A Mathematical Scheme of Multi-User Receiver in W-CDMA Mobile Communication based on the Conjugate Gradient Method

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Abstract— This paper describes a new scheme of multi-user receiver to improve the capacity of Wideband CDMA system. This scheme is obtained by applying Conjugate Gradient Method (CGM) based on mathematical approach. CGM gives an iterative solution of a large-scale matrix under multiple access interference environments. The relation between transmitted and received signal is represented by sparse matrix; which has diagonal components almost equal to one and other components approaching to 0. This matrix equation is expected to be solved effectively by using CGM.

It is shown that the configuration is clearly simplified with equivalently increased capacity compared to the sequential configuration under single path and multi-path environment.

Key-Words: - multiple access interference (MAI), conjugate gradient method (CGM), system capacity, system simplification, multi-path environment, mobile communication

1 Introduction

The authors are continuing studies of multi-user detection schemes after the proponent of Wideband CDMA for the PCS in the North America. Serial and parallel configurations are included in this study with sufficient characteristics under the actual environments[1][2][3][4]. System and circuit configurations are simplified but they are not built as hardware because of complex and large configuration of circuits.

This paper proposes a novel scheme of multi-user receiver for W-CDMA based on mathematical approach, the Conjugate Gradient Method (CGM)[5].

The algorithm and the characteristics of the proposed scheme are shown in this paper with the comparison of the conventional single user detector. The proposed scheme is found to provide practically reduced complexity for a hardware development in near future.

2 Model of Multi-user Receiver

To simplify matters, single path is assumed as propagation model (no multipath) and the data modulation is binary phase-shift keying (BPSK).

Assuming there are K users in a single-path BPSK channel, the received baseband signal can be expressed as:

$$r(t) = \sum_{k=1}^{K} d_k(t) PN_k(t) d_k(t) + n(t) \tag{1}$$

where, $A_k(t)$, $PN_k(t)$, $d_k(t)$ are the amplitude, PN code waveform, and data of the k-th user, respectively. $n(t)$ is additive white Gaussian noise (AWGN).

The conventional receiver described in Eq.(1) is designed assuming single user. The receiver is a group of K correlators where the replica of PN code waveform is generated and then correlated with the received signal in a separate branch.

Using conventional receiver, the output of k-th user's correlator is

$$y_k = \frac{1}{T_b} \int^{T_b} r(t) PN_k(t) dt \tag{2}$$

$$=\frac{1}{T_b}\int^{T_b}\left(\sum_{i=1}^{K}\gamma_i(t)PN_i(t)d_i(t)+n(t)\right)PN_k(t)dt \quad (3)$$

$$=A_k d_k+\sum_{\substack{i=1\\ i\neq k}}^{K}\rho_{i,k}A_i d_i+\frac{1}{T_b}\int^{T_b}n(t)PN_k(t)dt \quad (4)$$

where, $\rho_{i,k}$ is the correlation value, defined as:

$$\rho_{i,k}=\frac{1}{T_b}\int^{T_b}PN_i(t)PN_k(t)dt=\begin{cases}1 & i=k\\ \rho_{i,k} & i\neq k\end{cases} \quad (5)$$

The properties of the correlations between PN codes determine the performance of user receiver. Autocorrelations (correlation between the same PN code waveform) need to be much larger than cross-correlations (correlation between different PN code waveform).

From Eq.(4), the following equation is obtained.

$$y_k=A_k d_k+MAI_k+z_k \quad (6)$$

While correlation with a certain user (desired signal) yields the recovered data, correlation with other users yields multiple access interference MAI_k and correlation with AWGN yields the noise z_k.

If Eq.(6) is rewritten in a matrix-vector model, output of conventional detector will be:

$$\begin{pmatrix}y_1\\ \vdots\\ y_i\\ \vdots\\ y_K\end{pmatrix}=\begin{pmatrix}1 & & & & \rho_{1,K}\\ & \ddots & & & \\ & & 1 & & \\ & & & \ddots & \\ \rho_{K,1} & & & & 1\end{pmatrix}\begin{pmatrix}A_1 & & & & 0\\ & \ddots & & & \\ & & A_{i1} & & \\ & & & \ddots & \\ 0 & & & & A_K\end{pmatrix}\begin{pmatrix}d_1\\ \vdots\\ d_i\\ \vdots\\ d_K\end{pmatrix}+\begin{pmatrix}z_1\\ \vdots\\ z_i\\ \vdots\\ z_K\end{pmatrix} \quad (7)$$

or

$$\boldsymbol{y}=\boldsymbol{RAd}+\boldsymbol{z} \quad (8)$$

Matrix R is a symmetric K x K correlation matrix for K users. Vectors $\boldsymbol{y}$, $\boldsymbol{d}$, and $\boldsymbol{z}$ contain the output of conventional receiver (matched filter), data, noise of K users. Matrix $\boldsymbol{A}$ is a diagonal matrix of received amplitudes.

3 Multi-user Receiver using CGM

As stated in section **2**, the properties of the correlations between PN codes determine the performance of WCDMA system. Mutual interference means that the correlation between different PN codes is not equal to zero or yields a value approaching to zero. MAI component needs to be eliminated or reduced in order to improve the performance of user receiver. While single user receiver neglects the existence of other users, multi-user receiver needs to solve the MAI problem by canceling interference from other users.

3.1 Conjugate Gradient Method (CGM)

As current multi-user receivers are complicated to be applied both in software and hardware, we need to find other kind of receivers, which is much simpler. One way is to calculate the inverse of correlation matrix. But when the number of subscribers increases, it would be difficult to solve the large-scale equation by calculation of inverse matrix.

CGM has been proven as an effective method for solving symmetric positive-definite sparse matrix system. The method proceeds by generating vector sequences of iteration value, residuals corresponding to the iteration value, and search directions used in updating the iteration value and residuals. Residuals, the error of received (signal) value and data estimation (iteration) value, needs to be minimized. Although the length of these sequences can become large, only a small number of vectors need to be kept in the memory. In every iteration on the method, two inner products are performed in order to update scalars that are defined to make the sequences satisfy certain orthogonal conditions. On a symmetric positive-definite linear system these condition imply that the distance to the true solution is minimized in some norm.

The CGM is an iterative solution method for simultaneous linear equations which has a positive-definite matrix A as follows:

$$\boldsymbol{Ax}=\boldsymbol{b} \quad (9)$$

The approximate solution x_{k+1} at the (k+1)-th adaptation step is obtained from x_k at the k-th step. The iterations x_{k+1} is updated in each iteration by an adaptation coefficient α_k of the search direction vector p_k:

$$\boldsymbol{x}_{k+1}=\boldsymbol{x}_k+\alpha_k\boldsymbol{p}_k \quad (10)$$

Correspondingly, the residual vectors r_k are updated as

$$\boldsymbol{r}_{k+1}=\boldsymbol{r}_k-\alpha_k\boldsymbol{Ap}_k \quad (11)$$

where

$$\boldsymbol{b}_k=\boldsymbol{Ap}_k\,. \quad (12)$$

The choice of $\alpha_k=\dfrac{(r_k,p_k)}{(p_k,Ap_k)}$ (13)

Minimizes $r_{k+1}A^{-1}r_k$ over all possible choices for α in equation.

The search directions are updated using the residuals

$$p_{k+1}=r_{k+1}+\beta_k p_k \quad (14)$$

where, the choice of β_k : $\beta_k = \dfrac{(r_{k+1}, r_{k+1})}{(r_k, r_k)}$ (15)

In this method, the residuals $r_0, r_1, ...$ are mutually orthogonal and the direction vectors $p_0, p_1, ...$ are mutually conjugate, that is :

$(r_i, r_j) = 0 \qquad (p_i, Ap_j) = 0 \quad i \neq j$

3.2 Algorithm of CGM

Conjugate Gradient Method [3] implements these following steps :

1) Estimate (approximate) the initial value.
2) For k = 0, the residual between real value and estimated value is :
   $$r_0 = b - Ax_0 \qquad (16)$$
3) The search direction vector p_k for k = 0 :
   $$p_0 = r_0 \qquad (17)$$
4) For k -th step, the adaptation coefficient is :
   $$\alpha_k = \frac{(r_k, p_k)}{(p_k, Ap_k)}$$
5) For ($k+1$)-th step: $x_{k+1} = x_k + \alpha_k p_k$
6) $r_{k+1} = r_k - \alpha_k A p_k$
7) Calculation for β_k : $\beta_k = \dfrac{(r_{k+1}, r_{k+1})}{(r_k, r_k)}$
8) The search direction vector p_k for ($k+1$)-the step :
   $$p_{k+1} = r_{k+1} + \beta_k p_k$$
9) $k = k+1$, and back to step (4) until $r_{k+1} \ll \varepsilon$ (in the simulation, the threshold is set to 10^{-4}.)

The configuration of the proposed scheme is shown in Fig.1. According to Eq. (1), step (4) to (8) is repeated until small residual value of r is achieved. If this condition is achieved, the calculated value x approaches the real data valued.

The adaptive solution converges to certain point for solution with *n* time steps. The number of iteration is saturated at a certain point where the performance could not be increased clearly furthermore.

4 Evaluation

4.1 Evaluation Condition

The simulation will show the improvement of performance by CG Receiver compared to Conventional Receiver. And also, the enhancement of performance by the combination of CG Receiver and Bit Error Correction is shown.

The evaluation of this simulation is conducted in the following conditions:

a. Transmission system: baseband model
b. PN Code : Gold Code (Code length: 2^8-1 bit)
c. Spreading gain = 64
d. Power control
e. Single path (simple mode)

The number of simultaneously available users for a system is defined as the capacity.

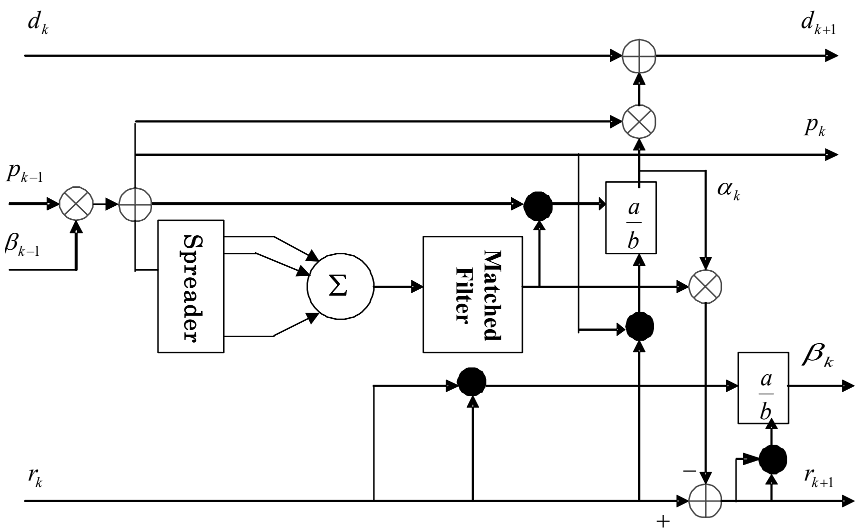

Fig.1 Configuration of the Conjugate Gradient Receiver (CG Receiver).

4.2 Characteristics of CG receiver

Characteristics of this CG receiver is shown in Fig.2. The vertical and horisontal axes are Bit Error Rate (BER) and sigal to noise ratio E_b/N_0. The capacity is assumed to be 50 users. The parameter is number of iteration steps toward convergence. The residual error of convergence corresponds to BER. E_b/N_0 was found to be 5 dB to BER 10^{-3} by 4 steps of iteration.

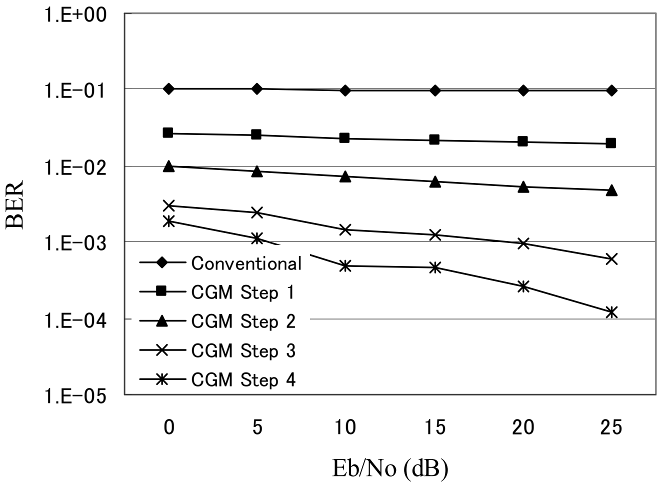

Fig. 2 Characteristics of CG receiver Capacity is assumed as 50 users.

4.3 Effect of Noise

The performance is shown in Fig.3 and Fig.4. Figure 3 corresponds to the characteristics without additive noise to channel. Figure 4 corresponds to the case with Gaussian noise. The noise level is 10 dB higher than the signal level.

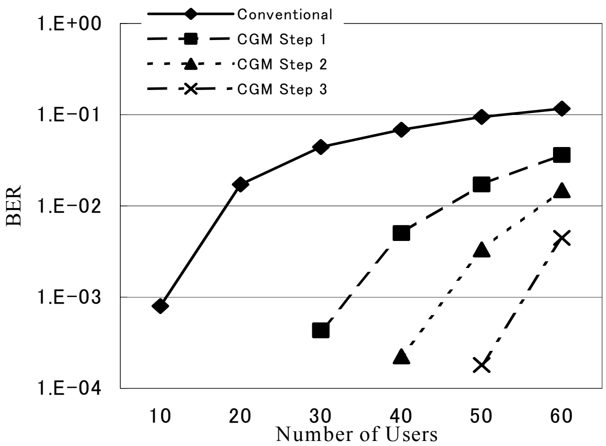

Fig. 3 BER characteristics shows the improvement made by CG Receiver.

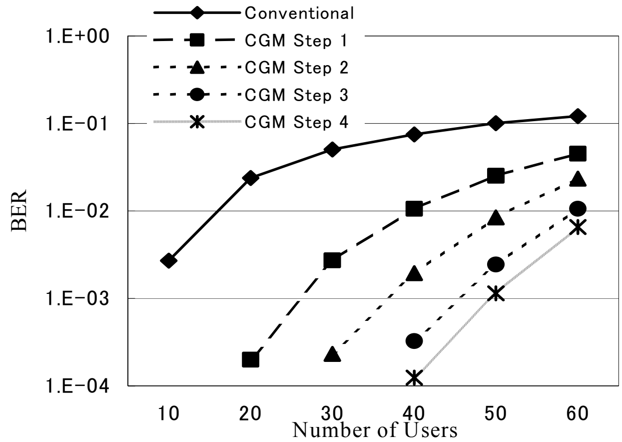

Fig.4 BER characteristics in noise environment.

Comparing Fig.3 and 4 with 3 iteration steps, the capacities are 55 and 45 users respectively at the points BER = 10^{-3}. This system is found robust against Gaussian noise. By increasing number of iteration steps, the capacity can be improved till 50 users.

4.4 Effect of Forward Error Correction

Forward Error Correction (FEC) coding technology is applied to get greater capacity of channels. FEC applies 1-bit error correction (Hamming Code) and receiver.

By applying FEC, user capacity could again be increased to approximately 64 users for CGM Step 4 (4^{th} iteration). This number is much greater compared to the conventional receiver (17 users).

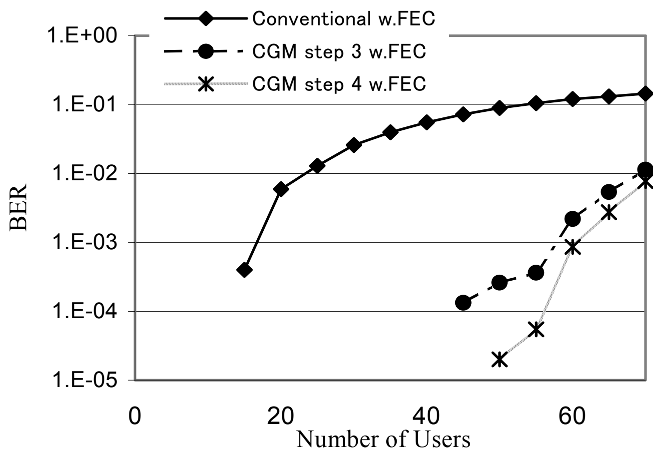

Fig.4. BER characteristics in noise environment with FEC.

5 Conclusion

New configuration has been given to compose a multi-user receiver applied to CDMA.

This configuration is obtained based on the CGM in mathematics to solve a sparse matrix with large number of dimension. The increase of capacity of CDMA receiver is now estimated about three times using simplified circuit configuration.

The configuration is sufficiently simplified with equivalent capacity enhancement compared to conventional configuration

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