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Multi ultrasonic actuators with some degree of freedom

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Abstract: Usually numerical modelling and simulation of multicomponent piezoelectric actuators lead to the large number of recurred calculations with different geometrical parameters of the actuator and to the problems concerning the changes of modal shape sequences during recurred calculations. The exchanges in the modal shape sequence prevents automation of solving different problems. For this reason an improved algorithm for solving the problems related to the changes in the modal shape sequences has been proposed in the paper, also, the dependence of eigenfrequencies on various parameters of multicomponent piezoelectric actuators and problems related with the exchanges in the modal shapes sequence has been analyzed.

Key-Words: piezoelectric actuator, modal shape, multicomponent oscillations

1 Introduction

Whole modern technical areas (radiotechnology, acoustic, vibrotechnology, wave-technologies) are based on the application of different vibration processes. The development of new device groups that use piezoactive materials for high frequency oscillation transformation into a continuos multidirectional motion provides an opportunity to extend the field of creating time constant positioning drivers, micromanipulators, micropumps and other drives [7]. The performance of these devices strongly depends on the features of the actuator, which is the main part of the piezomechanical system. The synthesis of needful oscillation fields of the actuator can be obtained optimizing the geometrical parameters, the vector of poliarization and the topology of excitation zones of the actuator. The piezoelectric effect and the hysteresis effect play an important role in the dynamical behavior of these actuators [6]. So it is very important to know what modal shape will be excited when modelling piezoelectric actuators.

The functioning principle of most piezoelectric actuators is based on the excitation of higher resonance frequencies. The numerical analysis of such piezoelectric actuator is usually tied to a particular modal shape. While performing numerical analysis, when only the geometrical parameters of a piezoelectric actuator change, a problem arises that is related to the change in the modal shape sequence. Since vibration devices usually function at one of their modal frequencies, as the modal shape sequence changes, the problem solution usually does not converge, and the numerical analysis becomes meaningless.

2 Construction of Piezo Actuators

The performance of ultrasonic motors strongly depends on the features of the actuator, the main part of any piezomechanical system. There are some basic shapes of piezo actuators such as beam, plate, cylinder, disc, ring and etc. Various constructions of actuators are used in order to achieve a particular law of movement of the actuator and the final link of the kinematics pair Fig.1 [7]. The actuators shaped like beams and plates are mostly used in ultrasonic motors.

The characteristics and types of the excited multicomponent oscillations of the piezoelectric actuator depend on its geometrical parameters, boundary conditions and direction of the polarization vector. The topology of electrodes and geometrical parameters of the actuator define the direction of the excited oscillations. In order to achieve suitable characteristics of the oscillations, particular geometrical parameters of the actuator must be calculated.

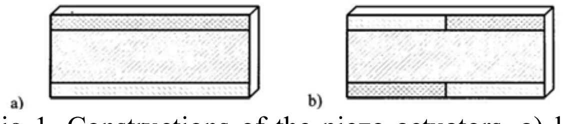

Fig 1. Constructions of the piezo actuators. a) k=l, n=l; b) k=l, n=2 [9, 10].

Many different types of multicomponent oscillations can be excited using these actuators: longitudinal-flexural, longitudinal - torsional and etc. Using the variable vector of polarization, three and four component oscillations of the beam shaped actuator can be achieved [1].

3 Problem Definition

Since the analysis of multidimensional piezoelectric actuator cannot be performed without considering the vibration device, most often the problems of piezoelectric actuator research are solved in an integral fashion taking into account the whole device.

The formal algorithm for solving the problem looks as follows:

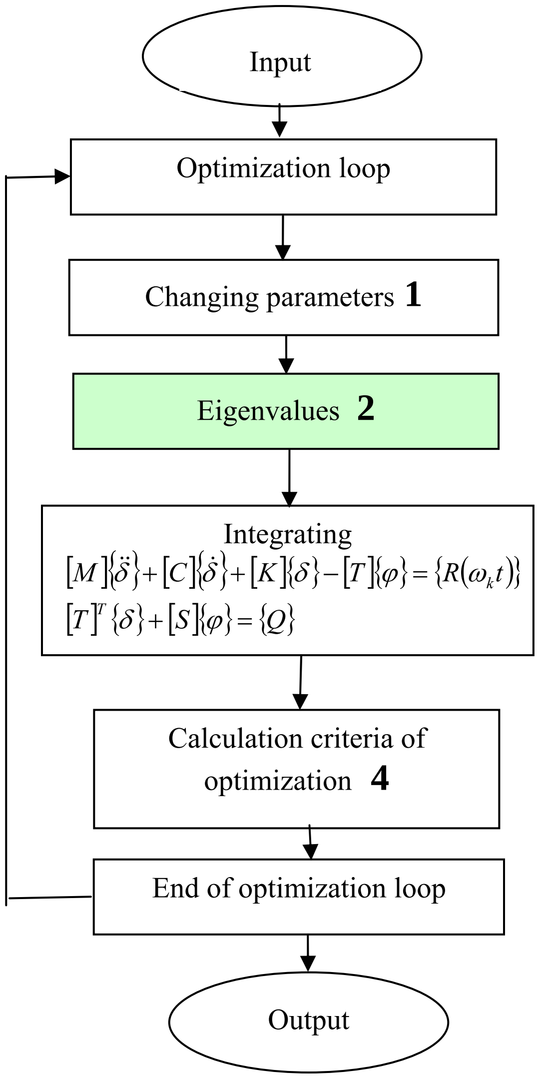

Fig. 2 The structure of the general calculation algorithm.

In the case under consideration, at the first stage (changing parameters) the geometrical parameters of a piezoelectric actuator are changed. At the second stage (eigenvalues) a matrix of eigenvalues is formed, whose every column describes a corresponding modal shape; the first column describes the first modal shape, the second column – the second shape and so on. While changing the geometrical parameters of a piezoelectric actuator the change in the modal shape sequence has been observed.

Exchanges in the modal shape sequence could be determined analyzing various constructions of piezoelectric actuators. For example, let's consider longitudinal – flexural oscillations of the beam actuator. Using this type of actuator, changes of the modal shape sequence could be found analytically [8].

Using the technical oscillation theory of the beam the longitudinal oscillations are found by solving the second order differential equation [2, 4]:

$$m(x)\frac{\partial^2 \xi}{\partial^2 t} - \frac{\partial}{\partial x}\left(Es_i \frac{\partial \xi}{\partial x}\right) = 0 \qquad (1)$$

Longitudinal oscillations of the beam can be expressed as follows [2, 4]:

$$\omega_k = \frac{k}{2l}\sqrt{\frac{E}{\rho}} \qquad (2)$$

E - the Jung modulus; k - the mode number of the longitudinal oscillations; l - the length of the beam; ρ mass density.

Flexural oscillations of the beam are found by solving the second order differential equation [2, 4]:

$$\frac{\partial^2}{\partial x^2}\left(EI\frac{\partial^2 \xi_{len}}{\partial x^2}\right) + \rho S \frac{\partial^2 \xi_{len}}{\partial^2 t} = 0 \qquad (3)$$

Flexural oscillations of the beam are described by the expression [2, 4]:

$$\omega_n = \frac{\pi h\,(n+0.5)^2}{4l^2}\sqrt{\frac{E}{3\rho}} \qquad (4)$$

h - the height of the beam; n – the mode number of the flexural oscillations;

If certain values of k and n are defined, then h/l ratio of the beam could be calculated. From the equations (2) and (4) following equation could be obtained:

$$\frac{h}{l} = \frac{2\sqrt{3}\,k}{\pi\,(n+0.5)^2} \qquad (5)$$

As an example:

$$k=1;\ n=2 \ \Rightarrow\ \frac{h}{l}=0.1765 \qquad (6)$$

But h/l ratio could be changed, for example, increasing or reducing the height of the beam. In this case k value remains the same, but n value changes. This means that the sequence of modal shapes changes when the geometrical parameters of the beam vary. For example, when the length and height ratio is $0.5<l/h<3$ we have an ordinary modal shape sequence and in other case second and third modal shape of two dimensional actuator changes. Calculations of multidimensional piezoelectric actuators, of course, have the same problem, but to solve it analytically is very difficult or even impossible [8]. Modal shape sequence exchanges often causes fatal errors in the calculation process well as unexpected errors in results [5]. And it means that at Eigenvalues stage of solving the problem an incorrect value of w_k (natural frequency) can be chosen. Then other stages of solving the problem become meaningless because the actuator made to the parameters obtained is defective. Hence the main problem is to choose a suitable value of w_k.

In most cases piezoelectric actuators are resonance systems that operate in the first or higher resonance frequency. The synthesis of the needful field of oscillations must be obtained by using a particular shape and dimensions of actuator and also certain geometry of the locations of excitation zones. Equations (7) fully define the piezoeffect [4]:

$$\begin{cases} \{\sigma\} = \left[c^E\right]\{\varepsilon\} - [e]^T \{E\} \\ \{D\} = [e]\{\varepsilon\} + \left[\ni^S\right]\{E\} \end{cases} \tag{7}$$

where $[c^E]$, $[e]$, $[\ni^S]$ – the matrix of stiffness for a constant electric field; the matrix of the piezoelectric constant; the matrix of dielectric constant evaluated at the constant strain, respectively; $\{\sigma\}$, $\{\varepsilon\}$, $\{D\}$, $\{E\}$ – the vectors of stress, strain, electric induction and electric field, respectively.

Various kinds of resonance oscillations of actuators - longitudinal, flexural, rotational, shear and so on - could be obtained using a different geometry of electrodes [1]. In order to achieve required resonance oscillations of the actuator, particular electrodes must be excited.

Analysis of the piezoelectric actuator must be carried out appreciating the electric occurrence in the system. Based on FEM, every node of the element has one additional DOF used for electric potentials in FEM modeling. The solution applied for the equations of motion, suitable for the actuator, can be derived from the principle of minimum potential energy by means of variation functional [4]. The basic dynamic FEM equation of motion for piezoelectric transducers that are fully covered with electrodes can be expressed as[11]:

$$\begin{aligned} &[M]\{\ddot{\delta}\} + [C]\{\dot{\delta}\} + [K]\{\delta\} - [T]\{\varphi\} = \{R(\omega_k t)\} \\ &[T]^T\{\delta\} + [S]\{\varphi\} = \{Q\} \end{aligned} \tag{8}$$

where [M], [K], [T], [S], [C] - the matrices of mass, stiffness, electro elasticity, capacity and damping, respectively; $\{\delta\}$, $\{\varphi\}$, $\{R\}$ - the vectors of nodes displacements, potentials and external mechanical forces, respectively.

Here:

$$[K] = \int_V [B]^T \left[c^E\right] [B]\, dV \tag{9}$$

$$[T] = \int_V [B]^T [e] \left[B_E\right] dV \tag{10}$$

$$[S] = \int_V \left[B_E\right]^T \left[\ni^s\right] \left[B_E\right] dV \tag{11}$$

$$[M] = \rho \int_V [N]^T [N]\, dV \tag{12}$$

$$[C] = \alpha [M] + \beta [K] \tag{13}$$

where [B], $[B_E]$ – the matrices of geometry used for evaluation of displacements and potential, respectively; [N] – the function of the shape used for evaluation of the mass matrix. The damping matrix [C] is derived using mass and stiffness matrices by assigning constants α and β.

Usually only the first equation from system (8) is used in the modeling process, because it is considering that the current source is powerful enough and ensures defined values of the electric potentials.

Solving dynamic equations using normalized co-ordinates, the vector of nodes displacements {u} is expressed as a superposition of modal shapes with weight coefficients [3]:

$$\{u\} = [Y]\{z\} = \{y_1\} z_1 + \ldots + \{y_q\} z_q \tag{14}$$

Based on the equation (14) the following expression of dynamic equation of the actuator could be obtained [3]:

$$\begin{aligned} &\ddot{z}_k + 2\omega_k c_k \dot{z}_k + \omega_k^2 z_k = \{y_k\}^T \{F_k\} \\ &k = 1, 2, \ldots, q \end{aligned} \tag{15}$$

Usually piezoelectric actuators operate a in certain resonance mode and the system of the equations (15) could be solved using the dynamical reduction method. This means that only one equation, which corresponds to a certain modal shape, could be solved. But in this case the calculation algorithm couldn't be statically tied with the definite equation number k, because its value becomes indefinite when geometrical parameters or boundary conditions change. This is the main reason why errors appear hindering automation of the numerical experiments. So it's necessary to identify the modal shape and find the corresponding number of the column in matrix [Y] during recurred calculations [8].

4 An Algorithm of Modal Shape Identification

Dominating coefficients is the way to define the type of oscillations of the actuator and to sort modal shapes by the dominating type of the oscillations for example longitudinal, flexural or torsion.

When the modal frequencies analysis of multicomponent actuators is done using FEM, dominating components of the oscillations can be found referring to the energetic method of the oscillation analysis, because amplitudes raised to the second power are proportional to the energy of the oscillations [8]. In that way the ratios (dominating coefficients) of the components of amplitudes in all directions can be found, and the direction with the maximum of amplitudes can be defined.

Let's calculate the following sum [7]:

$$S_p^b = \sum_{i=1}^{r} \left(A_{ip}^b \right)^2, \quad r = \frac{q}{p} \tag{16}$$

The dominating coefficients of the model can be expressed as follows [7]:

$$m_{jk}^b = \frac{S_j^b}{S_k^b}, \quad j \neq k \tag{17}$$

The physical meaning of dominating coefficients is the ratios between different oscillation energy components in the directions of coordinate axes. The sum S_k^b defines the energy of the oscillation of the b natural frequency in k direction and the dominating coefficient m_{jk}^b defines the relation of the oscillation energies in i and j directions of b natural frequency. Dominating coefficients have the following characteristic [7]:

$$m_{kj}^b = \frac{1}{m_{jk}^b}, \quad j \neq k \tag{18}$$

Based on dominating coefficients we could determinate the type of dominating oscillations and also define the level of correlation of multicomponent oscillations as follows:

$$m_{jk}^b = \tau \quad \Rightarrow \quad \lg \tau = 1, 2, 3 \ldots \tag{19}$$

Dominating coefficients is the method to define the type of oscillations of the actuator and to sort modal shapes by dominating type of the oscillations for example longitudinal, flexural or torsional. In order to finally identify the modal shape additional characteristic criteria must be applied, because the values of dominating coefficients vary when the geometrical parameters change. These criteria are the nodes points or lines number of the modal shape. During calculations the number of node points or lines could be found referring to the sign of the oscillations amplitude alternating around the equilibrium attitude (Fig. 3).

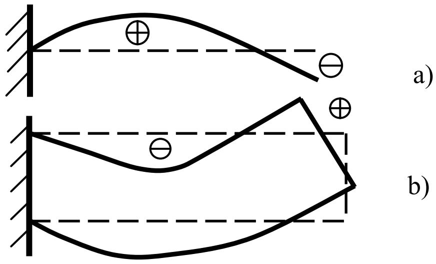

Fig. 3 Modal shape identification of piezo actuators: a) beam, b) plate.

The exchanges in the modal shape sequence are a general case problem concerning not only piezoelectric actuators, but also with all mechanical structures [8].

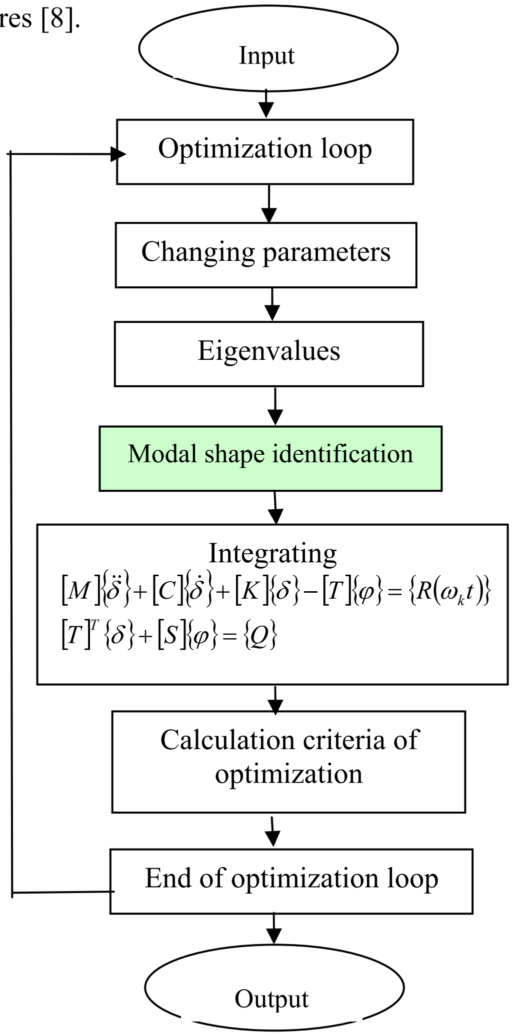

Fig. 4 The improved structure of the general calculation algorithm.

So, when solution of the mechanical problem includes dynamic equations solving during recurred calculation, it is proposed to make calculations using the improved structure of the general calculation algorithm and to add the stage of modal shape sequence identification as is shown in Fig. 4.

5 Conclusions

While changing the geometrical parameters of piezoelectric actuators the change in the modal shape sequence has been observed.

Identification of modal shapes sequence is necessary step in order to automate numerical experiments of multicomponent piezoelectric actuators. An algorithm of modal shape identification has been proposed that could be applied to all mechanical structures. This algorithm must be used as an additional stage in FEM software.

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