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Optimal Path Planning for Flexible Redundant Robot Manipulators

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Abstract: - Vibration is one of the most important resources of error in motion of tip of flexible robot manipulators. Although much work has been done in the design of controllers for flexible manipulators to follow a specified tip trajectory, it has been done a little work in trajectory planning itself. For redundant robot manipulators, trajectory planning can be accomplished with the aim of optimizing some objective functions while tracing along a given tip trajectory. In the present work, the optimal trajectory-planning problem for a flexible redundant manipulator will be formulated as a two point boundary value problem (TPBV) by globally minimizing the elastic deformation of flexible links during the motion.

Key-Words: Manipulator - Redundant - Flexible - Vibration - Optimization - Path Planning

1 Introduction

Precise control of a robot manipulator is an important aspect of robot performance. In the past, this has led to the design of heavy and stiff links and large motors. Hence most existing manipulators have very low payload to total weight ratio. However, to increase productivity by fast motion and to have less energy consumption, robot arms are required to have light and consequently flexible structures.

The main difficulty which arises from flexible structures is vibration. This vibration leads to an error in motion of the manipulator tip along a given trajectory and especially after reaching the manipulator to the desired end goal point. Since this vibration needs additional settling time before any task can be begun, an effective method should be employed to reduce the vibration and strain energy of the robot manipulator.

Although there has been much work in the area of controlling flexible manipulators to follow a desired tip trajectory [1], little work has been done in choosing the trajectory itself. Such a choice is necessary either for point to point motion or for resolving the redundancy of robots with more degrees of freedom than necessary to follow the given tip trajectory. Meirowitch and Chen used LQR control to reduce the elastic disturbance for a flexible redundant robot [2]. Eisler et al. considered joint torque bounds and determined minimum time trajectories to minimize tip tracking error for a flexible redundant robot [3]. However these works have been based on minimizing a cost function independent of elastic deflections and velocities.

Redundancy, which is widely used in robot manipulators to achieve additional performance while tracing a given end-effector trajectory, can be used for vibration reduction of flexible robot manipulators. Shigang analyzed the effects of initial configuration in vibration reduction of flexible robots with kinematic redundancy [4]. Although the problem of optimizing the joint motion for rigid, redundant manipulators has already received much attention from researchers, there has been a little work concerning flexible redundant manipulators.

Optimal path generation by minimizing an objective function evaluated locally, "local optimization", involves less complexity and requires less computational effort, while minimizing an objective function evaluated along the entire path, "global optimization", results in more complexity, but more desirable manipulator performance. In the previous works, some methods have been developed on optimal path generation for flexible redundant manipulators by local optimization of some objective functions [5], and in the present work we will propose a method for optimal path generation by global minimization of elastic deformation along a specified path.

The approach developed in this paper formulates the trajectory planning problem for a flexible redundant manipulator by a two point boundary value problem (TPBV) where globally minimizes the elastic deformation during the motion. The remainder of the paper is organized as follows. First, the equations of motion for a general flexible robot manipulator are briefly presented. The formulation of optimal trajectory planning will be developed in the third section. Then different numerical schemes for solving two-point boundary value problems will be defined and finally a numerical example will be solved by the presented formulation at the end.

2 Dynamic Modeling

Dynamic equations of a general flexible robot can be derived using different methods such as finite element or assumed mode methods. It can be shown that dynamic equations for a flexible robot manipulator can be written as:

$$\mathbf{M}_{11}(\boldsymbol{\theta},\boldsymbol{\Phi})\ddot{\boldsymbol{\theta}} + \mathbf{M}_{12}(\boldsymbol{\theta},\boldsymbol{\Phi})\ddot{\boldsymbol{\Phi}} + \mathbf{d}_1(\boldsymbol{\theta},\dot{\boldsymbol{\theta}},\boldsymbol{\Phi},\dot{\boldsymbol{\Phi}}) = \boldsymbol{\tau} \quad (1)$$

$$\mathbf{M}_{12}^T(\boldsymbol{\theta},\boldsymbol{\Phi})\ddot{\boldsymbol{\theta}} + \mathbf{M}_{22}(\boldsymbol{\theta},\boldsymbol{\Phi})\ddot{\boldsymbol{\Phi}} + \mathbf{d}_2(\boldsymbol{\theta},\dot{\boldsymbol{\theta}},\boldsymbol{\Phi},\dot{\boldsymbol{\Phi}}) = \mathbf{0} \quad (2)$$

Where $\boldsymbol{\theta}$, $\dot{\boldsymbol{\theta}}$ and $\ddot{\boldsymbol{\theta}}$ are vectors of joint angles, velocities and acceleration, $\boldsymbol{\Phi}$, $\dot{\boldsymbol{\Phi}}$ and $\ddot{\boldsymbol{\Phi}}$ are vectors of generalized coordinates, velocities and accelerations describing deformation behavior of the flexible links and $\boldsymbol{\tau}$ is the actuator torque vector. $\mathbf{d}_1$ and $\mathbf{d}_2$ are terms involving damping, stiffness, centrifugal, corilois of rigid and flexible coupling, and gravitational effects.

3 Trajectory Optimization

For redundant manipulators the number of joints is more than the number of degrees of freedom that is required to follow a desired tip trajectory. In optimal path generation approaches, the trajectory of joints is designed to optimize some objective function along the entire trajectory. In this paper we will find the optimal trajectory of a redundant, flexible robot by globally minimizing a function of flexible coordinates in the form of:

$$g(\boldsymbol{\Phi}) = \boldsymbol{\Phi}^T \mathbf{C} \boldsymbol{\Phi} \quad (3)$$

where $\mathbf{C}$ is a positive definite weight matrix.

To minimize a function evaluated along the entire path, the standard methods of calculus of variations such as Hamiltonian approach for optimal control problems may be employed. Optimal trajectory of the manipulator is determined by globally minimizing $g(\boldsymbol{\Phi})$, subject to some constraints. The first constraint is the motion of the tip of the manipulator along a specified trajectory:

$$\mathbf{X}_e = \mathbf{X}(t) \quad (4)$$

Also we can see that (2) represents another constraint. Essentially this is a differential constraint relating rigid acceleration $\ddot{\boldsymbol{\theta}}$ and elastic acceleration $\ddot{\boldsymbol{\Phi}}$. It should be noted that for general manipulators this constraint is non integrable. In motion planning problems, (1) is not a constraint because the torque $\boldsymbol{\tau}$ is not considered in these problems, but only the motion of joints is determined for a specified tip trajectory and once the accelerations and the current state are known, (1) may be employed to obtain actuator torques. It is apparent from (2) that elastic coordinates are not controlled directly. Thus the problem of motion planning is to calculate joint acceleration vector $\ddot{\boldsymbol{\theta}}$ which globally optimizes some objective functions such as (3), subject to the differential constraint (2), tip trajectory constraint (4) and suitable boundary conditions:

$$\min J = \int_0^T \boldsymbol{\Phi}^T \mathbf{C} \boldsymbol{\Phi} \quad s.t$$

$$\mathbf{M}_{12}^T(\boldsymbol{\theta},\boldsymbol{\Phi})\ddot{\boldsymbol{\theta}} + \mathbf{M}_{22}(\boldsymbol{\theta},\boldsymbol{\Phi})\ddot{\boldsymbol{\Phi}} + \mathbf{h}_2(\boldsymbol{\theta},\dot{\boldsymbol{\theta}},\boldsymbol{\Phi},\dot{\boldsymbol{\Phi}}) = 0\,, \quad (5)$$

$$\mathbf{X}_e = \mathbf{X}(t)$$

It should be noted that the tip position of the manipulator is a function of joint angles; In other words the constraint (4) can be written as:

$$\mathbf{X}_e = \mathbf{h}(\boldsymbol{\theta}) = \mathbf{X}(t) \quad (6)$$

The state of the manipulator can be defined as:

$$\mathbf{x} = [\boldsymbol{\theta}^T, \dot{\boldsymbol{\theta}}^T, \boldsymbol{\Phi}^T, \dot{\boldsymbol{\Phi}}^T]^T \quad (7)$$

As can be seen, (2) is a differential constraint of degree 2. We can state the constraints (2) and (6) as a differential constraint of degree 1 with a suitable transformation. Then the constraints of the problem can be summarized by the state dynamics equation:

$$\dot{\mathbf{x}} = \mathbf{f}(\mathbf{x},\mathbf{u}) \quad (8)$$

where (2) and (6) have been used to obtain $\ddot{\boldsymbol{\theta}}$ and $\ddot{\boldsymbol{\Phi}}$ as functions of $\mathbf{x}$ and a control vector $\mathbf{u}$.

If the number of joint angles is n, the number of elastic coordinates is r and $\mathbf{X}_e$ is a vector of dimension s, (2) and (6) represent $r+s$ constraint equations. We can differentiate (6) twice to have:

$$\mathbf{J}(\boldsymbol{\theta})\ddot{\boldsymbol{\theta}} = \ddot{\mathbf{X}}(t) - \dot{\mathbf{J}}(\boldsymbol{\theta})\dot{\boldsymbol{\theta}} \quad (9)$$

where $\mathbf{J}$ is the Jacobian matrix:

$$\mathbf{J}(\boldsymbol{\theta}) = \frac{\partial \mathbf{h}(\boldsymbol{\theta})}{\partial \boldsymbol{\theta}} \quad (10)$$

It is clear that (2) and (9) are linear with respect to $\ddot{\boldsymbol{\theta}}$ and $\ddot{\boldsymbol{\Phi}}$, and may be used to obtain them as functions of $\mathbf{x}$. It is evident that we have $n+r$ unknowns but $r+s$ equations, so if we define:

$$\mathbf{u} = [\ddot{\theta}_1, \ddot{\theta}_2, \ldots, \ddot{\theta}_{n-s}]^T \quad (11)$$

where $n-s$ is the number of degrees of robot redundancy, by this definition it will be possible to state $\ddot{\boldsymbol{\theta}}$ and $\ddot{\boldsymbol{\Phi}}$ as functions of $\mathbf{x}$ and $\mathbf{u}$, and as a consequence the derivative of the state vector $\mathbf{x}$ can be stated as a function of $\mathbf{x}$ and $\mathbf{u}$ as in (8).

Another important problem that we should consider is boundary conditions. In the present problem we want to obtain the optimal joint trajectory from known initial conditions. In other word the vector $\mathbf{x}$ is assumed to be known at time 0. Therefore we can state the problem by:

$$\min J = \int_0^{t_f} F(\mathbf{x},\mathbf{u})dt \quad s.t$$

$$\dot{\mathbf{x}} = \mathbf{f}(\mathbf{x},\mathbf{u}) \;, \quad \mathbf{x}(0) = \mathbf{x}_0 \quad (12)$$

It should be noted that in the present case the final time t_f is fixed. Following the LQR theory developed for optimal control [6], we can choose a general form for F:

$$F(\mathbf{x},\mathbf{u}) = \mathbf{x}^T\mathbf{Q}\mathbf{x} + \mathbf{u}^T\mathbf{R}\mathbf{u} \tag{13}$$

Where $\mathbf{Q}$ and $\mathbf{R}$ are constant weighting matrices, $\mathbf{Q}$ is a positive semi-definite and $\mathbf{R}$ is a positive definite matrix. If our aim is minimization of some objective function in the form of (5), we should define $\mathbf{Q}$ as the followings:

$$\mathbf{Q} = diag(\mathbf{0},\mathbf{0},\mathbf{C},\mathbf{0}) \tag{14}$$

While if our aim is minimizing the joint velocities along the entire path we can choose:

$$\mathbf{Q} = diag(\mathbf{0},\mathbf{C},\mathbf{0},\mathbf{0}) \tag{15}$$

To obtain differential equations governing the optimal trajectory, the differential constraint is incorporated into the problem formulation by augmenting F using the Lagrange multiplier method. So we can state (12) by

$$\min J = \int_0^{t_f} [F(\mathbf{x},\mathbf{u}) + \boldsymbol{\lambda}^T(\mathbf{f}(\mathbf{x},\mathbf{u}) - \dot{\mathbf{x}})]dt \tag{16}$$

Where $\boldsymbol{\lambda}$ is the vector of Lagrange multipliers or costates. For an optimal trajectory we should have:

$$\delta J = 0 \tag{17}$$

δJ can be derived from (16) and to obtain differential equations and associated boundary conditions we should have:

$$\delta J = \int_0^{t_f} [(\frac{\partial F}{\partial \mathbf{x}} + \boldsymbol{\lambda}^T \frac{\partial \mathbf{f}(\mathbf{x},\mathbf{u})}{\partial \mathbf{x}} + \dot{\boldsymbol{\lambda}}^T)\delta\mathbf{x} + \delta\boldsymbol{\lambda}^T(\mathbf{f}(\mathbf{x},\mathbf{u}) - \dot{\mathbf{x}}) + (\frac{\partial F}{\partial \mathbf{u}} + \boldsymbol{\lambda}^T \frac{\partial \mathbf{f}(\mathbf{x},\mathbf{u})}{\partial \mathbf{u}})\delta\mathbf{u}]dt - \boldsymbol{\lambda}^T\delta\mathbf{x}\Big|_0^{t_f} = 0 \tag{18}$$

Thus the equations governing the optimal trajectory will be developed as:

$$\dot{\boldsymbol{\lambda}}^T = -\frac{\partial F}{\partial \mathbf{x}} - \boldsymbol{\lambda}^T \frac{\partial \mathbf{f}}{\partial \mathbf{x}} \tag{19}$$

$$\dot{\mathbf{x}} = \mathbf{f}(\mathbf{x},\mathbf{u}) \tag{20}$$

$$\frac{\partial F}{\partial \mathbf{u}} = -\boldsymbol{\lambda}^T \frac{\partial \mathbf{f}(\mathbf{x},\mathbf{u})}{\partial \mathbf{u}} \Rightarrow 2\mathbf{u}^T\mathbf{R} = -\boldsymbol{\lambda}^T \frac{\partial \mathbf{f}(\mathbf{x},\mathbf{u})}{\partial \mathbf{u}} \tag{21}$$

Control vector $\mathbf{u}$ can be obtained as a function of $\mathbf{x}$ and $\boldsymbol{\lambda}$ from the last equation because $\mathbf{R}$ is a non-singular matrix and $\mathbf{f}$ is a linear function of $\mathbf{u}$. So we can integrate (19) and (20) to obtain states and costates from initial time 0 to final time t_f. Also the boundary conditions satisfy the expression:

$$\boldsymbol{\lambda}^T\delta\mathbf{x}\Big|_0^{t_f} = 0 \tag{22}$$

It should be noted that the state vector $\mathbf{x}$ is unknown at final time t_f, so (22) leads to:

$$\mathbf{x}(0) = \mathbf{x}_0 \quad , \quad \boldsymbol{\lambda}\Big|_{t_f} = 0 \tag{23}$$

As can be seen the problem of optimal trajectory planning for a flexible redundant robot manipulator tracing along a specified tip trajectory with minimum elastic deflections leaded to a two-point boundary value problem. The solution of the two-point boundary value problem defined by (19) and (20) associated with the given initial and final conditions for states and costates (23) will yield the optimal trajectory of the manipulator.

Note that by choosing $\mathbf{R}$ to be a positive definite matrix, we will have the positive definite Hessian and consequently the solution derived by the aforementioned scheme will be a unique global minimum. Also it should be noted that when the aim is not minimization of $\mathbf{u}$, as for the present problem, $\mathbf{R}$ should not be made very large to affect the solution.

4 Numerical Methods

There are some methods for solving two-point boundary value problems. The finite-difference method, shooting method, Rayleigh-Ritz method, collocation method and dynamic programming [7] are some of these methods. The important difference between these methods is in how well the differential equations and the boundary conditions of the problem can be satisfied. In finite-difference methods the derivatives in the differential equations of the problem are approximated by finite differences between the solution at discrete times [8]. In some problems as for the present case to have an accurate solution with finite-difference method a very high number of discrete times may be required. In shooting methods the differential equations of the problem are solved accurately and the solution is obtained within integration tolerances. This is done by initially approximating some of the boundary conditions and an iterative method is used to update the initial approximations to converge to a solution which satisfies the given boundary conditions. The error in the known boundary conditions expressed as a function of the initial approximations can be used to modify the initial approximations of the boundary conditions and a nonlinear optimization method is used to update the initial guesses to reduce the error. In Rayleigh-Ritz method the solution of a problem is approximated by a linear combination of some basis functions [8], so a good experience in choosing the appropriate basis functions is very essential to obtain an accurate solution by this method. Collocation method is a method similar to the Rayleigh-Ritz method in which dynamics constraints are satisfied at discrete times. Comparing these methods, we employed collocation method for solving the two point

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boundary value problem described in the previous section.

5 Numerical Example

As an example we consider optimal path generation for a two degree of freedom planar manipulator (Fig. 1) with the aim of minimization of the elastic deflections of the flexible links along the entire path. The tip of the manipulator is to move along a trajectory constrained by the equation:

$$x = 2\cos(t) \tag{24}$$

The above constraint can be written in the form of (6) as:

$$x = l_1 \cos(\theta_1) + l_2 \cos(\theta_1 + \theta_2) = 2\cos(t) \tag{25}$$

We have a two degree of freedom manipulator that should move along a trajectory constrained by only one constraint, so we have a robot with 1 degree of redundancy. Link 1 is a rigid link of length 2 m and link 2 is a flexible link of length 4 m, Young's modulus E=71 GPa and moment of inertia of the cross section I= 50×10-12 m^4. Two links have the same density of 2710 kg/m^3. A dynamic model for this manipulator can be developed using finite element or assumed mode method. Here, only transverse displacement of the flexible link 2 is considered in the model.

In order to make a comparison between the results, we have accomplished trajectory planning for two different problems. In the first problem, an objective cost function of elastic deflections is used to minimize elastic vibrations along the entire path, while in the second problem we have used a cost function of joint velocities in the form of:

$$\min J = \int_0^{t_f} (\dot{\theta}_1^{\,2} + \dot{\theta}_2^{\,2}) dt \tag{26}$$

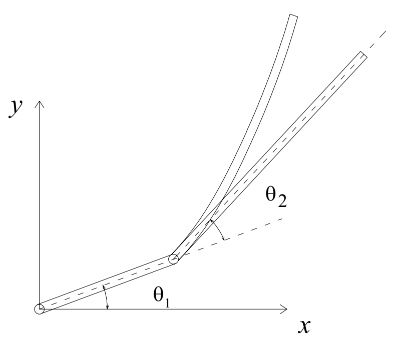

Fig. 1 A two link flexible manipulator

For the first problem matrix **Q** is obtained from (14) and for the second problem it is obtained from (15) and identity matrix with appropriate dimension may be substituted for **C** in (14) and (15). In the present example, vector **u** is a one dimensional vector, because we have one degree of redundancy. So the one dimensional weight matrix **R** is made positive definite but not large enough to affect the solution. Here we consider:

$$\mathbf{R} = 10^{-6} \tag{26}$$

Now we should specify the initial conditions $\mathbf{x}_0$ from which the optimal trajectory will be initiated, we consider:

$$\theta_1(0) = \pi/3$$
$$\theta_2(0) = 0.27 \tag{27}$$
$$\dot{\boldsymbol{\theta}} = \mathbf{0} \quad \boldsymbol{\Phi} = \mathbf{0} \quad \dot{\boldsymbol{\Phi}} = \mathbf{0}$$

It is apparent that the initial conditions for joint angles satisfy the constraint (25). Also the initial joint velocities, elastic deflections and elastic rates are considered to be zero. For the first problem the error in tracking the specified path (24) is shown in Fig. 2. It is seen that the error is very small and the tip of the manipulator moves along the specified path with an acceptable accuracy. The elastic deflection of the end point of link 2 is shown in Fig. 3 and as is shown, the maximum tip deflection is less than .04% of the length of the flexible link 2. Also the configurations of the manipulator as tracing the specified trajectory are shown in Fig. 4. The amount of the cost function of elastic deflections (J_1) and the cost function of joint velocities (J_2) calculated in this problem are:

J_1= 6.5527e-11
J_2= 0.0990

For the second problem the error in tracking the specified path (24) is shown in Fig. 5. It is seen that the tip of the manipulator moves along the specified path with a small error. The elastic deflection of the end point of link 2 is shown in Fig. 6. The maximum tip deflection is more than 10% of the length of the link 2. The amount of the cost function of elastic deflections (J_1) and the cost function of joint velocities (J_2) calculated in this problem are:

J_1= .0015
J_2= .0959

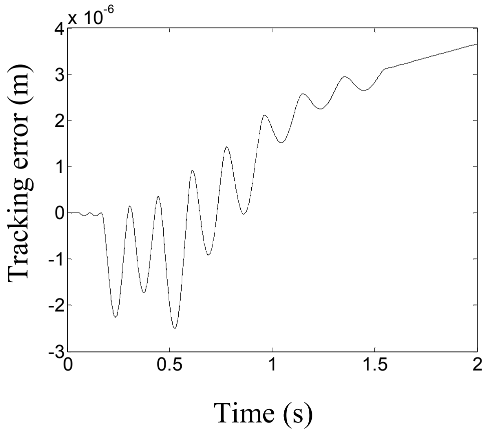

Fig. 2 Error in tracking the desired path (first problem)

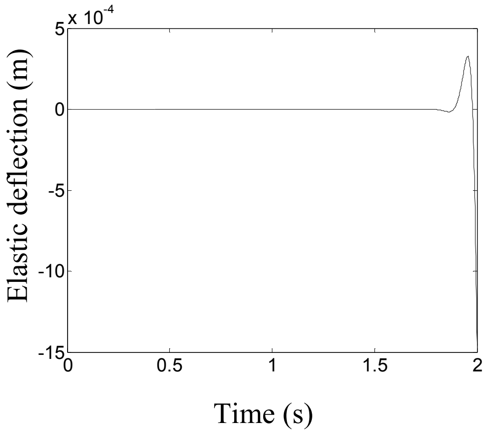

Fig. 3 Elastic deflection of the end point of link 2 (first problem)

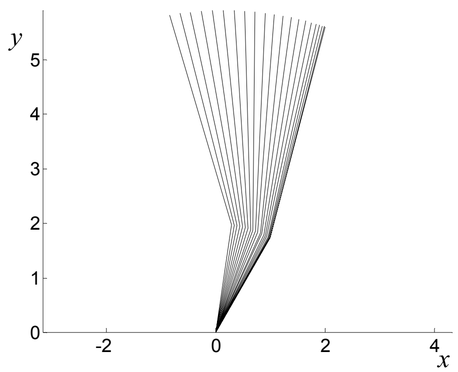

Fig. 4 Manipulator configurations (first problem)

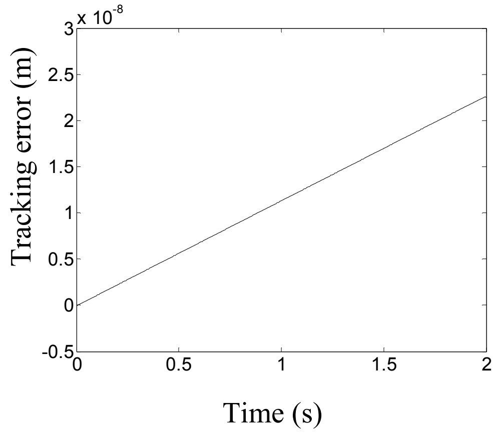

Fig. 5 Error in tracking the desired path (second problem)

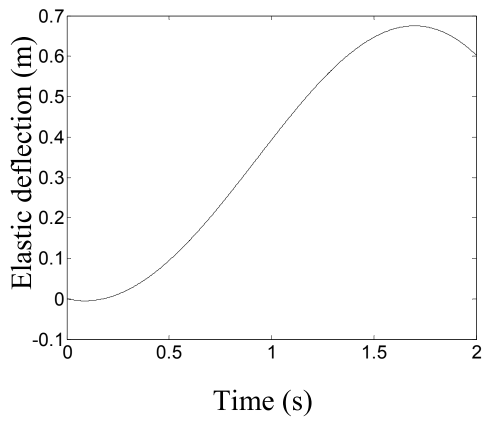

Fig. 6 Elastic deflection of the end point of link 2 (second problem)

It is apparent that as we expected J_1 calculated in the first problem is less than J_1 obtained in the second problem, Also the amount of the cost function J_2 in the second problem is less than J_2 obtained in the first problem.

6 Conclusion

An approach for solving the optimal path planning problem for a flexible redundant robot manipulator has been presented. The optimal path has been generated by globally minimizing an objective cost function dependent on the elastic deformations of the flexible links of the manipulator. The presented methodology leaded to a two-point boundary value problem and the collocation method has been employed for solving this problem. The efficiency of the proposed approach has been shown by a simple numerical example of optimal path planning for a two degree of freedom manipulator with link flexibility. This approach is applicable to a general flexible redundant manipulator for minimization of different objective functions.

References:

[1] W. J. Book. Structural flexibility of motion systems in the space environment, *IEEE Transactions on Robotics and Automation*, vol. 9, no. 5, pp. 524-530, October 1993.

[2] L. Meirowitch and Y. Chen. Trajectory and control optimization for flexible space robots, *Journal of Guidance, Control, and Dynamics*, vol. 18, no. 3, pp. 493-502, May-June 1995.

[3] G. R. Eisler, R. D. Robinett, D. J. Segalman, and J. D. Feddema. Approximate optimal trajectories for flexible-link manipulator slewing using recursive quadratic programming, *Transactions of the ASME, Journal of Dynamic Systems, Measurement and control*, vol. 115, pp. 405-410, September 1993.

[4] Y. Shigang. Weak-vibration configurations for flexible robot manipulators with kinematic redundancy, *Journal of Mechanism and Machine Theory,* vol. 35, pp. 165-178, 2000.

[5] X. Zhang, Y. Q. Yu. Motion control of flexible robot manipulators via optimizing redundant configurations. *Journal of Mechanism and Machine Theory,* vol. 36, pp. 883-892, 2001.

[6] S. M. Shinners. *Modern Control System Theory and Design*. John Wiley & Sons, 2nd edition, 1998.

[7] D. P. Bertsekas. *Dynamic Programming and Optimal Control*. Athena Scientific, 2nd edition, 1995.

[8] R. L. Burden and J. D. Faires. *Numerical analysis*. PWS-Kent Publishing, Boston, 6th edition, 1997.