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# On the causality integration in the design of axis drive control

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*Abstract:* - This paper addresses the problem of modelling and control synthesis for a class of electromechanical systems, which can be accurately decomposed into lumped constant parameters. The control design methodology presented in our former publications is based on a "natural" representation of the process and on two complementary inversion principles leading to a state control structure. This paper generalizes the concept of inversion and proposes a systematic method of control synthesis for the electromechanical processes, starting from a model based on the concept of localized energies and described by the Causal Ordering Graph (COG). We demonstrate that some control solutions, conventional or particular, can be expressed theoretically. The main interest is the systematic deductive aspect of the methodology, aiming at a faithful copy of the desired trajectories.

*Key-Words:* - Causal Ordering Graph (COG), control strategy, modelling formalism

## 1 Introduction

This article is related to the problems of the elaboration of control strategies for the new generation of positioning systems submitted to high dynamical demands. Among those systems, one finds the machine tools, the robots, the manoeuvre systems and, to a lesser extent, all computer controlled electromechanical processes. The main objective is obviously to reduce the production time, which necessarily implies a reduction of the system inertia and an increase of efforts (stresses) imposed on the overall mechanical structure. These evolutions lead the concerned machines to a dynamic behaviour, which cannot be neglected any more by the control law. For example, the mass saving makes the elastic character of the mechanical parts to become predominant, which implies a greater sensitivity to oscillatory modes and thus a deterioration of the accuracy [1].

These remarks show that the control laws have to be designed by taking into account the specific character of the axes. In particular, the flexible part is a sensitive element, which complicates the control design. Thus, adapted procedures on the effort modulation have to be defined for a correct positioning. The number of coupled problems is significant and, with such a multi-domain context, formalisms and methods are impossible to circumvent to establish a generic control for this kind of system.

If the components belong to a class of systems for which it is possible to locate representative objects of the internal energy transformations, one can determine a structural modelling based on knowledge of the physical phenomena. One of the major interests of this formalism, which we call the Causal Ordering Graph (COG), is the behaviour interpretation, since, by principle, all energy is transferred, accumulated or dissipated. The fundamental objective of the COG tool is to propose, by a graphical way and thanks to some simple principles, a control law synthesis taking into account the physical transfers of the process. Several articles have been published, demonstrating the advantage of the COG tool and its particular properties [2-5]. The design methodology of the control law presented in these articles is based on a "natural" representation of the process and on two complementary principles of inversion (direct and indirect), which leads to a state controller structure.

This paper generalizes the concept of inversion and proposes a systematic method of control synthesis for electro-mechanical processes, starting from a model based on the concept of localized energies.

The next sections are organized as follows. In Section II, we provide a brief overview of the Causal Ordering Graph, which will be used in the next developments. In Section III, we describe the control structure synthesis using the inverse model principle. The command generation aspect is discussed in Section IV. We conclude with some remarks in Section V.

## 2 The Causal Ordering Graph (COG)

The Causal Ordering Graph is made of several graphical processors, which are attached to different objects located in the studied process. The evolution of these objects is characterized by a transformation relation between influencing quantities and influenced quantities. This relation is induced by the principle of causality governing the energetic relation of an object or group of objects. In concrete terms, the output of a processor only depends on present and past values of the inputs. Such a formulation expresses causality in integral form and there are many significant electrical and mechanical examples. It is undeniable that this approach is close to that of links graphs such as the Bond Graphs [6-7], but it differs from them by the analysis process, which is based only on integral causality. The COG is a tool that structures the synthesis of a state model, but that aims at maintaining, for a given system, a representation that remains as close as possible to the physical reality. Obviously, the resulting model will undergo the consequences of the neglected elements and of the explicit and implicit assumptions induced by the physical interpretation of the constitutive objects.

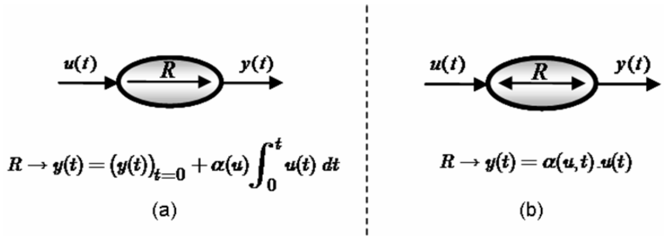


Fig.1 - COG symbolisms: (a) causal relation, (b) rigid relation.

In general, to express the transformation relations by means of state equations is the best warranty against physical misinterpretations. To simplify the presentation, we will only retain two complementary definitions of the integral causality: (a) If an object accumulates information, causality is internal: the output is necessarily a function of the energy state, the relation then oriented is known as causal. Time and the initial state are implicit inputs and are not represented. (b) If an object does not accumulate information, causality is external, i.e. determined by its environment. The output is an instantaneous function of the input, and the relation, which is not oriented, is then known as rigid. Fig. 1 gives the selected symbolism to differentiate between the two kinds of processors. Several electromechanical processes can be considered as an assembly of lumped and identified objects: sources, neutral or passive elements accumulating energy, dissipative elements, coupling elements.

# 3 Controller synthesis using the concept of the inverse model

## 3.1 Introduction

Which way should be followed to impose a given behaviour to a process?

To control a process means to impose the trajectory of one of the power components by using the other components for the tuning procedure. For example, the speed control of an axis is obtained by a modulation of the imposed force. This concept can be extended thus: "Since the effect induced by the cause is known, we only need to create the right cause in order to have the right effect". This aphorism expresses causal inversion.

In this section, we present a control synthesis methodology based on a model of the process expressed by the COG formalism. This study leads to a general solution whose degraded or simplified forms correspond to well-known control solutions (industrially used conventional cascaded control loops, state controller, model based control).

## 3.2 Inversion principle

The inversion of the relation associated with a processor determines a control relation, which is itself associated with another processor. This processor models the control which must then be materialized. Fig. 2 gives an illustration of the total procedure, which implicitly shows the importance of the modelling quality.

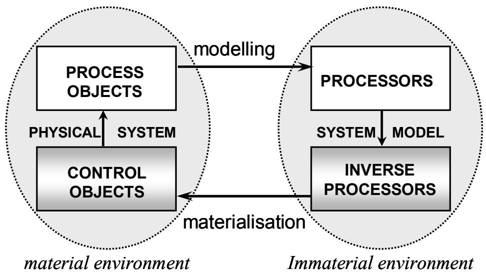


Fig.2 - Methodology of control definition.

In the general case, the assembly of several objects forming a mono-variable process (single input $u$) leads to a relation $f$ such as

$$R \to y(t) = f(u,t)\,, \tag{1}$$

where $f$ is represented by a linear $n$ order differential equation:

$$\sum_{i=0}^{n} a_i \frac{d^i y(t)}{dt^i} = u(t)\,. \tag{2}$$

Assuming that $v_{i+1}(t) \triangleq \dfrac{d^{i+1} y(t)}{dt^{i+1}}$ leads to the following equation:

$$\frac{d^i y(t)}{dt^i} = \left(\frac{d^i y(t)}{dt^i}\right)_{t=0} + \int_0^t v_{i+1}(t)\,dt\,. \tag{3}$$

Noting $y_{ref}$ the reference trajectory for $y$, one determines the input command $u_{reg}$:

$$R_c \to u_{reg}(t) = g(y_{ref}, t)$$
$$\text{if } g = f^{-1} \text{ and } u = u_{reg}, \text{ then } y(t) \to y_{ref}(t). \tag{4}$$

It appears that the function *f* must be invertible (open loop stable system), which means that the trajectory *y* must be known beforehand and *n* order piecewise continuously differentiable.

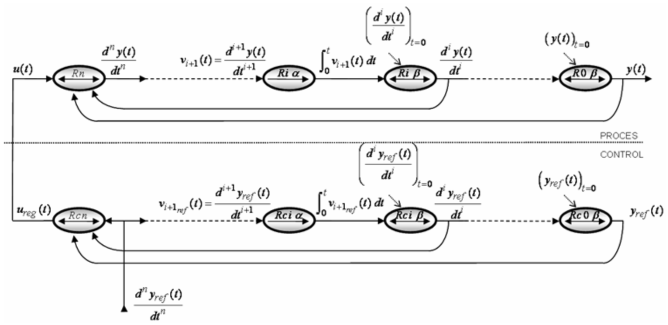


Fig. 3. Inversion principle: direct model based control.

As the process, the control model is represented by a linear *n* order differential equation:

$$u_{reg}(t) = \sum_{i=0}^{n} a_i \frac{d^i y_{ref}(t)}{dt^i} \tag{5}$$

Assuming that $v_{i+1_{ref}}(t) \triangleq \dfrac{d^{i+1} y_{ref}(t)}{dt^{i+1}}$ leads to:

$$\frac{d^i y_{ref}(t)}{dt^i} = \left(\frac{d^i y_{ref}(t)}{dt^i}\right)_{t=0} + \int_0^t v_{i+1_{ref}}(t)\, dt\,. \tag{6}$$

If $n \neq 0$, we obtain the direct model based control depicted in Fig. 3, which highlights several requirements:

1) The trajectory must be generated so that the *n*th time derivative exists.

2) The initial conditions must be known in advance.

We note that the process model results from a graphic interpretation of (2). That requires a study of the trajectory in order to observe the required continuity conditions as described in [8]. This formulation and these natural properties are found in the formalism of the flat systems, allowing the direct comparison of the system states with the state to be controlled and its successive time derivatives. The following questions are then raised concerning the variations between the model and its process, and thus by inversion between those of the prefilter and those of the real process, the disturbance rejection performances, the elaboration of the reference trajectory corresponding to the *n*th derivative (order of the selected model) of the state to control. Practically, this kind of control structure is almost never explicitly used.

If case $n = 0$, there is an instantaneous relation between the input and the output. In this particular case, the solution is easier to implement and so is currently met. It consists of a control law using direct inversion. Fig. 4 illustrates this principle, which results in determining the input command $u_{reg}$, starting from the reference trajectory $y_{ref}$. This principle is applicable to any rigid bijective relation (dissipators, neutral operators), but the limits can be found in the case of irreversible non-linearities such as saturation, which corresponds better to a topological modification of the system.

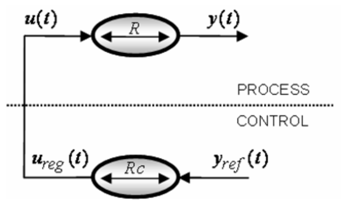


Fig. 4. Direct inversion principle ($n = 0 \longrightarrow R_c = 1/R$).

### 3.3 Indirect inversion principle

The indirect inversion principle, illustrated in Fig. 5, is the measure feedback principle, which aims at minimizing the tracking error $e = y_{ref} - y$ by indirect causal inversion. The command relation is expressed as

$$Rc \to u_{reg} = g\left(c(y_{ref} - y), t\right), \tag{7}$$

$c \in \mathbb{C}$ representing the regulation function, which is designed to ensure the control stability and to minimize the tracking error. We can note that if $|c| \to \infty$ and $u = u_{reg}$ then $y(t) \to y_{ref}(t)$.

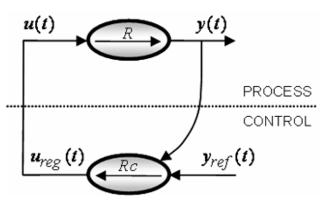


Fig. 5. Indirect inversion principle.

We can note that the process response tracks the reference independently of the a priori knowledge of the process. Moreover, if the loop gain tends towards infinity, this principle is theoretically independent from the nature of the relation. Thus, it is also applied to badly known nonlinear relations, since the command variable results from the difference between the controlled state and the imposed trajectory for this state. Actually, the accumulation of energy in the process leads to delays in the system response, most of the time implying oscillatory or unstable behaviours. Moreover, the increase in the loop gains widens the bandwidth as much as the measurement noise frequencies. The tuning limit is then reached and so are the maximum dynamic performances. For these reasons, the concept of infinite gain value is fallacious and it is necessary to elaborate more complex functions. The most traditional case is the Proportional and Integral regulator; the PI leads to an infinitely large amplification of the tracking error in stationary stages, but makes it possible to ensure stability thanks to the decrease of this amplification with the frequency.

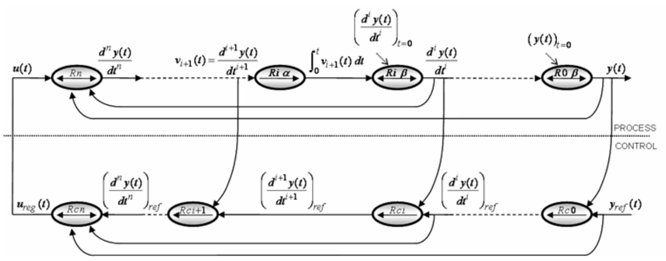

Fig. 6. Generalized indirect inversion principle: the cascaded loops control structure.

One can generalize the indirect inversion principle by measuring the whole system states (Fig. 6). The main interest is that all the states are controlled even if the previously announced limits exist. Then, we have a cascaded loops structure where the $R_{ci}$ relation (9) elaborates the reference for the closest interior loop.

$$\begin{cases} Rci \rightarrow \left(\dfrac{d^{i+1}y(t)}{dt^{i+1}}\right)_{ref} = c_i(t)\left[\left(\dfrac{d^i y(t)}{dt^i}\right)_{ref} - \dfrac{d^i y(t)}{dt^i}\right] \\ Rcn \rightarrow u_{reg}(t) = \displaystyle\sum_{i=0}^{n} a_i \left(\dfrac{d^i y(t)}{dt^i}\right)_{ref}, \ i = 0 \text{ to } n\text{ - }1. \end{cases} \tag{8}$$

This architecture is close to the industrial architecture for the axis drive control of machine tools [9]

### 3.4 Combination of the two inversion principles

The loop gain value cannot be infinite, for the previously mentioned reasons. A solution would consist in coupling the two inversion principles, each one bringing its own advantages. If the process model is perfectly identified in the entire working domain (signals amplitude and frequency), then the indirect inversion is useless. However, it is well known that a model is imperfect by nature. On the other hand, we can decide that the synthesized model of the process is the wished one, which satisfies the user. Thus, we introduce the concept of "Reference State Model based Control" (RSMC). Under these conditions, the model is elaborated for a specified application. Starting from the *n*th time derivative (*n* being the model order), the controller determines the successive reference states until the reference of the state to control (for example the position of an axis). The states are controlled so that the total command is the sum of the elementary commands and the process states converge towards those of the model (Fig. 7).

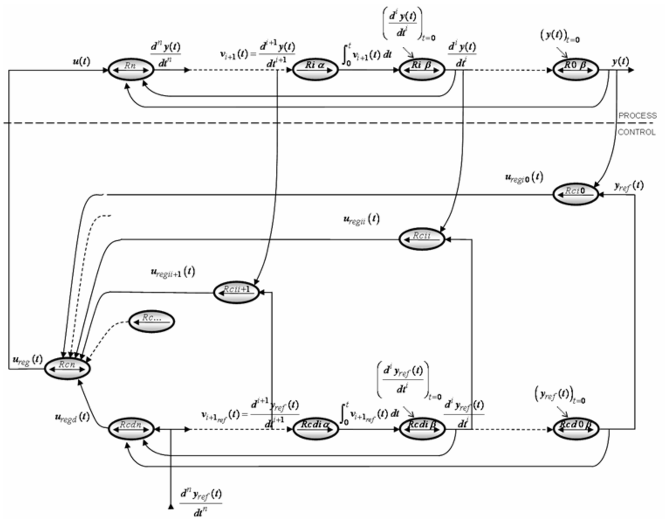

Fig. 7. Combination of the two inversion principles: the Reference State Model based Control structure.

In Fig. 7, the command relations are:

$$\begin{cases} Rcdn \rightarrow u_{regd}(t) = \displaystyle\sum_{i=0}^{n} a_{mi} \dfrac{d^i y_{ref}(t)}{dt^i} \\ Rcii \rightarrow u_{regii}(t) = c_i(t)\left(\dfrac{d^i y_{ref}(t)}{dt^i} - \dfrac{d^i y(t)}{dt^i}\right). \\ Rcn \rightarrow u_{reg}(t) = u_{regd}(t) + \displaystyle\sum_{i=0}^{n-1} u_{regii}(t) \end{cases} \tag{9}$$

From equations (2) and (10), we can write:

$$\begin{aligned} \sum_{i=0}^{n} a_i \frac{d^i y(t)}{dt^i} &= \sum_{i=0}^{n} a_{mi} \frac{d^i y_{ref}(t)}{dt^i} \\ &+ \sum_{i=0}^{n-1} c_i(t)\left(\frac{d^i y_{ref}(t)}{dt^i} - \frac{d^i y(t)}{dt^i}\right) \end{aligned} \tag{10}$$

If $a_{mi} = a_i$ (perfect identification) and $c_i = 0$ (direct inverse control) then $\dfrac{d^i y(t)}{dt^i} = \dfrac{d^i y_{ref}(t)}{dt^i} \quad \forall a_i$.

If $a_{mi} \neq a_i$ and $c_i \rightarrow \infty$ (RSMC), then $\dfrac{d^i y(t)}{dt^i} \rightarrow \dfrac{d^i y_{ref}(t)}{dt^i}$, $\forall a_i$.

If $c_0 \rightarrow \infty$, then $y(t) \rightarrow y_{ref}(t)$ and $\dfrac{d^i y(t)}{dt^i} \rightarrow \dfrac{d^i y_{ref}(t)}{dt^i}$, $\forall a_i$.

It is undeniable that we deal with the state control according to their respective references elaborated by a specified model fixing the behaviour. Obviously, in an ideal universe, the model would be the exact copy of the process, the model and process states would be equal at every moment and there would be no place for regulation. But we do not take into account the exogenous disturbances, in particular those which we do not control and those which are not directly observable. Under these conditions, the closed loop regulation can not be suppressed.

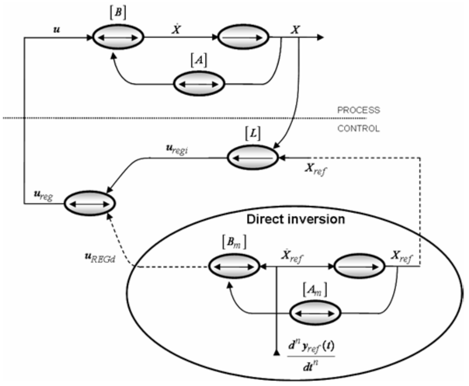


Fig. 8. Combination of the two inversion principles: State space control.

### 3.5 State feedback control

Previously, we showed that all the process states were described with the Causal Ordering Graph. Advanced controllers mainly use the state space formalism to define the various elements of the control structure. These controllers are characterized by the fact that their synthesis is not based only on the error, the time derivative and the time integral of available measurements, but on all the states governing the dynamic behaviour of the system. Based on the RSMC (Fig. 7), we can deduce the controller using the usual representation in state space form (Fig. 8), with the relations:

$$\begin{cases} \text{Process: } \dot{X} = AX + Bu \\ \text{Command: } u = u_{reg} = u_{regd} + u_{regi} \\ \text{Model: } \dot{X}_{ref} = A_m X_{ref} + B_m u_{regd} \\ \text{Indirect inversion: } u_{regi} = L\left(X_{ref} - X\right) \end{cases} \tag{11}$$

We can thus show that the suppression of the direct command leads to a traditional state feedback control. It should be noted that the cascaded loop controller (Fig. 6) is another form of state feedback controller. In the particular case of a mono-variable process governed by a second order differential relation, the equations (11) become:

$$\begin{cases} \text{Process: } \begin{Bmatrix} \dot{y} \\ \ddot{y} \end{Bmatrix} = \begin{bmatrix} 0 & 1 \\ -\dfrac{a_0}{a_2} & -\dfrac{a_1}{a_2} \end{bmatrix} \begin{Bmatrix} y \\ \dot{y} \end{Bmatrix} + \begin{bmatrix} 0 \\ \dfrac{1}{a_2} \end{bmatrix} u \\ \text{Command: } u = u_{reg} = u_{regd} + u_{regi} \\ \text{Model: } \begin{Bmatrix} \dot{y}_{ref} \\ \ddot{y}_{ref} \end{Bmatrix} = \begin{bmatrix} 0 & 1 \\ -\dfrac{a_{m0}}{a_{m2}} & -\dfrac{a_{m1}}{a_{m2}} \end{bmatrix} \begin{Bmatrix} y_{ref} \\ \dot{y}_{ref} \end{Bmatrix} + \begin{bmatrix} 0 \\ \dfrac{1}{a_{m2}} \end{bmatrix} u_{regd} \\ \text{Indirect inversion: } u_{regi} = \begin{bmatrix} c_0 & c_1 \end{bmatrix} \left( \begin{Bmatrix} y_{ref} \\ \dot{y}_{ref} \end{Bmatrix} - \begin{Bmatrix} y \\ \dot{y} \end{Bmatrix} \right) \end{cases} \tag{12}$$

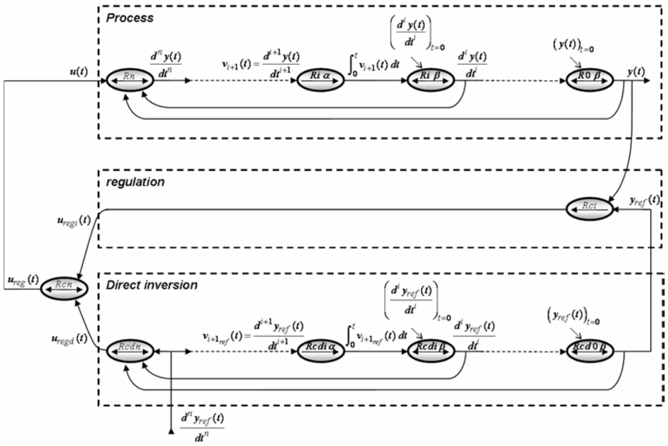


Fig. 9. Combination of the two inversion principles: Reference model based control.

### 3.6 Reference Model based Control

In this paragraph, we show that it is possible to degrade the previous structure while preserving the overall performances. The formalization of the indirect controller shows that if the loop gain for the controlled state is infinite, it is not useful to maintain the regulation of its successive time derivatives, provided that the system is stabilized. In practice, this method supposes the use of a more complex regulator such as a bi-quadratic filter to prevent instability and to maintain a certain damping level. Validations reveal that this kind of controller makes it possible to easily reach good performances because of the dominating contribution of the direct action (feedforward). Then, the regulation brings the complementary part of the command contributing to the good behaviour of the process in comparison with the model.

The Causal Ordering Graph of Fig. 9 is based on the one of Fig. 7. The reference input is still the *n*th time derivative of the controlled state. The regulated command is the combination of a direct inversion of the process and only one feedback loop on the controlled state. Equation (13) gives the expression of $u_{reg}$:

$$\begin{cases} Rcdn \rightarrow u_{regd}(t) = \displaystyle\sum_{i=0}^{n} a_{mi} \frac{d^i y_{ref}(t)}{dt^i} \\ Rci \rightarrow u_{regi}(t) = c(t)\left[y_{ref}(t) - y(t)\right] \\ Rcn \rightarrow u_{reg}(t) = u_{regd}(t) + u_{regi}(t) \end{cases} \tag{13}$$

The established structure corresponds to the Reference Model based Control (RMC). The advantage of this control structure is to impose the behaviour of the process according to the model without requiring an unphysical behaviour. The regulator $c(t)$ is defined so as to respect the expected performances.

# 4 Concluding remarks

This paper proposes a systematic method of control structure synthesis based on the inversion of an electromechanical process model using the concept of localised energies. This study is applied to a structure without unstable zero, which is described by the Causal Ordering Graph. The proposed approach does not claim to solve all the problems, but it effectively contributes to clarifying the control synthesis and help us think about the model structure adapted to a given physical system. The main advantage is to avoid an immediate mathematical development, which could quickly become hermetic. Thus, the model must initially be elaborated with the help of the fundamental laws of physics taking into account natural causality.

We have demonstrated that some classical control structures are formally expressed, such as the direct inversion principle or the cascaded loop control. Others, more particular and less industrial, such as the behaviour model control, the Reference State Model based Control and the Reference Model Control are defined.

The interest is to show the systematic deductive aspect of this approach, aiming at a faithful copy of the desired trajectories. Moreover, a better comprehension of the control, elaborated with a physical model respecting the natural causality of the process, makes it possible to avoid the traps of hasty investigations as well as of the trial-and-error methods. The suggested methodology prevents us from making the serious errors sometimes associated to empirical methods.

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