A Stochastic Approach to MiniUAVs Control Design

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Abstract: - A novel approach based on the application of an expressly tailored random search algorithm (Stochastic Gain Synthesis) to the design of a MiniUAV control system is presented in this paper. This methodology, joined to the well known Stochastic Robustness Analysis concept, may be applied to the design of control systems granting adequate level of robustness according to plant characterization and stability criteria. Due to the unconventional dynamics and its sensitivity to geometric, inertial and aerodynamic characteristics, control system design for MiniUAVs represents a challenging task whenever stability and performance requirements related to aircraft flying qualities have to be fulfilled. The robustness characteristics of the designed controller play a key role in granting adequate level of tolerance to plant uncertainties, environmental changes and flight condition variations within the planned mission profile.

Key-Words: - randomized algorithm, stochastic robustness, control design, mini UAVs.

1 Introduction

In the last decade the term MAVs (Micro Aerial Vehicles) has been used to define flying objects characterized by physical sizes less than 150mm, in length, width or height; they can be considered as *aerial robots, as six-degree-of freedom machines whose mobility can deploy a useful micropayload to a remote or otherwise hazardous location where it may perform any of a variety of missions* [1]. The concept of uninhabited aerial vehicles of reduced dimensions, able to perform mission profiles not compatible to any existing piloted platform, was extended to greater systems and it nowadays incorporates the so called MiniUAVs (maximum dimension up to 2 m).

These platforms have been the subject of considerable interest and development over the last years. A large number of successful designs has been generated for either research or commercial purposes by several universities, industries, and government-funded institutions. Even if fixed wing MiniUAVs can be considered as miniaturized aircraft so that conventional design issues and procedures can be extended to this family of aerial vehicles, flight dynamic analysis and preliminary control system design are discouraged by the inaccurate prediction of system dynamic response, affected by many model uncertainties (aerodynamics, propulsion system, actuator dynamics, ...) [2]. The reduced dimensions lead to highly nonlinear system behavior and unconventional dynamics (in terms of natural frequencies and damping ratios). In general the inertial characteristics of these platform yield to too high short period natural frequencies that cause abrupt responses to longitudinal commands. The conventional control system design techniques are often not suitable to provide the natural frequency and the damping ratio to fulfill stability and performance requirements. Differently from the large scale aircraft case, nowadays platform stabilization and loop closures (when available) have to be tuned with flight experiments during the development of the flying prototype. For that reason, Micro and Mini Aerial Vehicles represent an interesting challenge for the control design. These systems characterized by unconventional inertial characteristic are very sensitive to changes in flight condition (velocity), aerodynamic data (stability and control derivatives), geometric and inertial data. The design of a robust flight control system able to provide the system of a proper level of tolerance to environmental changes and platform manufacturing inaccuracies is necessary for such platforms.

A Stochastic Gain Synthesis methodology (SGS) is herein proposed. This methodology, joined to the well known Stochastic Robustness Analysis (SRA) concept, may be applied to the design of control systems granting adequate level of robustness according to plant characterization and stability criteria. The SRA, based on Monte Carlo simulation, consists of a numerical procedure, for evaluating the effects of plant parameter uncertainties on the aircraft stability and performance metrics. In [3] the concept of stochastic sensitivity of the stability characteristics was firstly introduced and applied to reduced order models of the aircraft dynamics, by analyzing the effects of aerodynamic uncertainties. The probability of instability definition was related to the system robustness to parameter uncertainties. Afterwards, due to the need for a further insight to the effects of uncertainties on system stability, the attention was focused on the graphical methods for a better comprehension of the robustness issues, leading to the introduction of the stochastic root loci [4]. In [5] the stability robustness concept was then extended to performance robustness, including within the evaluation method the metrics related to performance specifications for aircraft flying qualities. The application of random search to the synthesis phase was implemented for a basic benchmark problem (mass-spring-mass system) in [6] by using the random search results as a standard for comparison with genetic algorithm search.

The innovative aspect of the presented approach is based on the application of an expressly tailored random search algorithm to the design of a MiniUAV control system. Even if the developed methodology is applicable to different piloted and not piloted aircraft configuration and can be considered as a general approach, a Mini UAV platform is a representative and interesting case study due to its abovementioned unconventional dynamic characteristics.

The present paper is organized as follows: Section 2 outlines the MiniUAV reference platform, while Section 3 details the mathematical model of the aircraft dynamics as implemented within the proposed algorithm. Section 4 is focused on the control design methodology proposed (the SGS algorithm) and its integration with stochastic robustness analysis methods. Finally Section 5 presents the details of the application to the MiniUAV platform as reference case study.

2 The aerial platform

The MicroHawk configuration [7] is characterized by a conventional layout: it is a fixed wing, tailless integrated wing-body configuration, tractor propeller driven (Fig. 1).

Figure 1: MicroHawk configuration layout.

Different scaled versions of the MicroHawk configuration were designed and successfully fit to several mission profile needs. The Italian research Ministry (MIUR), in the frame of the project Cofin 2004, has recently co-founded a project aimed to design and develop a platform for environmental monitoring, involving applications such as fire detection and prevention, industrial areas reconnaissance, and natural disaster monitoring [8]. A scaled version of the MicroHawk configuration named MH1000 (Fig. 2), characterized by 1 m maximum dimension and a total weight of approximately 1.5 kg, was developed to carry on-board sensors and cameras in order to provide the user real time information about the territory under monitoring (Fig. 3).

Figure 2: MH1000 miniUAV.

Since different target users and multipurpose mission profiles have to encountered for, the mission requirements include capability of both remotely piloted flight and autonomous flight. Flight testing of the platform is an undergoing activity, aimed to evaluate and optimize the aircraft performances and to fix the high quality miniature camera position in an attempt to piloting the platform in out-of-sight mission.

The aerial platform is characterized by an average flight speed of about 55 km/h and it aims to achieve a flight endurance not less than 40 min.

Figure 3: Mission profile and operating scenario.

3 The mathematical model

The aerial platform dynamics is modeled by a complete 6 DoF nonlinear mathematical model [9]. The equations of motion, both for point mass center of gravity dynamics and for attitude dynamics, are written with reference to the body-axes reference frame, i.e. a vehicle body-fixed system, having origin at the vehicle center of gravity and axes aligned with vehicle reference directions. The rigid body assumption is made, neglecting structural flexibility: this assumption is commonly applied for general flight simulation application, as attention is focused on trajectory analysis and overall aircraft performances, and it is further supported in this context of simulation of a mini

uninhabited aerial vehicle, characterized by small dimensions and weight. The flat and nonrotating Earth assumption is also applied and it does not affect model fidelity for low speed flight simulation, interesting small areas. The point mass equations for the aircraft center of gravity dynamics are as follows:

$$
\begin{bmatrix} U \\ \dot{V} \\ \dot{W} \end{bmatrix} = \frac{1}{m} \begin{bmatrix} F_{XA} \\ F_{YA} \\ F_{ZA} \end{bmatrix} + \begin{bmatrix} F_{XT} \\ F_{YT} \\ F_{ZT} \end{bmatrix} - \begin{bmatrix} 0 & -R & Q \\ R & 0 & -P \\ -Q & P & 0 \end{bmatrix} \begin{bmatrix} U \\ V \\ W \end{bmatrix} + \begin{bmatrix} T_{VB} \end{bmatrix}^{-1} \begin{bmatrix} 0 \\ 0 \\ g \end{bmatrix}
$$

where U, V, and W are the linear velocity components, P, Q, and R are the angular velocity components. Aerodynamic and propulsive forces are represented by FA and F_T respectively. The $[T_{VB}]$ matrix, representing the rotation matrix from body to local navigation reference system, is involved in the definition of the gravitational terms.

The equations governing the attitude dynamics can be expressed as follows:

$$
\begin{bmatrix} P \\ \dot{Q} \\ \dot{R} \end{bmatrix} = \begin{bmatrix} I_{XX} & 0 & -I_{XZ} \\ 0 & I_{YY} & 0 \\ -I_{XZ} & 0 & I_{ZZ} \end{bmatrix}^{-1} \begin{bmatrix} M_{XA} \\ M_{YA} \\ M_{ZA} \end{bmatrix} + \begin{bmatrix} M_{XT} \\ M_{YT} \\ M_{ZT} \end{bmatrix} - \begin{bmatrix} 0 & -R & Q \\ R & 0 & -P \\ -Q & P & 0 \end{bmatrix} \begin{bmatrix} I_{XX} & 0 & -I_{XZ} \\ 0 & I_{YY} & 0 \\ -I_{XZ} & 0 & I_{ZZ} \end{bmatrix} \begin{bmatrix} U \\ V \\ W \end{bmatrix}
$$

where aerodynamic and propulsive moments are represented by M_A and M_T , respectively.

The vehicle attitude is modeled by the Euler kinematical relationships:

$$
\begin{bmatrix} \dot{\Phi} \\ \dot{\Theta} \\ \dot{\Psi} \end{bmatrix} = [H(\Phi)] \begin{bmatrix} P \\ Q \\ R \end{bmatrix}
$$

where the Euler angles vector consists of roll (Φ) , pitch (Θ) and yaw (Ψ) angles.

The kinematic relations for the center of gravity coordinates in a geographic system having origin at the vehicle center of gravity are as follows:

$$
\begin{bmatrix} \dot{X}_V \\ \dot{Y}_V \\ \dot{Z}_V \end{bmatrix} = [T_{VB}] \begin{bmatrix} U \\ V \\ W \end{bmatrix}
$$

The complete mathematical model is hence represented by twelve, coupled, nonlinear, ordinary differential equations. The aircraft maneuver response is obtained by applying a numerical integration algorithm, based on a fourth order Runge-Kutta scheme, to solving simultaneously the complete system.

The aerial platform is characterized by the aerodynamic model and the propulsive system model. The former one is based on an experimentally derived dataset of stability and control derivatives. The aerodynamic forces (F_A) and moments (M_A) are obtained by modeling the dependencies on flight condition (dynamic pressure), aerodynamic angles and control surface deflections:

$$
F_A = \frac{1}{2} \rho V^2 S C_{F_A}(\alpha, \beta, P, Q, R, \delta_a, \delta_e)
$$

$$
M_A = \frac{1}{2} \rho V^2 S I C_{M_A}(\alpha, \beta, P, Q, R, \delta_a, \delta_e)
$$

where *l* represents a reference length.

As to the propulsive system module, linear relationships were applied to model the voltage supply and current drain trend of variation for a DC motor-based propulsion. The propeller performance are estimated by implementing the blade element theory to compute propulsive forces (F_T) and moments (M_T) at a given regime of rotation:

$$
F_T = f(\alpha, \beta, V, \rho, n_P, r_P, c_P, \alpha_{c_P}, \alpha_T)
$$

$$
M_T = g(\alpha, \beta, V, \rho, n_P, r_P, c_P, \alpha_{c_P}, \alpha_T, b_X, b_Y, b_Z)
$$

The propeller model is directly related to the power control (throttle command) while the DC motor modeling relationships provide information on the supply conditions and the energy source (battery) status. The aerodynamic and propulsive load models are then combined to the vector equations to obtain the complete aircraft model formulation.

Figure 3: Mathematical model architecture.

 Fig. 3 shows the data flow within the mathematical model architecture.

The main control commands are represented by the stick input and the throttle input. The former one is directly related to the control surfaces deflection, that, according to the present aerial platform, are aileron and elevator for longitudinal and lateral-directional control, respectively. The stick input enters the aerodynamic model determining, together with aircraft states, the aerodynamic loads $(F_A,$ MA). On the other hand, the throttle input represents the power control and, as a consequence, it is involved in the computation of the propulsive loads (F_T, M_T) .

4 The control design methodology

A methodology based on the application of randomized algorithms (RAs) [10] to design a robust flight control system is here proposed. Different issues of designing and testing closed loop system are encountered for and can be divided into three phases: gain synthesis, stability analysis and performance analysis. These steps of the methodology are strongly dependent by each other and strictly related to the operating flight conditions and to the flying qualities standards.

The RAs based methodology is applied to linear timeinvariant systems:

$$
\dot{x}(t) = Ax(t) + Bu(t) \qquad (1)
$$

$$
y(t) = Cx(t)
$$

where $x(t)$, $u(t)$, and $y(t)$ are state, control and output vectors and A, B, and C are state, control and output matrices.

The synthesis phase is based on the implementation of the Stochastic Gain Synthesis (SGS) algorithm (Fig. 4). It consists of a random search by Monte Carlo evaluations of a gain set able to fulfill some stability criteria of the closed loop system

 $\dot{x}(t) = (A - BKC)x(t)$

 $u(t) = -KCx(t)$

Figure 4: Stochastic Gain Synthesis algorithm.

The open loop dynamics, characterized by the state and control matrices, is evaluated with respect to some boundary parameters $q_1, q_2, ..., q_n$. Therefore n linear time invariant systems result as problem bounds:

$$
\dot{x}(t) = A(q_1)x(t) + B(q_1)u(t)
$$

\n
$$
\dot{x}(t) = A(q_2)x(t) + B(q_2)u(t)
$$

\n...\n
$$
\dot{x}(t) = A(q_n)x(t) + B(q_n)u(t)
$$

The proposed algorithm may be viewed as an alternative to the gain scheduling issue by bounding the gain searching problem according to operating conditions (flight speed and altitude) or can be implemented within fault tolerant control design applications by properly modeling the failure effects on the system behavior.

The SGS algorithm randomly generates a gain set to simultaneously close the loop of the n LTI systems and it evaluates the fulfillment of the stability criteria. Due to the simplicity of the algorithm structure, the set of criteria are defined by the user in terms of system eigenstructures in order to customize the desired dynamics by introducing an enlarged stability concept. It has to be noted that the more severe are the stability criteria the lower is the probability of the SGS algorithm to found a gain set fulfilling the requirements.

The gain set obtained as a result of the SGS algorithm are then tested in terms of robustness by applying them to close the nominal open loop of uncertain plants (Fig. 5). This second phase of the proposed methodology is based on the Stochastic Stability Robustness Analysis concept [4]. The state and control matrices of the uncertain plant are randomly generated within a defined interval:

$$
\dot{x}(t) = A(\Delta)x(t) + B(\Delta)u(t)
$$

where Δ represents the uncertain parameters vector. Structured parameter uncertainties are taken into account, including those related to flight conditions (dynamic pressure), aerodynamic data (stability and control derivatives), geometric and inertial data. The uncertainties related to the flight conditions can be ascribed to the real flight in a non-ideally-calm air and to the need to cover a portion of the flight envelope as large as possible. The uncertainties concerning the aerodynamic data can be related to experimental measurement errors or computational approximations due to numerical evaluation. The uncertainties in terms of geometric and inertial data may allow to take into account manufacturing inaccuracies.

Figure 5: Stability Analysis algorithm.

The robustness of the gain(s) obtained by the SGS algorithm can then be tested and related to an estimated probability of stability value:

$$
\hat{P}_{stab} = \frac{N_{good}}{N}
$$

where N represents the total number of Monte Carlo evaluations and N_{good} is the number of closed loop systems fulfilling the stability criteria.

Each gain set is characterized by a root locus highlighting the region of interest defined by the user and a probability of stability trend as a function of the uncertainty entity. In case of multiple gain set, the best fitting one will be selected according to the SSRA results.

The performance analysis phase is based on the Stochastic Performance Robustness Analysis and is aimed to test the compliance of the uncertain closed loop system to the frequency and time domain standard requirements (Fig. 6).

Figure 6: Performance Analysis algorithm.

As to the frequency domain, check of compliance and hence performance robustness analysis are carried out with reference to the standard requirements not included within the stability criteria applied within the synthesis phase. Finally, the time domain response of the aircraft to a maneuver is evaluated by referring to the standard requirements parameters, such as overshoot, raising time, settling time. Each gain set, as obtained by the SGS algorithm, is characterized by a probability of stability with reference to the performance metrics and a correlation can be founded between the probability of stability referred to different criteria.

5 The MH1000 case study

The MH1000 is presented as reference platform for the application of randomized algorithms to the control design and the robustness analysis of mini aerial vehicles. The case study here reported is based on the assumption of a decoupled dynamics and it is focused on the longitudinal plane dynamics assessment.

The nominal plant is described, according to Eqn. (1), by:

$$
A = \begin{bmatrix} -0.205 & 0.861 & -1.610 & -9.712 \\ -0.687 & -6.179 & 11.280 & -1.388 \\ 0.668 & -4.680 & -7.763 & 0.000 \\ 0.000 & 0.000 & 1.000 & 0.000 \end{bmatrix}
$$

$$
B = \begin{bmatrix} 6.112 \\ -48.575 \\ -448.840 \\ 0.000 \end{bmatrix}
$$

$$
x(t) = \{ u \quad w \quad q \quad \theta \}
$$

$$
u(t) = \{ \delta_e \}
$$

where the state components consist of the linear velocities *u* and *w* along the body axes X and Z, respectively, the pitch rate q and the pitch angle θ , and the control vector consists of the symmetrical elevon deflection.

The state and control matrices are obtained by numerical linearization about a reference trim condition $(V = 12.5 \text{ m/s})$; $h = 50$ m), within the experimentally defined flight envelope (9 m/s $<$ V $<$ 18 m/s; 5 m $<$ h $<$ 100 m). The MiniUAV is characterized by a dynamic behaviour strongly dependent on plant uncertainties and operating condition variations. In an attempt to design a control system having satisfactory performance within a flight envelope region as large as possible and according to the limited range of variation of altitude, flight speed was chosen as boundary parameter for the SGS algorithm implementation. Therefore, the problem was bounded in terms of two linear time-invariant systems, corresponding to the flight speed range limits ($V_{\text{lower}} = 11$) m/s; $V_{upper} = 15$ m/s).

The enlarged stability concept for the specific case study was defined according to the following set of criteria:

$$
Im(\lambda_i) \neq 0
$$

\n
$$
Re(\lambda_i) < 0
$$

\n4.0 rad/s $\omega_{SP} < 6.0$ rad/s
\n
$$
0.5 < \zeta_{SP} < 0.9
$$

\n1.0 rad/s $\omega_{PH} < 1.5$ rad/s
\n
$$
0.1 < \zeta_{PH} < 0.3
$$

\n
$$
\Delta \omega = \omega_{upper} - \omega_{lower} < \pm 15\%
$$

It has to be noted that the set of criteria is strictly related to the reference platform; it was defined according to a tradeoff between a thorough analysis of the nominal plant dynamics and the existing standard requirements [11].

Table 1 reports the gain set obtained as a result of the random search (N_{MCE} = 150000).

	${\bf k_u}$	$\mathbf{k}_{\mathbf{w}}$	${\bf k_0}$	Kθ
\mathbf{K}_1	0.0000	0.0092	0.0094	-0.0054
\mathbf{K}_2	-0.000355	0.0083	0.0144	-0.0044
$\mathbf{K_3}$	-0.000371	0.0095	0.0089	-0.0035
\mathbf{K}_4	-0.000419	0.0077	0.0129	-0.0043

Table 1: Gain set resulted from SGS algorithm $(N_{MCE} = 150000)$.

The closed loop system behaviour was then tested according to the Stochastic Robustness Analysis in terms of fulfillment of stability and performance requirements.

The controller robustness was evaluated taking into account the following flight condition variations and plant uncertainties:

$$
\Delta = \left\{ \rho, V, m, b, \overline{c}, S, I_{Y}, C_{X_{\alpha}}, C_{Z_{\alpha}}, C_{m_{\alpha}}, C_{Z_{q}}, C_{m_{q}}, C_{X_{\delta_{e}}}, C_{Z_{\delta_{e}}}, C_{m_{\delta_{e}}} \right\}
$$

including those related to geometric data (mass *m*, wingspan *b*, mean aerodynamic chord *c*, wing surface *S*), inertial data (moment of inertia I_Y), and stability and control derivatives. Stability robustness was evaluated by means of root locus based analysis (Fig. 7). A comparative analysis of the different gain set shows that, although the SGS and SSRA algorithm are based on the same stability criteria, the further insertion of plant uncertainties could be responsible for a decay of the closed loop dynamics within the selected flight speed range.

Each gain is correlated to an estimated probability of stability value (Table 2), that characterizes the level of matching to the desired dynamics.

Figure 7: Stability robustness analysis (for K₂-black and K₃-red; N_{MCE} = 2300) with evidence of the regions of interest **representative of the enlarged stability criteria**.

As a final step of the proposed methodology, the performance robustness analysis [5] was carried out as regard to the SPRA algorithm. The desired performance were defined accordingly to standard requirements [10], in terms of Control Anticipation Parameter (CAP), Bandwidth Criterion and time domain response characterization. As an example, the Stochastic Performance Robustness results concerning bandwidth and phase delay trend are reported in Fig. 8.

Figure 8: Performance robustness analysis (for K₂-black and K_3 -red; N_{MCE} = 2300) with evidence of the flying qualities level **defined by the standard requirements**.

Likewise for stability, performance robustness of the closed loop system could be related to an estimated probability. Probability values reported in Table 2 highlight the possible differences between SSRA and SPRA results: they can be ascribed to the dependence of the stability criteria on the user's experience and knowledge about the plant dynamics despite of the relationships of the performance metrics with the standard requirements.

The Stochastic Gain Synthesis algorithm, joined to the stochastic analysis, led to the identification of a gain set able to fulfill adequate robustness level of the augmented miniUAV.

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7 Concluding Remarks

The Stochastic Gain Synthesis methodology demonstrates a good flexibility of application together with a simplicity of implementation for aircraft control design purposes. The gain selection phase may be supported by a numerical procedure for testing the robustness to parameter uncertainties according to a fully stochastic approach. The proposed methodology demonstrates to be able to manage unconventional plants, solving some critical issues arising when classical/modern control techniques are applied. The proposed case study shows the SGS effectiveness when applied to a miniUAV platform. Adequate robustness characteristics were achieved by a trade-off analysis between the user defined stability criteria and the standard requirements fulfillment.

References:

- 1. W.R. Davis, MicroUAVs, *Presentation to the 23rd Annual AUVSI Symposium*, 1996.
- 2. F. Quagliotti, G. Guglieri, B. Pralio, L. Lorefice, Simulation Tools for Flight Dynamics Analysis in the Design of MAVs, *24th ICAS Congress*, Yokohama, Japan, 2004.
- 3. R.F. Stengel, Some Effects of Parameter Variations on the Lateral-Directional Stability of Aircraft, *Journal of Guidance, Control, and Dynamics*, vol. 3, no. 2, 1980.
- 4. R.F. Stengel, L.R. Ray, Stochastic Robustness of Linear Time Invariant Control Systems, *IEEE Transaction on Automatic Control*, vol. 36, no. 1, 1991.
- 5. L.R. Ray, R.F. Stengel, Stochastic Measures of Performance Robustness in Aircraft Control Systems, *Journal of Guidance, Control, and Dynamics*, vol. 15, no. 6, 1992.
- 6. C.I. Marrison, R.F. Stengel, Robust Control System Design Using Random Search and Genetic Algorithms, *IEEE Transaction on Automatic Control*, vol. 42, no. 6, 1997.
- 7. B. Pralio, G. Guglieri, F. Quagliotti, Design and Performance Analysis of a Micro Aerial Vehicle Concept, *2nd AIAA "Unmanned Unlimited" Systems Technologies, and Operations – Aerospace, Land and Sea Conference*, San Diego, USA, 2003.
- 8. A. Sanna, B. Pralio, Innovative Tool for Simulating and Controlling Mini Air Vehicles. *WSEAS Transaction on Information Science and Applications*, Issue 10, Vol. 2, 2005.
- 9. B. L. Stevens and F. L. Lewis, Aircraft Control and Simulation, John Wiley & Sons Inc., 2003.
- 10. R. Tempo, G. Calafiore, F. Dabbene, Randomized Algorithms for Analysis and Control of Uncertain Systems, Springer-Verlag, 2005.
- 11. Anonymous, Flying Qualities of Piloted Aircraft, MIL-HDBK-1797, Department of Defense, USA, 1997.