraft model, which describes its longitudinal dynamics, [3]:

$$
\begin{aligned}
\dot{x} &= \begin{pmatrix} -0.8 & -0.006 & -12 & 0 \\ 0 & -0.014 & -16.64 & -32.2 \\ 1 & -0.0001 & -1.5 & 0 \\ 1 & 0 & 0 & 0 \end{pmatrix} x + \\
& \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix} \omega + \begin{pmatrix} -19 & -3 \\ -0.66 & -0.5 \\ -0.16 & -0.5 \\ 0 & 0 \end{pmatrix} \sigma(u) \\
z &= \begin{pmatrix} 0 & 0 & 0 & 1 \\ 0 & 0 & -1 & 1 \end{pmatrix} x + \begin{pmatrix} 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix} \omega \\
y &= \begin{pmatrix} 0 & 0 & 0 & 1 \\ 0 & 0 & -1 & 1 \end{pmatrix} x
\end{aligned}
$$

In this example, the bounded controls $u_1 \in [-25, 25]$ and $u_2 \in [-25, 25]$ are considered.

By means of LMI toolbox for MathLab©, the linear inequalities for the $\|T_{z\omega}\|_\infty < \mu$ subject to $\|T_{u\vartheta}\|_\infty < \mu$ problem are solved. The following results are obtained:

$$\mu = 1.0125$$

$$\mathbf{A}_c = 1.0\times 10^4 \begin{pmatrix} -0.0674 & 0.0001 & -1.7866 & -2.3089 \\ -0.0024 & 0.0000 & 0.5904 & 0.2884 \\ -0.0006 & 0.0000 & -0.0008 & -0.0059 \\ 0.0001 & -0.0000 & -0.0018 & -0.0020 \end{pmatrix}$$

$$\mathbf{B}_c = 1.0 \times 10^4 \begin{pmatrix} 3.8947 & -1.9456 \\ -0.8909 & 0.5866 \\ 0.0043 & -0.0020 \\ 0.0038 & -0.0018 \end{pmatrix}$$

$$\mathbf{C}_c = \begin{pmatrix} 434.8067 & -0.0509 & -74.3562 & 195.8146 \\ 3.3372 & -0.0050 & -2.8796 & 11.3936 \end{pmatrix}$$

$$\mathbf{D}_c = \begin{pmatrix} -18.5431 & 9.8052 \\ 3.3972 & -3.1300 \end{pmatrix}$$

The compensation gain obtained is

$$\mathbf{E_c} = \begin{pmatrix} -18.9767 & -3.0047 \\ -0.6616 & -0.5139 \\ -0.1600 & -0.5000 \\ 0.0000 & -0.0000 \end{pmatrix}.$$

For the time analysis, a simulation was made. In order to evaluate the robustness with respect to changes in the saturation limits, an actuator with $u \in [-10, 10]$ was considered. Figure 1 shows the results for the actuator outputs without saturation, while the Figure 2 shows the actuator outputs with saturation and without compensation, and with compensation.

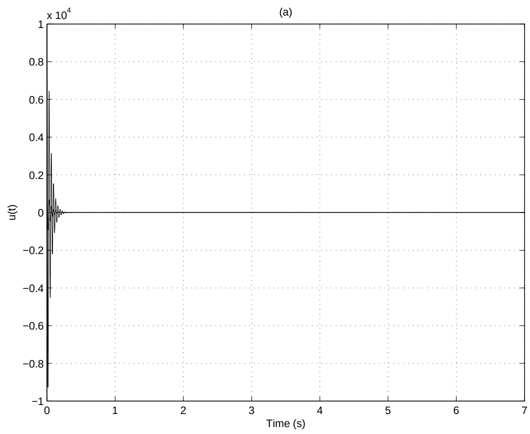

Figure 1: The actuator outputs without saturation.

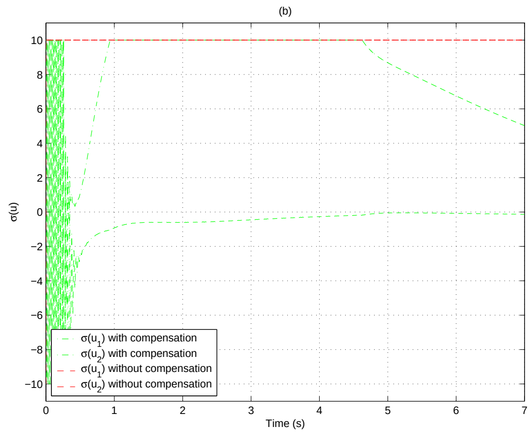

Figure 2: The actuator outputs with saturation.

For the case without saturation, it can notice that the control signal overcomes thoroughly the action limit of the actuator. In saturation, the compensation action allows to enter in the actuator limits, which guarantees the stability of the system in spite of changes in the saturation limits. The temporary behavior without saturation and with compensation are similar, while that for without compensation case the system is unstable due to the saturation, see Figure 3.

Just as it can be observed in the Figure 3, the compensation guarantees the system closed loop performance, in spite of the change in the saturation limits. In the without-compensation case the time response is deteriorated (unstable). This situation is

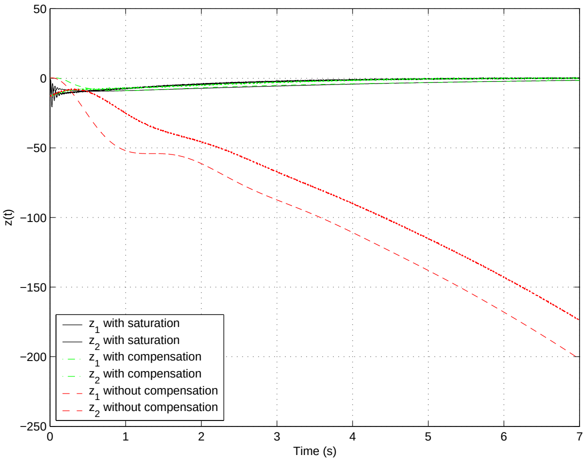

Figure 3: The controlled outputs.

important to evaluate for critical system such as the F-8 fighter aircraft. Thus, it is important to consider some robustness characteristics for designing the gain compensation matrix such as has been proposed.

5 Conclusion

An approach for robust *anti-windup* compensation design for multivariable systems has been presented. The technique is based on $\mathcal{H}_2$ and $\mathcal{H}_\infty$ norms characterization as Linear Matrix Inequalities (LMI). The robustness analysis is considered on the closed loop transfer matrix of the controlled output respect to the perturbation, and on the transfer function of the control signal respect to difference between the actuator output and the controller output. This difference is also considered as a perturbation. A mix the criteria in $\mathcal{H}_2/\mathcal{H}_\infty$ are studied, which allows to guarantee the robust stability and performance, in spite of unknown changes on the actuator saturation limits. The robust compensation gain design is obtained by means LMIs, which allows to describe a convex optimization problem. This problem can be solved in polynomial time by specialized algorithms.

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