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Planar motion estimation algorithm for region based coding

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Abstract: - In this paper, we proposed a randomised algorithm to estimate the motion parameters of a planar shape without knowing a priori point-to-point correspondences. By randomly searching points on two shapes measured at different times, we determine the centroids, after which the algorithm proceeds to determine the rotation by searching points on each shape that form congruent polygons.

Key-Words: - **M**otion estimation algorithm, rigid object motion, object tracking, motion parameters estimation

1 Introduction

Coding of video signals to be transmitted over the low bit rate has been the subject of research over the last fifteen years, with application in videophone, videoconference, and multimedia and is clear that the solution is in techniques based on region.

Motion estimation [7] deals with computation of motion parameters for objects from image sequences. Motion information is utilised for: segmentation, pattern recognition and tracking, scene interpretation and in image sequence processing. Motion estimation algorithms differ with respect to the type of a priori information and constrains about the motion field they used and, of course, the computational framework within which they perform the estimation. The main types of motion estimation algorithm are: hierarchical model-based [6], optical flow [1],[2], modelling the motion field as a vector Markov random field [11] and a parametric model for describe the change in the image plane coordinates [13],[14].

The main idea in region-based segmentation techniques is to identify various regions in an image that have similar features. On the other hand, many applications in image sequences compression require motion detection. In the following we use the features that are invariant under Euclidean motion to detect the motion parameters of planar shape without knowing a priori the point correspondences.

The present algorithm randomly searches points on each shape to estimate the centroid of each shape. After the centroids are estimated, the rotation is computed using random selection of points in each image frame subject to the constraint that the angle between vectors constructed by three non-collinear points in the shape are invariant. After possible point correspondences are identified, the algorithm proceeds to solve the rigid motion equation and voting for each angle in the parameter space. This process continues until a limit of iteration cycles or a peak is detected in parameter space.

The proposed algorithm uses random samples to solve the planar motion equation. The method computes the planar motion parameters by checking the congruence of pairs of circles generated in each frame [5]. This method is different comparing with Kalviainen [12], which is based on the idea of the randomised Hough transform. This method detects translations of centroids and rotations of mechanical axes of planar objects by random samples.

The aim of this paper is to develop a non-model-based algorithm to estimate the motion parameters in two-dimensional Euclidean space, which deals with the problem of point correspondences, and takes advantage of the linear constraint and the higher speed of the computation provided by the randomised sampling approach. The algorithm uses image boundary points that provided sufficient conditions to estimate the motion parameters.

2 Outline of the region based motion estimation algorithm

The algorithm for image sequence coding is based on the static adaptive segmentation [4], which takes into account the following proprieties of the human visual system:

- the edge and contour information plays an important role in the visual perception,

• the texture information has relative importance. Texture is associated with additional information. It influences our perception when taken together with contour information.

The diagram of the proposed algorithm is shown in fig. 1.

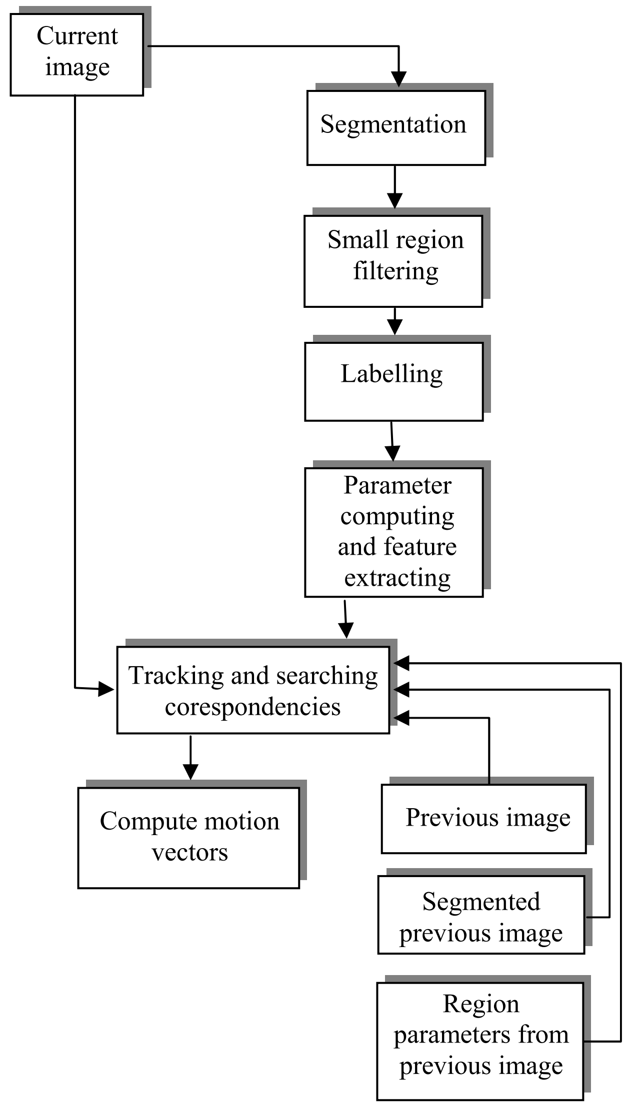


Fig. 1. Motion estimation algorithm

The main problem is to describe an object from sequences such as the additional information which must be transmitted be smaller. The proposed technique use a global search based on shape and position parameters.

The proposed algorithm consist of five steps:
1st step – image segmentation
2nd step – region growing and extract the features
3rd step – find correspondences between regions in current and previous frame
4th step – estimate translation, angle of rotation and scaling parameters
5th step – computes motions vector

3 Rigid object motion

The term rigid body [10], means an assembly of particles with fixed inter-particle distances. Thus, in kinematics of solid objects, the motion of an object is rigid if and only if the distance between any two points of the body is invariant with time. Rigid motions can be described as the sum of rotations and translations about an axis that is fixed in directions for short periods of time [8]. Let $\mathbf{R}^2$ be the two dimensional Euclidean plane, and by x and y the orthogonal coordinates on $\mathbf{R}^2$, so a vector on $\mathbf{R}^2$ can be expressed as:

$$\bar{x} = (x, y)^T \tag{1}$$

The rigid motion for a point on a planar shape *CR*, which is a finite closed set on $\mathbf{R}^2$ is defined by:

$$\bar{x}' = R\bar{x} + \bar{d} \tag{2}$$

where $\bar{d} \in \mathbf{R}^2$ is translation vector , and **R** is rotation matrix such that:

$$R^T R = I, \quad |R| = 1 \tag{3}$$

and the two-dimensional rotation matrix is defined as:

$$\mathbf{R} = \begin{pmatrix} \cos\theta & \sin\theta \\ -\sin\theta & \cos\theta \end{pmatrix} \tag{4}$$

In these conditions we define CR', which is result of applying the rigid transformation to *CR,* as:

$$CR' = \{\bar{x}' \;|\bar{x}' = R\bar{x} + \bar{d}, x \in CR\} \tag{5}$$

We compute the motion parameters from two shapes *CR* and *CR'* measure at times t_1 and t_2, respectively. Knowing three pairs of points which correspond, the equations (2) can be solved.

In the next, we show how we compute the rotation of an object, based on the randomly points in order to define a vector for an object in a sequences of images.

From properties of rigid motion on a plane examination, the following propositions will be stated:

Proposition 1. Setting g and g' to be centroids of CR and CR', respectively, the relations:

$$g' = g + a \tag{6}$$

holds. Where $g = (\bar{x}, \bar{y})^T$ and $g' = (\bar{x}', \bar{y}')^T$.

Proposition 2. For any triplet of vectors ***x, y, z***, $\|x - y\|$ (Euclidean norm) and $(x - z)^T (y - z)$ are invariant under Euclidean motion (translation and rotation).

THEOREM: Let *CR* be a planar shape; this means that *CR* is a finite closed set on $\mathbf{R}^2$. Denoting by ∂CR the boundary of *CR*, for a point g $\in$ *CR*, we define the set

$$K(r) = \partial CR \cap \{x \mid |x - g| = r, r > 0\} \qquad (7)$$

Let $|K(r)|$ be the number of elements of K(r). Then $|K(r)|$ is invariant under Euclidean motion.

PROOF:

For $K(r) \subset CR$ and **x, y, z** $\in$CR and $x', y', z' \in CR'$, setting $g' = g + a$, we obtain:

$$K'(r) = \partial CR' \cap \{x' \mid |x' - g'| = r, r > 0\} \qquad (8)$$

This leads to the conclusion that $|K(r)| = |K'(r)|$.

The theorem implies that in continuous space, we can solve the point correspondences problem by using $|K(r)|$ and $|K'(r)|$, since in most cases $|K(r)|$ and $|K'(r)|$ are finite. Thus, we select a pair of sets of polygon points from the intersection of the boundaries of the planar shapes and circles of which centers are the centroids g and g', respectively, with radius r.

4 Algorithm for motion estimation

Knowledge of the boundary of the shape is sufficient to compute motion parameters, so that the algorithm can be applied.

4.1 Centroid estimation

A region *CR* can be decomposed into non-overlapping parts, that is,

$$CR = CR_1 \cap CR_2,$$
$$CR_1 \subset CR, CR_2 \subset CR,$$
$$\mathrm{int}\, CR_1 \cap \mathrm{int}\, CR_2 = \varnothing \qquad (9)$$

where $\mathrm{int}\, CR$, is the interior of the region. Denoting by g, g_1 and g_2 the centroids of CR, CR_1 and CR_2, respectively, we obtain the following equation:

$$g = \frac{|CR_2|}{|CR|} g_1 + \frac{|CR_1|}{|CR|} g_2 \qquad (10)$$

where $|CR|$ is the area measure of the CR (i.e. number of pixels). Thus, the centroid of a digital image can be written as [8]:

$$(\bar{x}, \bar{y})^T = \left(\frac{1}{n} \sum_{i=0}^{n} x_i, \frac{1}{n} \sum_{i=0}^{n} y_i \right)^T \qquad (11)$$

where $(x_i, y_i)^T$, is the centroid expressed in pixels. Alternatively equation (12), which represents recursive approximations of equation (11), can be used.

$$g_{n+1} = \frac{n}{n+1} g_n + \frac{1}{n+1} x \qquad (12)$$

for $x \in CR$, where $g_0 = 0$.

Using a randomised approach we compute the centroid using equation (12) that will converge after a limit of iteration cycles to the true value of the centroid of the shape. Furthermore, by proposition 1 it is clear that the translation of the shape is translation of the centroid. The points are randomised selected from boundary points, because the centroid of the boundary shapes gives a good approximation of the centroid of the planar shape, as shown in Section 3.

4.2 Rotation estimation

After centroid computation, the algorithm proceeds to estimate the rotation angle. The points are obtained from the intersection between the boundary and the circle of radius *r*, with the property $r_1 \le r \le r_2$, where r_1 and r_2 denotes minimum and maximum distance between centroids and boundary, respectively.

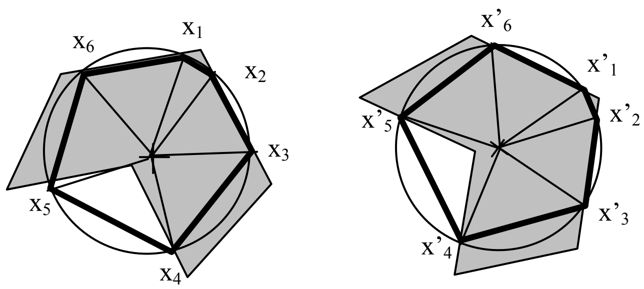


Fig. 2. Boundaries and polygons

Then we define the set of points $\bar{\mathbf{X}} = \{x_i\}_{i=1}^n$ and $\bar{\mathbf{X}}' = \{x'_i\}_{i=1}^n$ where $x_i \in K(r) \cap \partial CR$, $x'_i \in K'(r) \cap \partial CR'$, respectively. We determine a pair of congruent polygons of which vertices are on the circles centred at the shape centroids with radius r as is shown in figure 1. Using the property that ***X*** and ***X'*** are polygons with vertices lying on the circles, we develop an algorithm to estimate the rotation angle. Let a pair of sets be $\bar{\mathbf{X}} = \{\bar{x}_i\}_{i=1}^n$ and $\bar{\mathbf{X}}' = \{\bar{x}'_i\}_{i=1}^n$, where: $\bar{x}_i = x_i - g$ and $\bar{x}'_i = x'_i - g'$. Then $\bar{\mathbf{X}}$ and $\bar{\mathbf{X}}'$ are the polygon vertices on the circles with radius *r*. If x_i and x'_{i+k} correspond, we have the relation:

$$\bar{x}'_{i+k} = R\bar{x}'_i, \text{ for } i=1, 2, \ldots, n \qquad (13)$$

Thus, assuming equation (13), we develop an algorithm using the angles between vertices by setting:

$$\theta_i = \arccos\left(\frac{\bar{x}_i^T \bar{x}_{i+1}}{\|\bar{x}_i\|\|\bar{x}_{i+1}\|}\right) \text{ and,} \tag{14}$$

$$\Phi_i = \arccos\left(\frac{\bar{x}_i'^T \bar{x}_{i+1}'}{\|\bar{x}_i'\|\|\bar{x}_{i+1}'\|}\right) \tag{15}$$

We define a pair of lists of angles:

$$L(\bar{X}) = \langle \theta_1, \theta_2, ..., \theta_n \rangle \tag{16}$$

$$L(\bar{X}') = \langle \Phi_1, \Phi_2, ..., \Phi_n \rangle \tag{17}$$

If equation (13) holds for $1 \le j \le n$, then it follows that $\theta_k = \Phi_{j+k}$. Thus, an error criterion could be defined as:

$$E(j) = \sum_{k=1}^{n} \left|\theta_k - \Phi_{j+k}\right| \tag{18}$$

Then, a set of pairs $\{(\bar{x}_k, \bar{x}_{i+k}')\}_{k=1}^{n}$ which minimise E(j) is a candidate for correspondences. So, we introduce a second criterion to determine the correspondences between elements of $\bar{X}$ and $\bar{X}'$

$$E_1(j) = \sum_{k=1}^{n} (\alpha_k - \alpha_{k+1})^2 \tag{19}$$

If $E_1(j) \le \varepsilon$, for a small positive value, we adopt:

$$\alpha = \frac{1}{n}\sum_{k=1}^{n} \alpha_k \tag{20}$$

as an estimation of the rotation angle, where

$$\alpha_k = \arccos\left(\frac{\bar{x}_i^T \bar{x}_{i+k}'}{\|\bar{x}_i\|\|\bar{x}_{i+k}'\|}\right) \tag{21}$$

4.3 Scaling estimation

For region zooming, we define shape characteristics as [9]:

$$cf = \frac{perimeter^2}{4\pi \times area} \tag{22}$$

$$cf_1 = \frac{(m_{2,0} - m_{0,2})^2 + 4m^2_{1,1}}{(m_{2,0} + m_{0,2})^2} \tag{23}$$

For motion model we introduced a few restrictions: maximum translations of 16 pixels, maximum angle of rotation of 45°, and maximum zoom of 50%. These restrictions are necessarily in order to avoid false correspondences between regions. We use a global motion estimation. The searching algorithm [3], is illustrated in fig. 3.

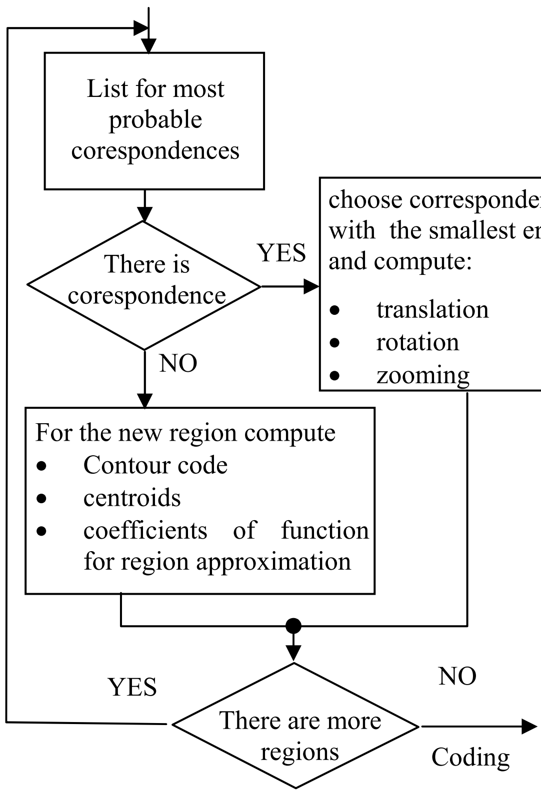
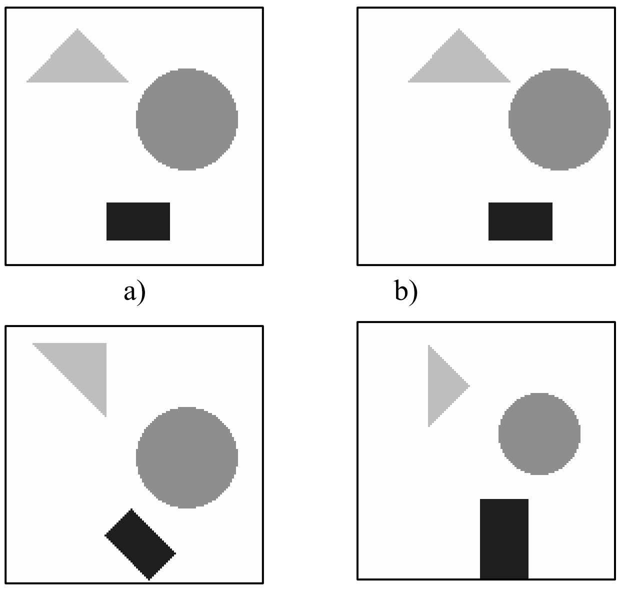


Fig. 3. Algorithm for region searching

5 Experimental results

The algorithm was tested using a binary image, with size 128 × 128, figure 4.

The centroid was detected with good accuracy, the error in the majority of cases is zero. In table 1 is presented the estimation using eq.(11), and eq.(12). The computed translation between objects from figure 4b and 4a is presented in table 1.



a) b) c) d)

Fig. 4. Test shape

The centroid was detected with good accuracy, the error in the majority of cases is zero. In table 1 is presented the estimation using eq.(11), and eq.(12). The computed translation between objects from figure 4b and 4a is presented in five-th row.

Table 1. Centroid and translation using centroids results

Objects / Parameters		triangle	circle	square
Centroid eq.(11)	x	50	100	80
	y	27	55	106
Centroid estimation eq. (12)	x	50	101	80
	y	27	55	106
Translation estimated	dx	15	10	15
	dy	0	0	0
Real translation	dx	15	10	15
	dy	0	0	0

To compute the rotation, we generate circles with radius r of which the center is g. The interval for searching radius affects the computation time.

Table 2. Rotation angle

Objects	Rotation Fig. 3c/ 3b	Estimation 1	Estimation 2
triangle	43°	43°	42.98°
square	40°	40°	40.85°

For complex transformation (translation, rotation, scaling) the results are presented in the third table 3.

Table 3. Moving estimation between figure 4.c and 4.d.

Objects / Moving vectors		triangle	circle	square
dx		4	0	1
dy		11	10	2
zoom		0,627	0,66	1,04
angle		- 45°	0	- 45,17°
Absolute errors	dx, dy	0	0	+1, - 1
	angle	0,01	0	4,83°
	zoom	0,127	0,001	0,06

The majority of the error is due to the image digitalisation, although round off error also contributes. The digitalisation affects location of the centroids, translation, boundary detection and zooming.

If the location of the centroids and translation are correct but there is inaccuracy in the boundary detection we obtain changes between corresponding points, leading to an error which depends by the circle radius used for polygon points generation.

The total error of the estimation of the rotation angle could be expressed as

$$E \approx 2\frac{\delta}{r}\frac{180}{\pi} \qquad (24)$$

where δ is the error, in pixels, and r circle radius, in pixels, used for polygon points generation. Settings $\delta = \pm 1$ and the average radius 50, we obtain $E \approx 5°$, which corresponds to the error of our experimental results, figure 3.

6 Conclusion

We showed that we can estimate the motion parameters using a randomised approach, without knowing a priori the point correspondences. The remaining problems are the accuracy, which depends on the image digitalisation and the performance for contour detection, and the performance of the algorithm.

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