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A problem of interpolating irregular robot trajectories

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Abstract: - In this paper we estimate the joints trajectories of a robot, when a fixed number of points, for the end-effector trajectory, are assigned. In particular we compare two methods. The first method utilizes the idea of approximating the graphics of a continuous function by polygonal lines, employing a system of functions, obtained by integrating the Haar wavelets. The second method is based on cubic spline interpolation.

Key-Words: - Wavelet analysis, numerical analysis, data set interpolation, robot trajectories.

1 Introduction

In the recent statistical literature, the implementation of interpolation techniques is an important task. In particular, Wavelet theory is almost indispensable when the function to interpolate is not-smooth (e.g., see [1] or [2]). On the other side, other techniques are used when the data to interpolate are quite regular, such as splines [3]. However, the problem of finding an optimal technique, when the function is only continuous, is again discussed. In fact, in these cases, the problem of approximating a function, by step functions, appears clearly to be inappropriate, such as describing it by polynomials. A possible answer to the question is given by Schauder (see [4] or [5]). He proposed to approximate the graphics of functions, by lines, obtained by integrating the curves of the Haar Wavelet system.

In our work we compare splines and Schauder methods, in order to interpolate empiric trajectories of the joints, in a robot; we consider the case in which some positions, concerning the end-effector trajectory, are fixed. This is a typical problem of the recent robotic literature (e.g., see [6]).

2 The model

Let us consider the end-effector motion, in a robot with n joints (and, therefore, $n-1$ links). Suppose that the end-effector position $P(t_i)$ ($i \in \{1, 2, \dots, N\}$) at time t_i must satisfy must interpolate the points $P_i + \xi_i$ ($i \in \{1, 2, \dots, N\}$), where ξ_i are iid normal random variables whose mean is zero and whose variance is relatively small. In this case, for any $j \in \{1, 2, \dots, n\}$ we calculate the position of the j-th joint. In this way let q_{ji} be the position of the j-th joint when the end-effector is at $P_i + \xi_i$. Therefore we calculate the minimum duration in order to transit from $P_i + \xi_i$ to $P_{i+1} + \xi_{i+1}$. In particular, for any j, we must unify q_{j1}, q_{j2}, …, q_{jN}, with a interpolating method. In particular, we must choose the method in order to preserve this value of the duration. Furthermore, we suppose that the motion law of the end-effector has at least a discontinuity in its derivative. This means that also the joints motion will not be smooth. In fact, for example, let us consider the case $n = 2$, in which the end effector law is two-dimensional $P(t) = (x(t), y(t))$ and it is described by the equations

$$x = l_1 \cos\theta_1 + l_2 \sin(\theta_1 + \theta_2)$$

$$y = l_1 \sin\theta_1 + l_2 \sin(\theta_1 + \theta_2),$$

where l_i is the length of the i-th link, θ_1 is the angle comprised between the x axis and the first link and θ_2 is the angle comprised between the two links. In this case we obtain

$$\theta_2 = \arccos\left(\frac{x^2 + y^2 - l_1^2 - l_2^2}{2l_1l_2}\right)$$

and

$$\theta_1 = \arctan\left(\frac{y}{x}\right) - \arctan\left(\frac{l_2 \sin\theta_2}{l_1 + l_2\cos\theta_2}\right).$$

These relations tell us that if $x(t)$ or $y(t)$ are not smooth at a certainly time, so it is also the motion law of the joints.

3 Mathematical background

3.1 Wavelet interpolation

Consider the two function defined on [0, 1]:

$$\varphi(x) = \begin{cases} 1, & x \in (0,1] \\ 0, & x \notin (0,1] \end{cases} \quad \psi(x) = \begin{cases} 1, & x \in \left[0, \frac{1}{2}\right] \\ -1, & x \in \left(\frac{1}{2}, 1\right] \end{cases}$$

ψ (mother wavelet) has its first moment equal to zero; φ is (father wavelet) orthonormal to ψ, according to the L^2 norm. Note that the set of all the functions, obtained by translating and dilating them, is complete and orthonormal: it is called Haar wavelet system. This system is utilized in order to approximate not regular functions square-integrable. A derivate method, valid for functions more regular, proposed by Schauder, consists substantially in replacing Haar system by their primitives (see [4] or [5]). More specifically, let

$$\Delta(t) = \begin{cases} 2t, & t \in \left[0, \frac{1}{2}\right) \\ 2(1-t), & t \in \left[\frac{1}{2}, 1\right] \end{cases}$$

be the function obtained by integrating ψ between 0 and t. Furthermore for any integer k let,

$$\Delta_n(x) = \Delta(2^j x - k),$$

for $j > 0$, $n = k + 2^j$ and $0 < k < 2^j$.

Note that, if $j = 1$, then $k = 1$, if $j = 2$, then $k \in \{2, 3\}, \ldots$ and, if $j = j_1$, then $k \in \{2^{j_1 - 1}, \ldots, 2^{j_1}\}$. Furthermore, let

$$\Delta_0(t) = \begin{cases} t, & t \in (0,1] \\ 0, & t \notin (0,1] \end{cases}$$

be the function obtained by integrating φ between 0 and t. Now, consider a function f defined on [0, 1]. By Parseval theorem, we have that

$$f(x) = f(0) + \sum_{n=0}^{\infty} c_n \Delta_n(x), \tag{1}$$

where $c_0 = f(1) - f(0)$ and

$$c_n = f\left(\frac{k + 1/2}{2^j}\right) - \frac{1}{2}\left[f\left(\frac{k}{2^j}\right) + f\left(\frac{k+1}{2^j}\right)\right].$$

For this reason, the system $\{1, \Delta_0, \Delta_1, \ldots, \Delta_n\}$ is a Schauder basis ([7] or [8]) for the Banach space C[0, 1] of the continuous function on [0, 1].

3.2 Spline interpolation

A spline is a constructed of piecewise polynomials which pass through a set of given points. In particular, the cubic spline, for a set of $n + 1$ points $y_0, y_1, y_2, \ldots, y_n$, is composed by pieces of the form

$$Y_i(t) = a_i + b_i t + c_i t^2 + d_i t^3, \tag{2}$$

where $t \in [0, 1]$ and $i \in \{0, 1, \ldots, n - 1\}$. We require that, for any interval $[y_i, y_{i+1}]$,

$$Y_i(0) = y_i = a_i, \quad Y_i(1) = y_{i+1} = a_i + b_i + c_i + d_i, \tag{3}$$

$$Y'_i(0) = b_i, \quad Y'_i(1) = b_i + 2c_i + 3d_i. \tag{4}$$

Furthermore, for any $i \in \{1, \ldots, n - 1\}$, we must have

$$Y_{i-1}(1) = Y_i(0), \quad Y'_{i-1}(1) = Y'_i(0) \tag{5}$$

and

$$Y''_{i-1}(1) = Y''_i(0). \tag{6}$$

Finally, we require the boundary conditions

$$Y''_0(0) = Y''_n(0) = 0. \tag{7}$$

From conditions (3)-(4), we obtain

$$a_i = y_i,$$

$$c_i = 3(y_{i+1} - y_i) - 2b_i - 2b_{i+1}$$

and

$$d_i = 2(y_i - y_{i+1}) + b_i + b_{i+1}$$

Finally, from conditions (5)-(7) we obtain that the vector $(b_0, b_1, \ldots, b_n)$ satisfies the following tridiagonal system

$$\begin{pmatrix} 2 & 1 & & & & \\ 1 & 4 & 1 & & & \\ & 1 & 4 & 1 & & \\ \ldots & \ldots & \ldots & \ldots & \ldots & \ldots \\ & & & 1 & 4 & 1 \\ & & & & 1 & 2 \end{pmatrix} \begin{pmatrix} b_0 \\ b_1 \\ b_2 \\ \ldots \\ b_{n-1} \\ b_n \end{pmatrix} = 3 \begin{pmatrix} y_1 - y_0 \\ y_2 - y_0 \\ y_3 - y_1 \\ \ldots \\ y_n - y_{n-2} \\ y_n - y_{n-1} \end{pmatrix}.$$

4 Application of the interpolation methods in studying trajectory regularity

Let us suppose that the position $f(t)$ of the i-th joint is a function of the time, between 0 and 1. In fig.1 is depicted the graphic of $(t, f(t))$, formed by $2^6 + 1$ points, derived from a not regular trajectory of the end-effector.

Suppose that only $h = 2^5 + 1$ equidistant points of the trajectory are known (Fig.2). In this case, by equation (1), we may know an approximation $f_{WAV}(t)$ of $f(t)$, on $2h + 1 = 2^6 + 1$ points, by utilizing the Shauder system. We must suppose that $2^{j_1} + 1 = h$. At the same time, we may obtain an approximation $f_{SPL}(t)$ of $f(t)$ by utilizing the cubic spline interpolation. Fig.3 shows the wavelet interpolation for f on $2h + 1$ points. As showed by Fig. 3, qualitatively the wavelet interpolation is better than cubic spline (Fig. 4).

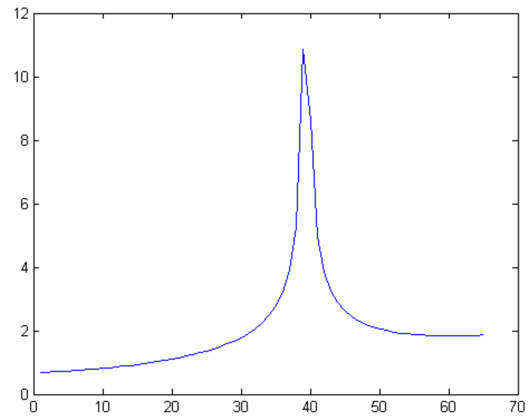


Fig.1 *Original trajectory* ($h = 2^6 + 1$).

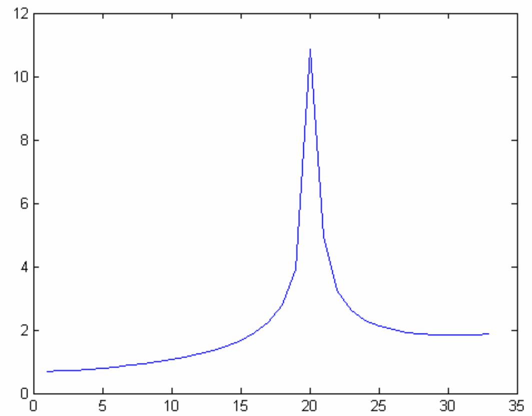


Fig.2. *Known trajectory* ($h = 2^5 + 1$).

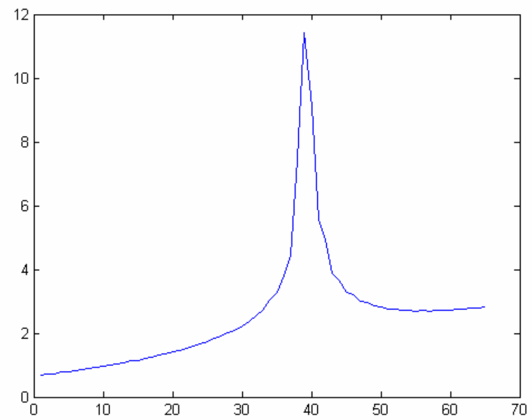


Fig.3. *Reconstruction of the trajectory by Wavelet interpolation* ($h = 2^6 + 1$).

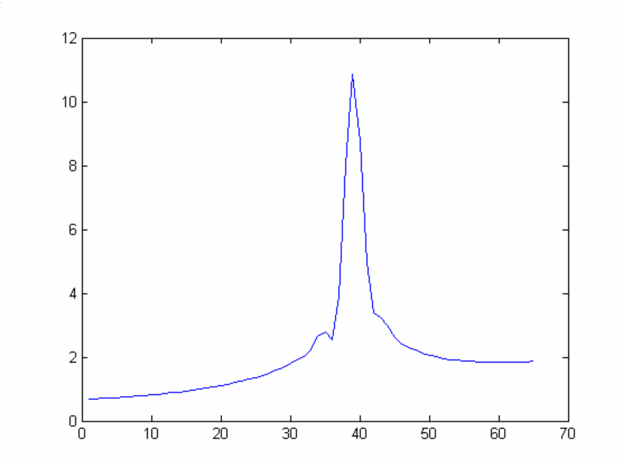


Fig.4. *Reconstruction of the trajectory by cubic spline interpolation* ($h = 2^6 + 1$).

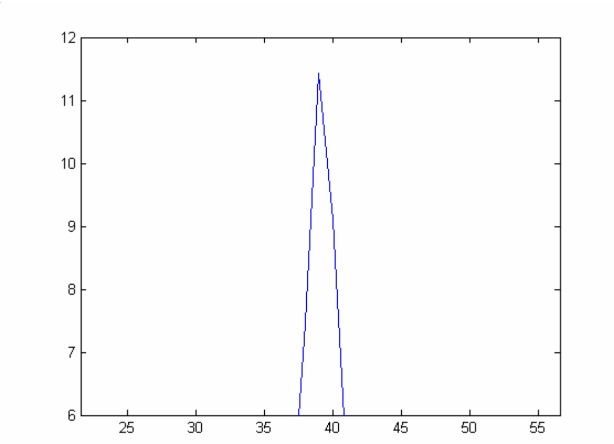


Fig.5. *Detail of the test function by Wavelet interpolation* ($h = 2^6 + 1$): *the reference parameters are* $a = 0.5$ *and* $b = 0.6$.

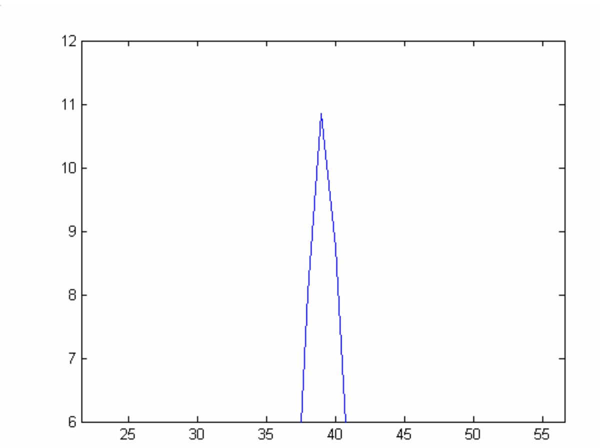


Fig.6. *Detail of the test function by cubic spline* ($h = 2^6 + 1$): *the reference parameters are* $a = 0.5$ *and* $b = 0.6$.

5 Efficiency of the wavelet interpolation

In order to test the performance of the Schauder interpolation, we consider the test function of the form

$$f(t) = t^2 + a|\,t - 0.6|^{-b},\ \ t \in [0, 1]. \quad (8)$$

The Figg. 5-6 shows that the maximum of the function, estimated by means of wavelet method, is higher than the other one estimated by spline. According to this first observation, the wavelet interpolation appears more appropriate: in fact $f(t)$ by (8) diverges as t tends to 0.6.

Now, set l be the arclenght of the original signal, supposed composed by $2^6 + 1$ points; furthermore let l_{WAV} and l_{SPL} be the arclenght of the graphics ($2h$ points) calculated respectively by wavelet and by spline interpolation, starting from a known graphic of $h = 2^5 + 1$ points.

Now, let us estimate the quantities

$$e^0(a,b) := \frac{|\,l - l_{SPL}\,|}{l} - \frac{|\,l - l_{WAV}\,|}{l} \qquad (9)$$

$$e^p(a,b) := \frac{\sqrt[p]{E\left(|\,f - f_{SPL}\,|^p\right)}}{\sqrt[p]{E\left(|\,f\,|^p\right)}} - \frac{\sqrt[p]{E\left(|\,f - f_{WAV}\,|^p\right)}}{\sqrt[p]{E\left(|\,f\,|^p\right)}} =$$

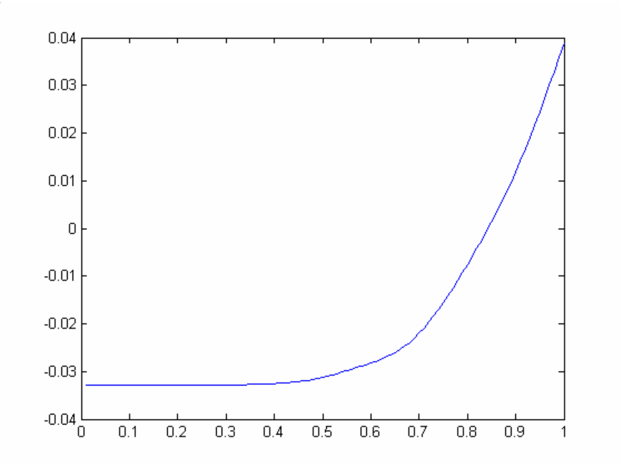
$$= \frac{\|\,f - f_{SPL}\,\|_p}{\|\,f\,\|_p} - \frac{\|\,f - f_{WAV}\,\|_p}{\|\,f\,\|_p}$$

(10)

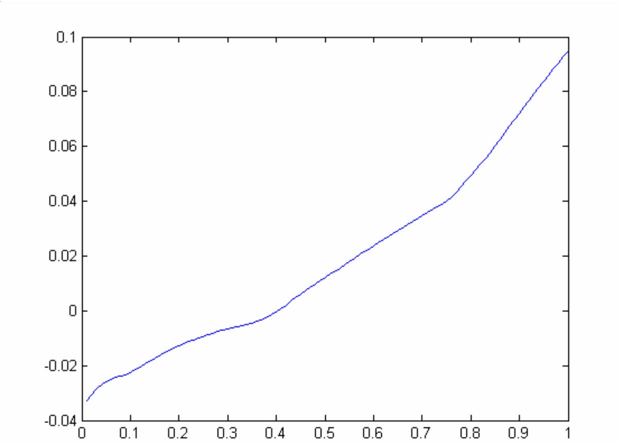
and

$$e^\infty(a,b) = \frac{\|\,f - f_{SPL}\,\|_\infty}{\|\,f\,\|_\infty} - \frac{\|\,f - f_{WAV}\,\|_\infty}{\|\,f\,\|_\infty}, \quad (11)$$

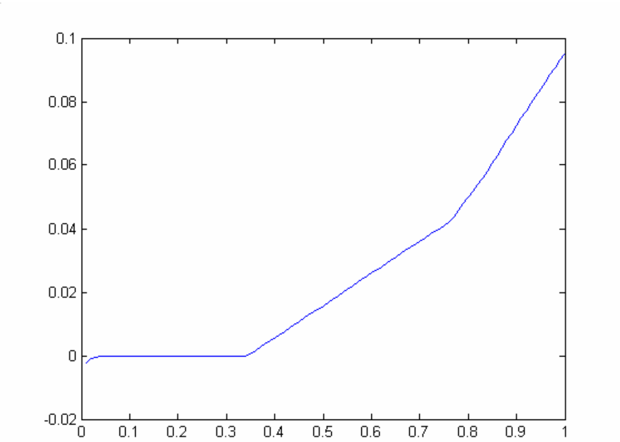
as a, b and p varying. Figg. 7-9 show that, for any a and p and for $b \cong 1$, e^p is positive. The meaning is summarized as follows: when the function is “not smooth” *enough* the wavelet interpolation must be preferred to the spline one.



a = 0.005

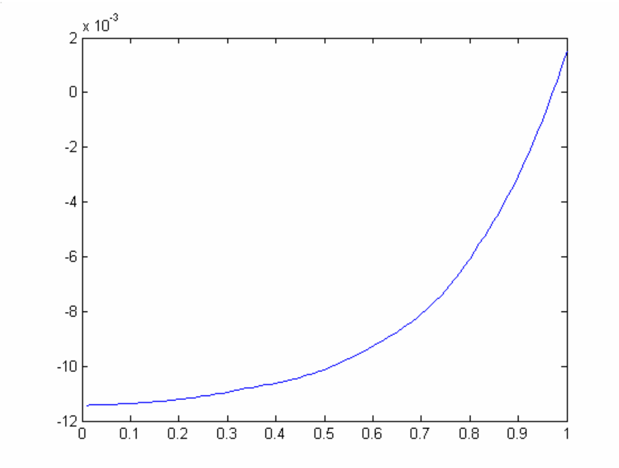


a = 1

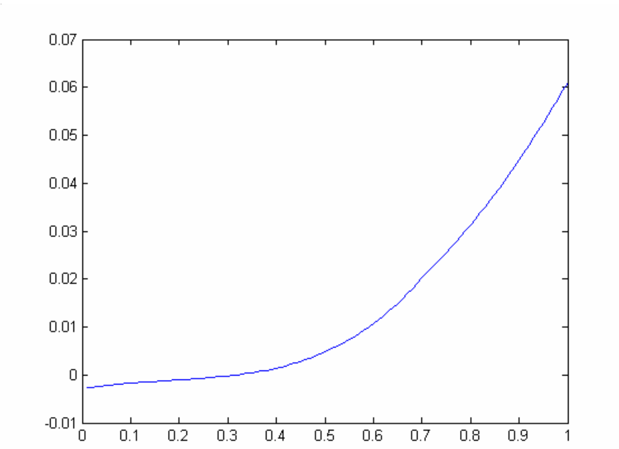


a = 300

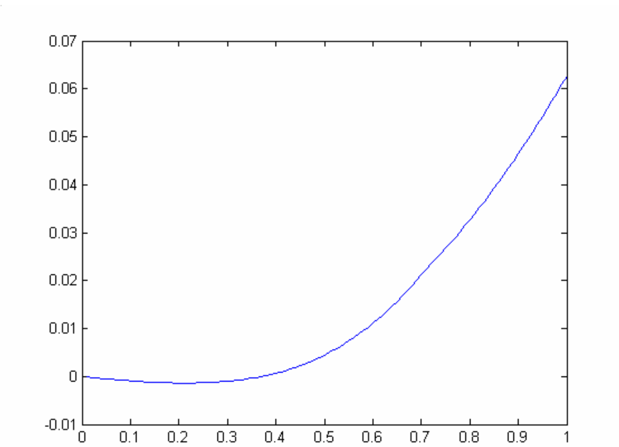
Fig.7. *Plot of e^0, as a function of b, for some value of a*.



a = 0.005

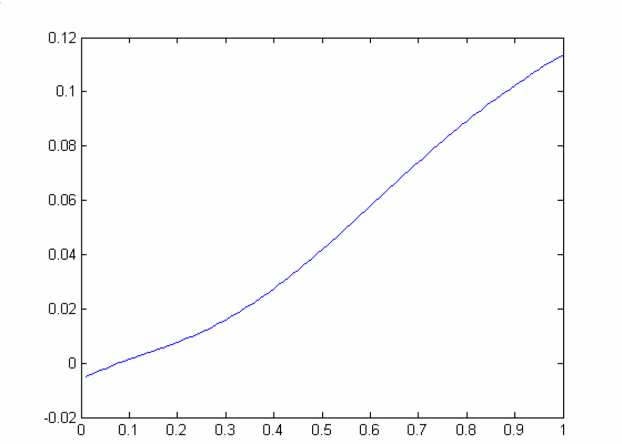


a = 1

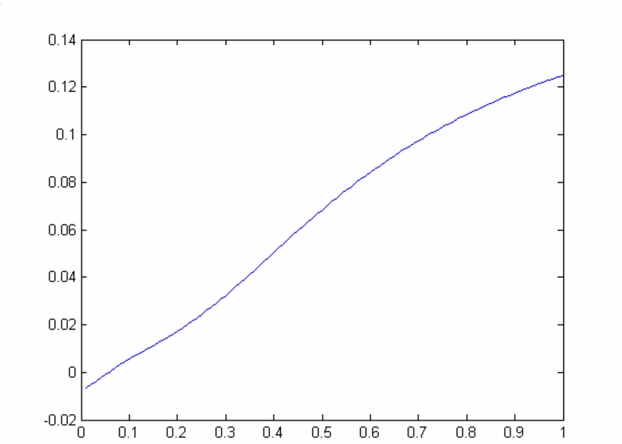


a = 300

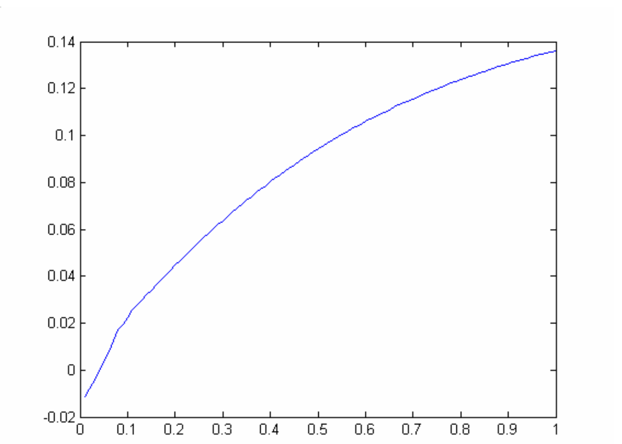
Fig.8. *Plot of e^1, as a function of b, for some value of a*.



$p = 2$



$p = 3$



$p = \infty$

Fig.9. *Plot of* e^p*, as a function of b, for some value of p and for a* = 1.

6 Conclusions

We proposed a methodology in order to interpolate a set of data points when their graphics are not smooth. It is based on a Schauder basis derived by integrating the Haar wavelet system. Subsequently, the desired interpolated function is obtained as a linear combination of the element of such basis: the combination coefficients are function of the data set points assigned. In particular, the proposed method seems to be efficient in the cases in which the data set to interpolate is not smooth. Our attention focuses out on the problem of estimating the end-effector and joints trajectories, in a robot, having supposed that only a limited set of the end-effector trajectory is note.

Starting from the results derived from this work, with reference to the model described on paragraph 2, it will be necessary to investigate further in order to make the estimated end-effector law $P(t)$ more consistent.

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