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Hybrid Estimators for Multivariable Systems with Variable Parameters

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Abstract: - The properties of system with variable parameters are described in the nominal operations regimes by means of the state variables system and by means of the normal or mixed difference equations system. The estimators are designed according to these descriptions. The control performance improvement is evaluated.

Key-Words: - System with variable structure, multivariable linear discrete time incremental estimator, hybrid system.

1 Introduction

The control performance of each real control loop can often change during the operation partly in consequence of the controlled systems properties changes (parameters and structure changes) and partly in consequence of the input disturbances changes. The whole domain of the possible operation regimes of the controlled system can be described by means of the finite number of the nominal regimes. The groups of the optimally tuned controller and of the optimally tuned estimators in the control system with the variable structure are designed for each nominal operation regime in advance (during the designing of the control system). The most suitable controller and estimator are switched on into the control operation. The choice is based on the estimator error.

The controller and controlled system may be of different types: continuous [3] or discrete [6], linear or nonlinear, SISO [1] to [5] or MIMO [6], with [3] or without the time delay and even logical, fuzzy [2] and hybrid.

The control with variable structure using the designed incremental discrete time estimator makes the control processes to be very near to the optimum in the whole domain of the operation regimes. The optimisation process is very fast in comparison with the usual adaptation methods. The control with variable structure described in this paper is planned to be used in the power system control.

2 The Estimator Design

The design of the MIMO estimator is extended from the SISO estimator [4]. The estimator is discrete time and incremental. The signals in the estimator scheme correspond to the time increments of the corresponding signals of the controlled system model. See Fig.1.

The feed back loop from the estimator output signals is replaced by the increments of the real controlled system outputs $\boldsymbol{y}_S$.

The controller, which should be connected with the estimator, is tuned for the controlled system model with the measurable state variables. The state description or the difference equations description of the controlled system is supposed to be available. Arbitrary controller can by used (continuous, discrete, PID or state type, logical, fuzzy, …).

2.1 The block structure of the controlled system

The controlled system consists of several blocks, whose outputs are the different controlled variables y. There are no interactions between the state variables of the different blocks.

The state description of the e.g. two dimensional controlled system of the fourth order is (the vector $\boldsymbol{y}$ is measurable)

$$\boldsymbol{x} = z^{-1}\mathbf{A}\,\boldsymbol{x} + z^{-1}\mathbf{B}\,\boldsymbol{u}\,,\ \ \boldsymbol{y} = \mathbf{C}\,\boldsymbol{x}\,, \tag{1}$$

where

$$\mathbf{A} = \begin{bmatrix} a_{11} & 0 & 0 & 0 \\ 0 & a_{22} & 0 & 0 \\ 0 & 0 & a_{33} & 0 \\ 0 & 0 & 0 & a_{44} \end{bmatrix},\ \mathbf{B} = \begin{bmatrix} b_{11} & 0 \\ b_{21} & 0 \\ 0 & b_{32} \\ 0 & b_{42} \end{bmatrix}, \tag{2}$$

$$\mathbf{C} = \begin{bmatrix} 1 & 0 & 1 & 0 \\ 0 & 1 & 0 & 1 \end{bmatrix},\ \boldsymbol{u} = \begin{bmatrix} u_1 \\ u_2 \end{bmatrix},\ \boldsymbol{y} = \begin{bmatrix} y_1 \\ y_2 \end{bmatrix}.$$

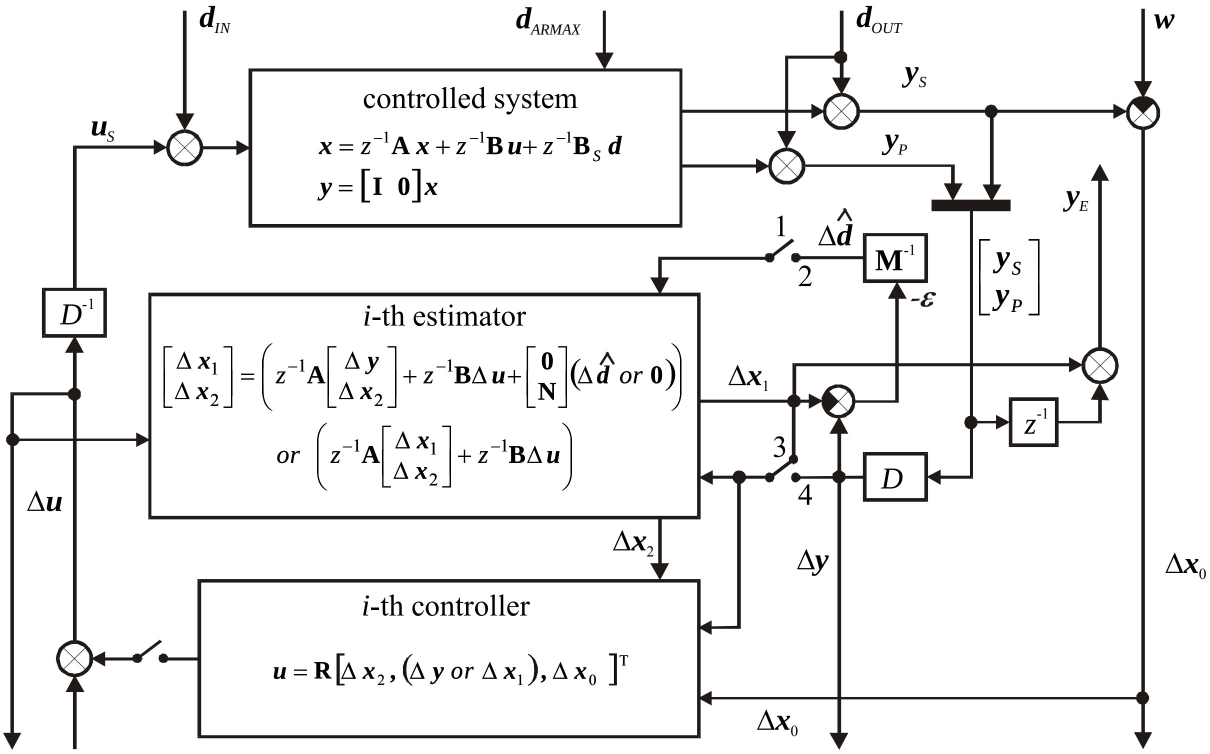

Fig.1. The estimator model designed according to the state variables description and connected in the variable structure control system, $\mathbf{B}_S = \begin{bmatrix} \mathbf{M} \\ \mathbf{N} \end{bmatrix}$, $D = 1 - z^{-1}$,

contact position: 1 for E_{ARX} and E_{OUT}, 2 for E_{IN} and E_{ARMAX}, 3 for E_{OUT}, 4 for E_{IN}, E_{ARX} and E_{ARMAX}.

The vector $\boldsymbol{x}$ is linearly transformed to the vector with the variables $\boldsymbol{y}$

$$\begin{bmatrix} \boldsymbol{y} \\ x_3 \\ x_4 \end{bmatrix} = \mathbf{T}\,\boldsymbol{x} = \begin{bmatrix} \mathbf{C} & & & \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \boldsymbol{x}\,,\; \mathbf{T} = \begin{bmatrix} 1 & 0 & 1 & 0 \\ 0 & 1 & 0 & 1 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \quad (3)$$

and after inserting in relation (1)

$$\begin{bmatrix} \boldsymbol{y} \\ x_3 \\ x_4 \end{bmatrix} = \mathbf{T}\,\boldsymbol{x} = z^{-1}\mathbf{TAT}^{-1}\begin{bmatrix} \boldsymbol{y} \\ x_3 \\ x_4 \end{bmatrix} + z^{-1}\mathbf{TB}\,\boldsymbol{u} =$$

$$= z^{-1}\begin{bmatrix} a_{11} & 0 & (-a_{11}+a_{33}) & 0 \\ 0 & a_{22} & 0 & (-a_{22}+a_{44}) \\ 0 & 0 & a_{33} & 0 \\ 0 & 0 & 0 & a_{44} \end{bmatrix} \begin{bmatrix} \boldsymbol{y} \\ x_3 \\ x_4 \end{bmatrix} +$$

$$+ z^{-1}\begin{bmatrix} b_{11} & b_{32} \\ b_{21} & b_{42} \\ 0 & b_{32} \\ 0 & b_{42} \end{bmatrix} \begin{bmatrix} u_1 \\ u_2 \end{bmatrix}. \quad (4)$$

The model scheme for the relation (4) is in Fig.2.

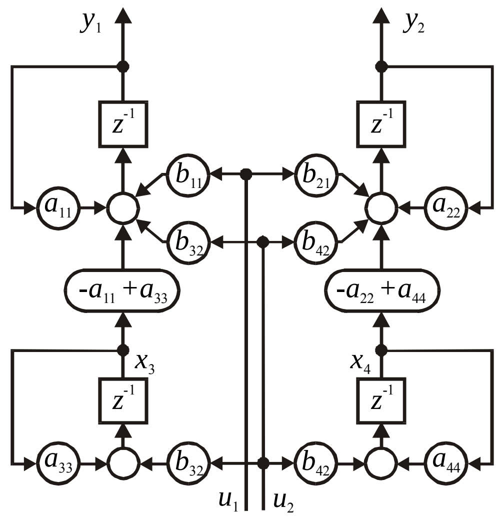

Fig.2. The model scheme of the blocks structure control system using state variables description (4).

Description (4) of the total system is of the fourth order. The system consists of two blocks. There are no interactions between the state variables from different blocks. For being nilpotent by means of the controlled variables y_{S1} and y_{S2} the coefficient matrix TAT^{-1} must fulfil the condition of non-zero coefficients in Fig.3. The matrix in (4) does not fulfil this condition.

$$\mathbf{A}=\begin{bmatrix} \mathrm{x} & \mathrm{x} & \mathrm{x} & \mathrm{x} \\ \mathrm{x} & \mathrm{x} & \mathrm{x} & \mathrm{x} \\ \mathrm{x} & \mathrm{x} & 0 & \mathrm{x} \\ \mathrm{x} & \mathrm{x} & 0 & 0 \end{bmatrix}$$

Fig.3. The configuration of the coefficients matrix **A** of two-dimensional system with admissible number of non-zero elements.

However it is possible to do that by means of the elimination of the variables x_3 and x_4 from the equations system for y_1 and y_2 (4)

$$\begin{aligned} y_1 &= z^{-1}a_{11}y_1 + z^{-2}\left(-a_{11}+a_{33}\right)\frac{b_{32}}{1-z^{-1}a_{33}}u_2 + \\ &\quad + z^{-1}b_{11}u_1 + z^{-1}b_{32}u_2 \,, \\ y_2 &= z^{-1}a_{22}y_2 + z^{-2}\left(-a_{22}+a_{44}\right)\frac{b_{42}}{1-z^{-1}a_{44}}u_2 + \\ &\quad + z^{-1}b_{21}u_1 + z^{-1}b_{42}u_2 \end{aligned} \tag{5}$$

and after arrangement

$$\begin{aligned} &\left(1-z^{-1}a_{11}\right)\left(1-z^{-1}a_{33}\right)y_1 = \\ &= z^{-1}\left(1-z^{-1}a_{33}\right)b_{11}u_1 + z^{-1}\left(1-z^{-1}a_{11}\right)b_{32}u_2 \,, \\ &\left(1-z^{-1}a_{22}\right)\left(1-z^{-1}a_{44}\right)y_2 = \\ &= z^{-1}\left(1-z^{-1}a_{44}\right)b_{21}u_1 + z^{-1}\left(1-z^{-1}a_{22}\right)b_{42}u_2 \,. \end{aligned} \tag{6}$$

The scheme of the controlled system model corresponding with (6) is in Fig.4. The system model can make the estimator nilpotent using the variables y_{S1} and y_{S2}.

2.2 The global structure of the controlled system

In the global structure, each state variable of the MIMO system can be influenced by another arbitrary state variable from the whole system and vice versa.

The formal state description is the same as for the block structure system (1). However the configuration of the matrices is different. In the case of a controlled system of the fourth order with two inputs and two outputs the matrices are e.g.

$$\mathbf{A}=\begin{bmatrix} a_{11} & 1 & 0 & 0 \\ a_{21} & 0 & 1 & 0 \\ a_{31} & 0 & 0 & 1 \\ a_{41} & 0 & 0 & 0 \end{bmatrix},\ \mathbf{B}=\begin{bmatrix} b_{11} & b_{12} \\ b_{21} & b_{22} \\ b_{31} & b_{32} \\ b_{41} & b_{42} \end{bmatrix}, \tag{7}$$
$$\mathbf{C}=\begin{bmatrix} 1 & 0 & 0 & 0 \\ c_1 & 0 & c_3 & 0 \end{bmatrix}.$$

The output variable y_2 is linearly transformed into the state vector e.g.

$$\begin{bmatrix} y_1 \\ x_2 \\ y_2 \\ x_4 \end{bmatrix} = \mathbf{T}\,\boldsymbol{x},\ \mathbf{T}=\begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ \mathrm{c}_1 & 0 & c_3 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}. \tag{8}$$

After inserting (8) and (7) to relation (1) the resulting state description is

$$\begin{bmatrix} y_1 \\ x_2 \\ y_2 \\ x_4 \end{bmatrix} = \mathbf{T}\,\boldsymbol{x} = z^{-1}\mathbf{TAT}^{-1}\begin{bmatrix} y_1 \\ x_2 \\ y_2 \\ x_4 \end{bmatrix} + z^{-1}\mathbf{TB}\,\boldsymbol{u} \tag{9}$$

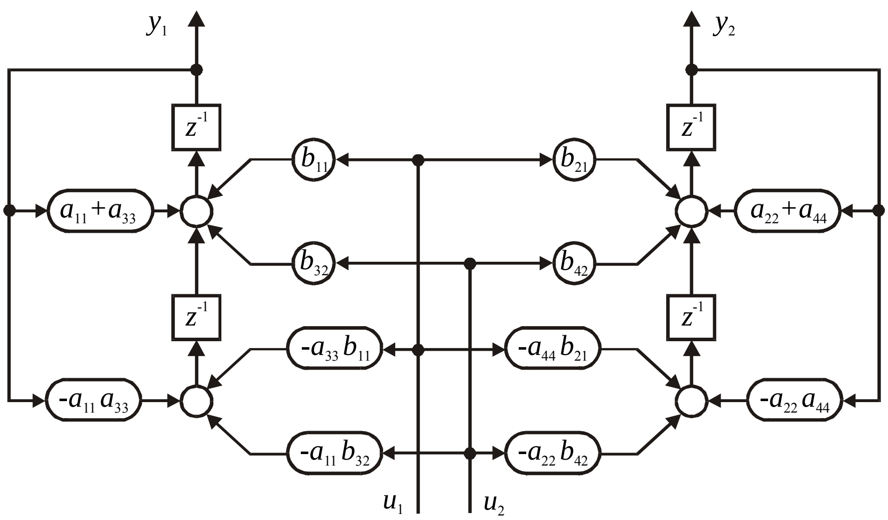

Fig.4. The model of the blocks structure controlled system using the normal difference equations description (6) (the total order is 4)

and the same in numerical form is e.g.

$$\begin{bmatrix} y_1 \\ x_2 \\ y_2 \\ x_4 \end{bmatrix} = z^{-1} \begin{bmatrix} 2.4261 & 1 & 0 & 0 \\ -1.4501 & 0 & 0.1 & 0 \\ -9.4450 & -7.5719 & 0 & 10 \\ -0.1353 & 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} y_1 \\ x_2 \\ y_2 \\ x_4 \end{bmatrix} + \tag{10}$$

$$+ z^{-1} \begin{bmatrix} 0.0018 & 0.0010 \\ 0.0130 & 0.0050 \\ 0.0738 & 0.4924 \\ 0.0005 & 0.0010 \end{bmatrix} \boldsymbol{u} .$$

After rewriting (10) to the general form, the description is

$$\begin{bmatrix} y_1 \\ x_2 \\ y_2 \\ x_4 \end{bmatrix} = z^{-1} \begin{bmatrix} a_{11} & a_{12} & 0 & 0 \\ a_{21} & 0 & a_{23} & 0 \\ a_{31} & a_{32} & 0 & a_{34} \\ a_{41} & 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} y_1 \\ x_2 \\ y_2 \\ x_4 \end{bmatrix} + \tag{11}$$

$$+ z^{-1} \begin{bmatrix} b_{11} \\ b_{21} \\ b_{31} \\ b_{41} \end{bmatrix} u_1 + z^{-1} \begin{bmatrix} b_{12} \\ b_{22} \\ b_{32} \\ b_{42} \end{bmatrix} u_2 .$$

The model scheme for the relation (11) is in Fig.5.

The description of the total system is of the fourth order.

The state variables of the system influence each other. The estimator system can be made nilpotent by means of the output variables y_{S1} and y_{S2} .

The state description is obtained by means of the physical and mathematical analysis as usual. It can also be obtained from the normal difference equations by means of the matrix inversion. The same can be done from the mixed difference equations after transformation of the mixed equations to the normal equations (by means of elimination of the other variables) at first.

<u>The mixed difference equations description</u>

The equations can be currently identified on the controlled system. Otherwise it is possible to calculate the equations by means of elimination of the unmeasurable variables from the state description (11)

$$\begin{aligned} y_1 &= z^{-1} a_{11} y_1 + z^{-1} a_{12} \left(z^{-1} a_{21} y_1 + z^{-1} a_{23} y_2 + \right. \\ &\quad \left. + z^{-1} b_{21} u_1 + z^{-1} b_{22} u_2 \right) + z^{-1} b_{11} u_1 + z^{-1} b_{12} u_2 , \\ y_2 &= z^{-1} a_{31} y_1 + z^{-1} a_{32} \left(z^{-1} a_{21} y_1 + z^{-1} a_{23} y_2 + \right. \\ &\quad \left. + z^{-1} b_{21} u_1 + z^{-1} b_{22} u_2 \right) + z^{-1} a_{34} \left(z^{-1} a_{41} y_1 + \right. \\ &\quad \left. + z^{-1} b_{41} u_1 + z^{-1} b_{42} u_2 \right) + z^{-1} b_{31} u_1 + z^{-1} b_{32} u_2 , \end{aligned} \tag{12}$$

The same programme in the MATLAB software package for calculating the normal difference equations from the step responses offers the mixed difference equations too and the result in the numerical form is

$$\begin{aligned} y_1 &= \left(2.4261\, z^{-1} - 1.4501\, z^{-2} \right) y_1 + 0.1\, z^{-2} y_2 + \\ &\quad + \left(0.0018\, z^{-1} + 0.013\, z^{-2} \right) u_1 + \\ &\quad + \left(0.001\, z^{-1} + 0.005\, z^{-2} \right) u_2 , \\ y_2 &= \left(-9.445\, z^{-1} + 9.627\, z^{-2} \right) y_1 - 0.7519\, z^{-2} y_2 + \\ &\quad + \left(0.0738\, z^{-1} - 0.09343\, z^{-2} \right) u_1 + \\ &\quad + \left(0.4924\, z^{-1} - 0.02786\, z^{-2} \right) u_2 . \end{aligned} \tag{13}$$

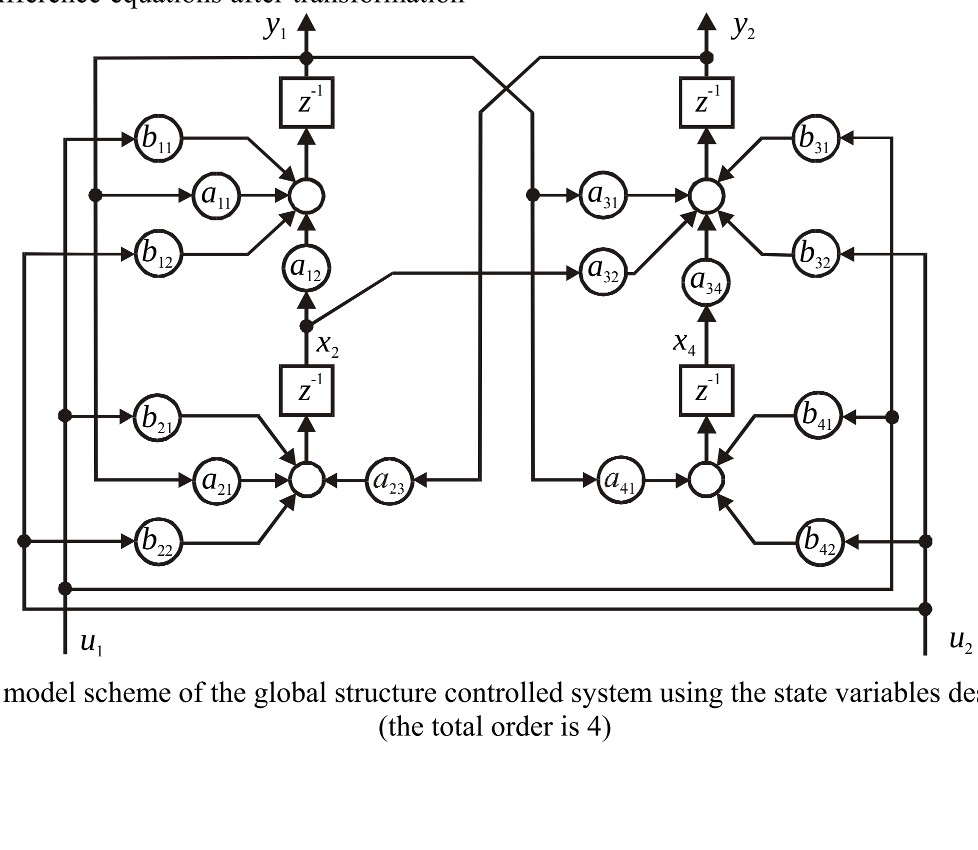

Fig. 5 The model scheme of the global structure controlled system using the state variables description (11) (the total order is 4)

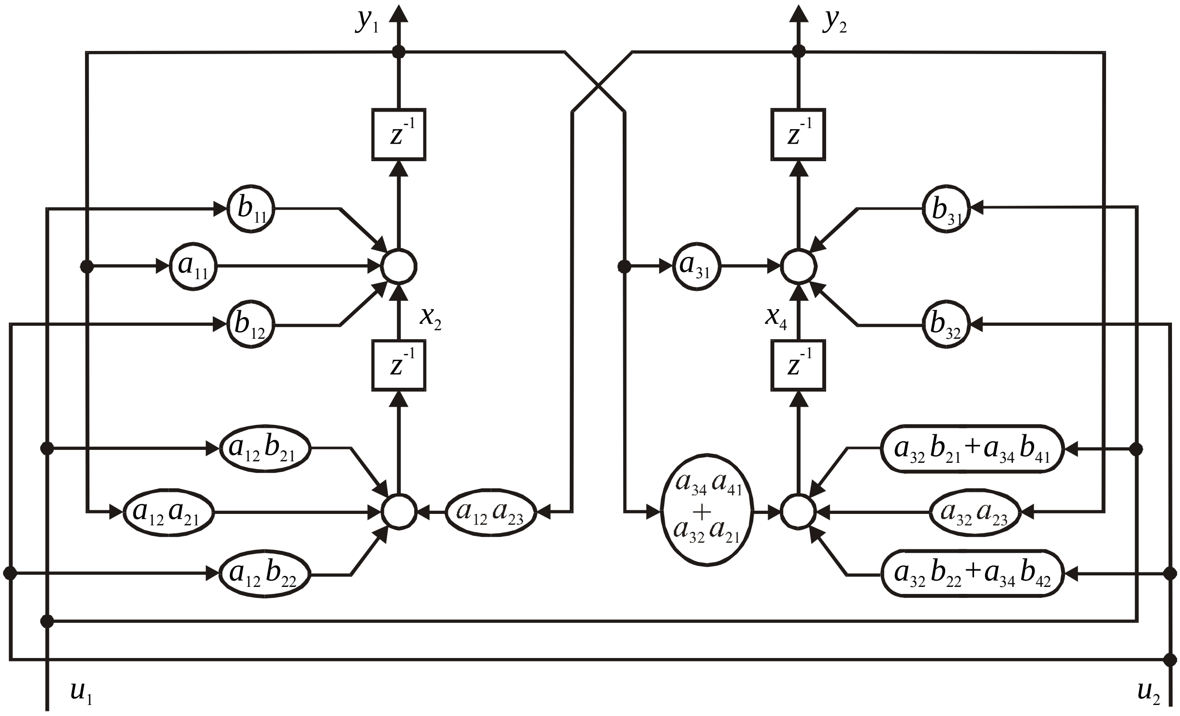

Fig.6. The model scheme of the global structure controlled system using the mixed difference equations description (12) (the total order is 4)

The corresponding scheme of model (12) is in Fig.6.

3 Problem Solution

The designed procedures were tested in the laboratories TU Liberec by means of the computer simulation. The control processes on the controlled system model with the measurable state variables and on the model with the estimator, designed according to the difference system descriptions, are compared.

The continuous model with the transfer functions

$$y_1 = \frac{1}{0.5s+1}u_1 - \frac{0.5}{1.5s+1}u_2 \ , \qquad y_2 = -\frac{0.5}{s+1}u_1 + \frac{1}{2s+1}u_2 \tag{14}$$

was chosen for the block structure controlled system. The corresponding discrete time state description (with the sampling interval 0.2 s) is

$$y_1 = \frac{z^{-1}}{1-z^{-1}a_{11}}b_{11}u_1 + \frac{z^{-1}}{1-z^{-1}a_{33}}b_{32}u_2 \ , \qquad y_2 = \frac{z^{-1}}{1-z^{-1}a_{22}}b_{21}u_1 + \frac{z^{-1}}{1-z^{-1}a_{44}}b_{42}u_2 \ . \tag{15}$$

The scheme in Fig.2 doesn't fulfil the nilpotent condition. After the arrangement into (5) and (6), the normal difference equations are obtained (6). They fulfil the nilpotent condition. The estimator is designed according to the model in Fig.4.

The discrete time model (10) (with the sampling interval 0.5s) was used for the global structure controlled system. The scheme of the model is in Fig.5. The procedure for normal and mixed difference equation in MATLAB software package was used (Fig.6).

The control processes reached by means of the control with the measurable state variables and with the estimator designed according to the state description and to the mixed difference equations are compared.

The control processes on the global structure controlled system are demonstrated in Fig.7, where the variables y, u belong to the controlled system with measurable state variables and the variables $\boldsymbol{y}_S$, $\boldsymbol{u}_S$ belong to the controlled system with the estimators designed according to the state variables (10) and to the mixed difference equations (12) descriptions (the total estimators orders are 4 and the processes $\boldsymbol{y}_S$ and $\boldsymbol{u}_S$ are very near in both cases).

If estimator E_{IN} - tuned for the disturbance $\boldsymbol{d}_{IN}$ - is used, all processes $\boldsymbol{y}$, $\boldsymbol{y}_S$ and $\boldsymbol{u}$, $\boldsymbol{u}_S$ are equal (on condition that the corrective feedback in stable).

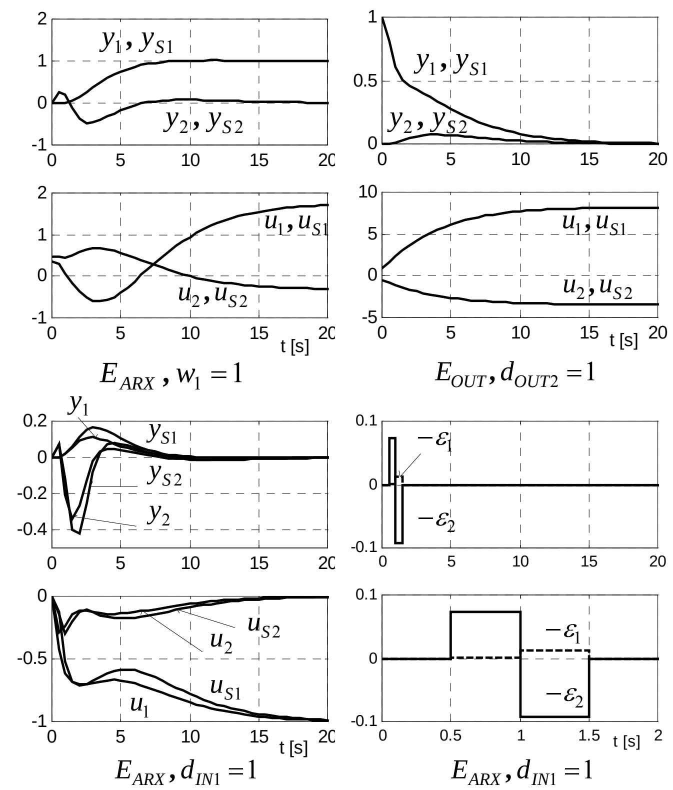

Fig.7. The control processes on the global structure controlled system (the total estimators orders are 4).

4 Conclusion

The multivariable estimator uses all suitable measurable variables on the controlled system. The real inner structure of the controlled system and that of the estimator need not be equal. The estimator is designed according to the description of the controlled system. The description by means of the state variables, by means of the transfer functions (corresponding with the normal difference equations system) and by means of the mixed difference equations system is used. The estimators designed by means of the state variables description are always of the same order as the controlled system. The estimators designed by means of the normal difference equations are of the equal order only if the controlled system has a block structure, otherwise they may be of higher order too.

The control processes with the estimators are equal to the processes on the controlled system with the measurable state variables only if the estimators are tuned for the input disturbance variable, which really has entered the controlled system and when the controllers are tuned for equal performance criterion. Otherwise the processes become different.

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