A Hierarchical Evolutionary-Deterministic Algorithm in Topological Optimization of Electrical Grounding Grids

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Abstract: - This paper proposes a Hierarchical Evolutionary-Deterministic Algorithm (HEDA) for designing square grounding grids. This algorithm performs the design by means of a hierarchical coupling of a real coded evolutionary algorithm and the Hooke-Jeeves algorithm. The design of the grounding grid is here formalized as a min-max problem. The maximization part is the search of the most dangerous point for a given topological structure, the minimization part is the optimization of the topological parameter (compression ratio) of the grounding grid. The solution of the system of partial differential equations related to the spatial distribution of the current field is carried out by the Galerkin method. The program gives good results in terms of accuracy and computational complexity.

Key-Words: - Min-max problems, Hierarchical algorithms, Evolutionary algorithms, Hooke-Jeeves algorithm, Galerkin Method, Soil testing, Unequally spaced grounding grids, Compression ratio

1 Introduction

When a grounding grid is designed, the most important caution is to ensure the safety level required by technical Standards, this level being correlated with the maximum value of the touch voltage. This value depends on geometrical and physical parameters such as the configuration of the grounding grid, the resistivity of the soil and the fault current that the grounding grid must leak. It is therefore important to design a grounding grid that does not generate on the soil surface any touch voltage higher than the maximum permissible value U_{Tp} , this value being fixed by IEEE Standards on the basis of the value of the time to eliminate the fault [1].

For a given amount the conductors, the problem of the grounding grid design can be seen as a topological optimization problem consisting in arranging the conductors in a way such that the maximum touch voltage generated is minimum.

Some studies proved that, using the same amount of conductor material, grounding grids having a higher conductors density in the peripheral area than in the central one (unequally spaced grounding grids) were more performing in terms of safety than the equally spaced structures (see [2], [3], [4], [5] ,[6] and [7]).

Even though it is nowadays very clear that, qualitatively, the peripheral should contain more conductors than the central one, the problem of the quantitative arrangement of the conductors is still topic of discussion. Some empirical rules are given in [2] and some automatic algorithms are given in [7] [8]. Moreover, in the latest years, on the basis of some observations on the symmetry of the current fields, an arrangement of the conductors according to exponential regularity has been proposed. The main advantage of this arrangement is that the optimal topological design consists in the search of only one parameter called compression ratio C.

This paper proposes an algorithm for designing optimal unequally spaced square grounding grids for given values of fault current, cross-section of the cylindrical conductors, burying depth and resistivity of the soil. This algorithm finds the optimal compression ratio C^* defined as that value which ensure that in the worst working condition the touch voltage is minimized.

2 Current Field Analysis

When a grounding grid leaks a fault current I_F, a current field is generated on the points P of the soil surface. In order to study this current field a system of partial differential equations must be set. Obviously this system depends on the geometry of the problem and in many cases it cannot be easily solved analytically. It is therefore fundamental to study the problem by means of a numerical method. Basically, in literature, two approaches to solve this problem have been proposed: the Finite Element

Method and the Galerkin method. The former is more accurate but much slower since it leads to a large scale linear system, the latter is less accurate but much faster. In fact the Galerkin method is widely used for simple soil problems (i.e. homogenous and layered soil) since it gives, in these cases, very reliable results in a short calculation time. The Galerkin method has been used in this paper since, as it will be shown, a computationally inexpensive tool was required.

Briefly the method consists of the following (see [9], [10], [11] and [12]). Let us consider a grounding system having any configuration, buried at a depth d and which leaks a fault current I_F. Each conductor constituting the grounding system must be divided into a suitable number of elementary parts (subareas). The fault current leaked by each cylindrical conductor (subarea) is simulated by a line current lying on the axis of the cylinder as shown in Fig. 1. Each subarea j is supposed to leak a subcurrent I_{Fj}, and it generates on the induced subarea i a voltage given by $R_{ij} I_j$ where the voltage coefficient R_{ij} takes into account the action of both the inducing subarea j and its electrical image j' on the induced subarea i [9]. The voltage coefficients R_{ij} between the subareas i and j can be easily evaluated by the following formula [13]:

$$R_{ij} = \frac{1}{L_j} \int_l G(r, r') dl_j \qquad (1)$$

where r and r' are the distances from the subarea i of the subareas j and j' respectively, L_j is the length of the inducing subarea j and G is the Green's function. The R_{ij} coefficients can be easily calculated as shown in [9]. The total voltage generated by all the N inducing subareas constituting the grounding system on the induced subarea i is given by:

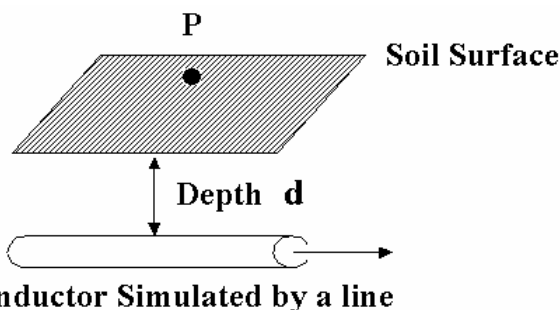$$U_i = \sum_{j=1}^{N} R_{ij} I_j \qquad i = 1, 2, \ldots, N \qquad (2)$$

Fig. 1: Buried conductor simulated by a line current

If we impose the equipotentiality of the grounding grid, that is if we impose $U_1 = U_2 = U_3 = \ldots = U_N = U_E$

where U_E is the earthing voltage, we can set the following system of linear equations:

$$\begin{cases} R_{11}I_{F1} + R_{12}I_{F2} + \ldots \ldots R_{1N}I_{FN} = U_E \\ R_{21}I_{F1} + R_{22}I_{F2} + \ldots \ldots R_{2N}I_{FN} = U_E \\ \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \\ R_{N1}I_{F1} + R_{N2}I_{F2} + \ldots R_{NN}I_{FN} = U_E \\ I_{F1} + I_{F2} + \ldots \ldots \ldots \ldots + I_{FN} = I_F \end{cases} \qquad (3)$$

where the fault current I_F leaked by the grid is known. The N subcurrents I_{Fj} and the earthing voltage U_E can be determined by solving the system (3). Once each subcurrent I_{Fj} is known, the contribution in terms of voltage given by the subarea j and by its electrical image j' to any point P of the soil can be calculated by the following formula:

$$U_j(P) = R_{P,j} I_{Fj} \qquad (4)$$

where R_{Pj} is the voltage coefficient between the inducing subarea j and the induced point P. R_{Pj} takes into account the action of both the inducing subarea j and its electrical image j' on the induced point P under examination. The R_{Pj} can be easily calculated as shown in [9]. The total voltage $U(P)$ generated in P by all the N subareas composing the grounding system can be calculated as the sum of the voltages generated in P by each subarea and its electrical image by means of the following formula:

$$U(P) = \sum_{j=1}^{N} U_j(P) \qquad (5)$$

Finally, the touch voltage can be calculated in each point P of the soil surface by the following formula:

$$U_T(P) = U_E - U(P) \qquad (6)$$

where the earthing voltage U_E has been determined by solving the system (3) and $U(P)$ is given by formula (5).

3 Conductors Arrangement

Let us consider a square grounding grid having N_c conductors running parallel to a side and N_c conductors running parallel to the other side. If the conductors are arranged according to an exponential regularity from the centre of the grounding grid, the distance D_n of the nth conductor from the $(n-1)$[th] conductor is given by [5] and [14]:

$$D_n = D_{max} C^n \qquad (7)$$

where: $n = 1, 2, \ldots, N_c/2$ if N_c is even, or $n = 1, 2, \ldots, (N_c-1)/2$ if N_c is odd; C is called compression ratio. It defines the arrangement of the conductors and varies in the interval $]0,1[$.

As can be seen from (7), the distances D_n are the terms of a geometrical progression having C as the ratio between two generic contiguous terms.

As to D_{max}, it can be obtained in the following way. If we indicate with L_s the length of the side of the square grounding grid, we can impose:

$$\sum_{n=1}^{N_c/2} D_n = D_{max} C \frac{1-C^{N_c/2}}{1-C} = \frac{L_s}{2} + C \frac{D_{max}}{2} \qquad (8)$$

if the number of conductors N_c is even

$$\sum_{n=1}^{(N_c-1)/2} D_n = D_{max} C \frac{1-C^{(N_c-1)/2}}{1-C} = \frac{L_s}{2} \qquad (9)$$

if the number of conductors N_c is odd.
From (8) and (9) it is possible to get D_{max}:

$$D_{max} = \frac{L_s(1-C)}{C(1+C-2C^{N_c/2})} \qquad (10)$$

if the number of conductors N_c is even

$$D_{max} = \frac{L_s(1-C)}{2C(1-C^{(N_c-1)/2})} \qquad (11)$$

if the number of conductors N_c is odd.

4 Problem Formulation and HEDA

As shown the Galerkin method is a tool to determine the touch voltage in a point P of the soil surface and if the conductors are arranged according to an exponential regularity each square grounding grid is represented by only one design parameter that is C. In this light, the grounding grid design problem can be formulated as the search of that parameter C^* which ensures that the maximum touch voltage $U_{T\text{-}max}$ on the point P^* is minimum. In a more formal way the problem can be expressed as the following:

$$\min_C \max_P U_T(P, C) \qquad (12)$$

In order to solve the problem in (12), a Hierarchical Evolutionary-Deterministic Algorithm (HEDA) is proposed. The HEDA consists of an evolutionary algorithm, solving the maximization problem, which returns the maximum value of touch voltage on the soil points as the objective function of the Hooke-Jeeves Algorithm which solves the minimization problem.

4.1 Maximization Problem Solution

To find the maximum touch voltage $U_{T\text{-}max}$ generated on the soil surface by a leaking grounding grid a steady state evolutionary algorithm has been implemented. This algorithm takes the touch voltage $U_T(P)$, calculated by the Galerkin method, as a fitness.
This evolutionary algorithm consists of the following. An initial sampling of N_P points P of the soil surface is done pseudo-randomly. Due to the continuous trend of the touch voltage on the soil

surface, a real encoding has been chosen where each point $P(x, y)$ is an individual having the coordinates x and y as genes. The Authors have chosen the arithmetic cross-over technique [15] and established that each individual thus generated has a 0.1 probability of undergoing a random mutation [16]. In each iteration, N_{cr} cross-over occur (where N_{cr} is a number chosen a priori). The choice of the pairs of chromosomes which undergo cross-over occurs selecting at random $2Ncr$ parents. A "naive" elitist survivor selection has been chosen: at each iteration a prefixed number equal to N_P of individuals having better fitness (higher value of U_T) must survive.
The rate at which the population improves is measured by calculating, in each ith iteration, the difference $\Delta U_{T\,i}$ between the maximum touch voltage obtained in the ith iteration and the average value between all the touch voltages $U_{T\,i}(P)$ calculated in the same iteration. The algorithm is stopped when at least one of the two following conditions occurs: the value $\Delta U_{T\,i}$ is smaller than a pre-arranged index of accuracy ϵ; the number of iterations N_{iter} reaches a pre-arranged value $N_{iter\text{-}max}$. By this criterion it is possible to determine the processing instant in which there is no need for the program to run since the population has reached such an evolution level that further iterations would involve an insignificant advantage in terms of accuracy.
Fig. 2 shows an example of the trend of the touch voltages $U_T(P)$ generated by a square grounding grid, this trend having been carried out by calculating the function $U_T(P)$ in a lot of points of the soil surface by the Galerkin method. As can be seen from Fig. 2, the maximum touch voltages are located, in this case, in the four corners of the grounding grid.
Fig. 3 show the behavior of the population during the iterations of the evolutionary algorithm applied to the previous grounding grid. In particular, Fig. 3c shows that after twenty iterations all the population of points P is overlapped in the corner point (0,0).
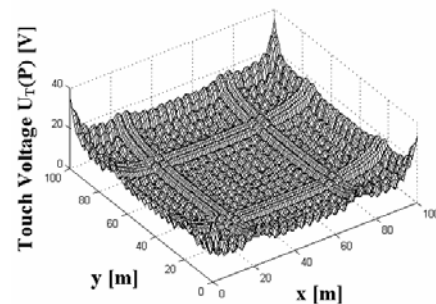

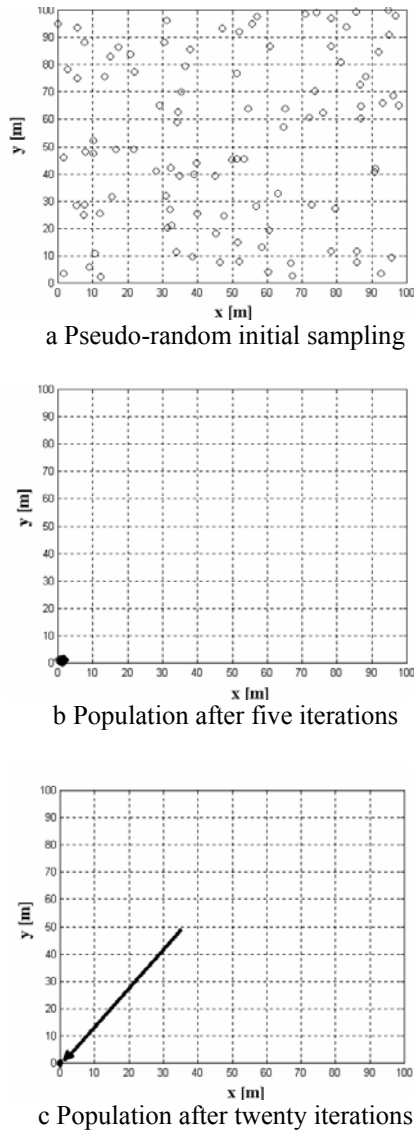Fig. 2: An example of the trend of the touch voltages

a Pseudo-random initial sampling

b Population after five iterations

c Population after twenty iterations

Fig. 3 Behavior of the population during the iterations of the evolutionary algorithm

4.2 Minimization Problem Solution

From what said above, it is clear that to each value of C corresponds an arrangement of conductors, that is a configuration of a square grounding grid. So, once the number of conductors, their length, cross-section and burying depth are fixed, we get a family of meshed square grounding grids, each grid being characterized by a different value of compression ratio C. For each grounding grid of the family under examination, it is possible to find the maximum touch voltage U_{T-max} by applying the evolutionary algorithm previously explained. The evolutionary algorithm carried out can be therefore considered as a tool to determine the objective function $U_{T-max}(C)$ to be minimized. Fig. 4 shows an example of the trend of $U_{T-max}(C)$.
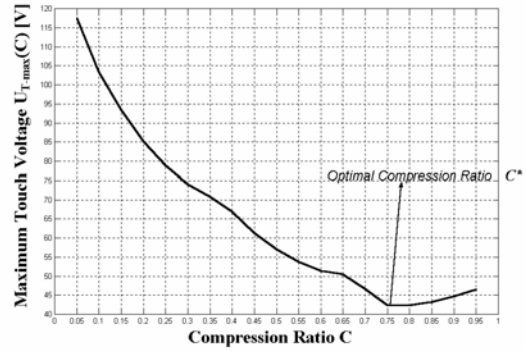

Fig. 4 Maximum touch voltages versus the compression ratio C

It is now clear that, in each family of grounding grids, there is a value of compression ratio C^*, such that the maximum touch voltage is minimum (see Fig.4). In order to minimize $U_{T-max}(C)$ in $]0,1[$ the Hooke-Jeeves algorithm has been applied (see [19], [20] and [21]). Briefly the algorithm consists of the following. The algorithm starts with a "base point" C =0.5 and an "initial pattern size" h=0.1. In the first phase of the algorithm, called the "exploratory move", the function is sampled at successive perturbations of the base point in the search directions. The sampling is managed by first evaluating $U_{T-max}(C +h)$ and only testing $C -h$ if $U_{T-max}(C +h) \geq U_{T-max}(C)$. The exploratory move will either produce a new base point C' or fail, meaning no more performing points are found. In the second phase of the algorithm, called "pattern move", rather than centre the following exploration at the new current best point, which would use some of the points that were examined in the previous exploration, the Hooke-Jeeves algorithm tries to move further. The algorithms centres the following exploratory move at $C+2h = C' + h$. If this second exploratory move fails to improve U_{T-max} , then an exploratory move with C' as the centre is tried. If that fails h is reduced (in our case halved) and the process is started over. The algorithm is stopped when h is smaller than a predetermined value.
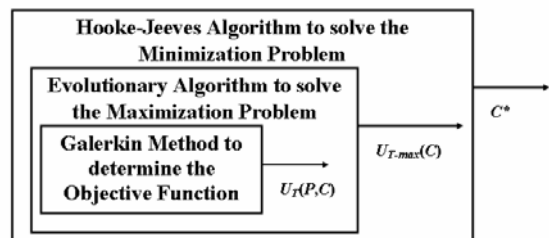Fig. 5 shows a graphical representation of the HEDA.

Fig. 5 Graphical representation of the HEDA

5 Numerical Results

The designing method proposed has been applied to various meshed square grounding grids and good results coinciding with the studies found in literature have been obtained (see [5], [6] and [14]). In this paper a meshed square grounding grid automatically designed by means of the method proposed is given as an example.

Table 1 and Table 2 give the parameters of the problem and the set of parameters chosen for the evolutionary algorithm, respectively. Fig. 6 shows the automatically designed grounding grid for a prefixed number N_c=8 of conductors for each side.

Table 1. Parameters of the problem

Grounding grid parameters	Symbols	Values
Side length	L_s	50 m
Cross-section of the conductors	S	50 mm^2
Burying depth	d	0.5 m
Resistivity of the soil	ρ	100 Ωm
Fault current	I_F	400 A

Table 2. Parameters of the evolutionary algorithm

Parameters	Symbols	Values
Size of the population	N_{ind}	100
Number of cross-over	N_{cr}	50
Mutation probability	P_m	0.1
Maximum number of iterations	$N_{iter-max}$	80
Index of accuracy	ϵ	0.01 V

Under these hypotheses the optimal compression ratio C^* has been found equal to 0.7151 and the corresponding maximum touch voltage value of U_{T-max} is 65.63 V. Fig. 6 shows the results of the designing method.
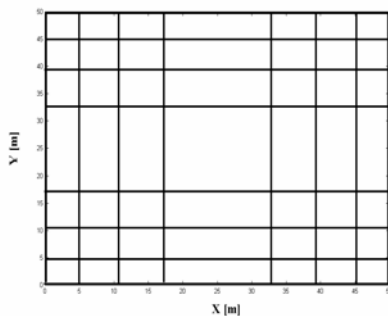

Fig. 6 Grounding grid designed by the HEDA

6 Discussion Around the Grounding Grid Designing Methods

The problem of grounding grid design has been discussed over the years using different approaches. The methods present in literature can be distinguished in two groups:

1) methods based on the compression ratio (see [5] [6] [14] [17] [18])

2) methods based on the optimization of an objective function without structural assumptions (see [3], [4], [7] and [8])

The 1) have the advantage of working in a continuous decision space ( ]0,1[ ), instead of a discrete one used by the 2), and therefore these methods allow us to consider an infinite set of the configurations of grounding grid. Besides, the 1) allow us to control the structural configuration working just with one parameter and therefore are computationally less expensive than 2). On the other hand the 1) assume that the conductors are arranged according to an exponential regularity and, due to this assumption, are less general than the 2).

Within the 1) this paper has the merit to propose a formal criterion for the determination of the optimal compression ratio. Instead of the empirical approaches given in literature, an efficient objective function (i.e. the maximum touch voltage U_{T-max} (C)) has been built and a deterministic optimization method has been implemented.

In [12] it is shown the design of a grounding grid whose size is 100 m x 60 m and made up of 7 conductors parallel at the long side and 11 parallel at the short side, the following set of parameters characterizing the problem:

ρ =100 Ω m d=0.5 m I_F=1 kA R_c=0.52 cm

where R_c is the radius of each conductor. In [14], for simplicity, the hypothesis that can be used only one compression ratio for the rectangular grounding grid has been done. Under the same hypothesis and the same parameters the method here proposed has been implemented. Table 3 shows the comparison of the results in terms of absolute value and Ground Potential Rise (GPR).

Table 3 Comparison of the results

Method	C^*	U_{T-max}	
		[V]	% GPR (599.3 V)
[14]	0.6	105.4768	17.60
HEDA	0.6613	93.2860	15.56

7 Conclusion

The HEDA is a completely automatic algorithm which allows to design square (or rectangular with single compression ratio) grounding grids. The HEDA optimizes the arrangement of the conductors in order to obtain the safest topology for a given grounding area and a given amount of conductor material. The method can be easily used also in the case of layered soil. The results show that a proper choice of the compression ratio can improve significantly the safety condition of a power system without increasing the cost of the grounding grid (see Table 3). The Authors think that a possible future development of this research can be formalizing a new model to design optimal rectangular meshed grounding grids by taking into account two compression ratios, according to the x and y directions, and their effect on the maximum touch voltages.

References:
[1] IEEE Standard 80 – 2000, *IEEE Guide for Safety in AC Substation Grounding.*

[2] L. Huang,, X. Chen, H. Yan, Study of Unequally Spaced Grounding Grids. *IEEE Trans. on Power Delivery*, Vol. 10, No 2: 1995, pp. 716-722.

[3] A. F. Otero, J. Cidras, C. Garrido, Genetic Algorithm Based Method for Grounding Grids Design, *Proc. IEEE International Conference on Evolutionary Computation.* 1998, pp. 120-123.

[4] M. C. Costa, M.L.P. Filho, Y. Marechal, J. L. Coulomb, J. R. Cardoso, Optimization of Grounding Grids by Response Surface and Genetic Algorithms. *IEEE Trans. on Magnetics*, Vol. 39, No 3, 2003, pp. 1301-1304.

[5] W. Sun, J. He, Y. Gao, R. Zeng, W. Wu, Q. Su, Optimal Design Analysis of Grounding Grids for Substations built in Non-Uniform Soil, *Proc. Intern. Conf. on Power System Technology*, Vol.3, 2000, pp. 1455-1460.

[6] S. S. El-Dessouky, M.A. El Aziz, A. Khamis,. An Accurate Design of Substation Grounding System Aid Expert System Methodology, *Conf. Record IEEE International Symposium on Electrical Insulation*, Vol. 2, 1998 pp. 411-414.

[7] F. Neri, A New Evolutionary Method for Designing Grounding grids by Touch Voltage Control, *Proc. IEEE Intern. Symp. on Industrial Electronics,* Vol. 2, 2004, pp. 1501-1505.

[8] F. Neri, A. Kononova, G. Delvecchio, M. Sylos Labini, A. Uglanov, A Hierarchical Evolutionary Algorithm with Noisy Fitness in Structural Optimization Problems in F. Rothlauf et al. eds. *Applications of Evolutionary Computation, LNCS*, Vol. 3449, pp. 610-616, Springer, 2005.

[9] M. Sylos Labini, A. Covitti, G. Delvecchio, C. Marzano, A Study for Optimizing the Number of Subareas in the Maxwell's Method. *IEEE Trans. on Magn.*, Vol. 39 No. 3, 2003, pp. 1159-1162.

[10] M. El Sherbiny, Efficient Method for Calculating the Grounding Resistance of a Buried Grounding Grid in a Homogeneous Soil, *Proc. Canadian Conf. on Electrical and Computer Eng.*, Vol.1, 1994, pp. 136-139.

[11] Y. L. Chow, M. M- Elsherbiny, M.M.A. Salama, Surface Voltages and Resistance of Grounding Systems of Grid and Rods in Two-Layer Earth by the Rapid Galerkin's Moment Method, *IEEE Transactions on Power Delivery*, Vol. 12, No. 1, 1997, pp. 179-185.

[12] Y. L. Chow, M. M- Elsherbiny, M.M.A. Salama, Efficient Computation of Rodbed Grounding Resistance in a Homogeneous Earth by Galerkin's Moment Method, IEE Proceedings on Generation, Transmission and Distribution, Vol. 142, No. 6, 1995, pp. 653-660.

[13] Yuan, J., Yang, H., Zhang, L., Cui, X. & Ma, X. 2000. Simulation of Substation Grounding Grids with Unequal potential. *IEEE Trans. of Magnetics*, Vol 36 No 4: 1468-1471.

[14] L. Hyong-Soo, K. Jung-Hoon, F. P. Dawalibi, J. Ma, Efficient Ground Grids Design in Layered Soils, *IEEE Trans. on Power Delivery*, Vol.13, No 3, 1998 pp. 745-751.

[15] Z. Michalewicz, C. Z. Janikow, Handling Constraints in Genetic Algorithms, *Proc. 4th Intern. Conf. on GAs*, 1991, pp. 151-157.

[16] Z. Michalewicz, *Genetic Algorithms + Data Structures = Evolutionary Programs,* Springer-Verlag, 1992.

[17] M. Sylos Labini, M. Castellitti, G. Delvecchio, F. Neri, A New Method for Designing Meshed Square Grounding Grids by Genetic Algorithms and Optimal Compression Ratio, *Proc. 8th Intern. Workshop on Optimisation and Inverse Problems in Electrical Eng.*, 2004, pp.57-58.

[18] J. He, Y. Gao, R. Zeng, W. Sun, J. Zou, Z. Guan, Optimal Design of Grounding Systems Considering the Influence of Seasonal Frozen Soil Layer, *IEEE Trans. on Power Delivery*, Vol. 20, No 1, 2005, pp. 107-115.

[19] R. Hooke, T. A. Jeeves, Direct Search Solution of Numerical and Statistical Problems, *Journal of ACM*, Vol. 8, 1961, pp. 212-229.

[20] F. Kaupe Jr., Algorithm 178: Direct Search, *Comm. of ACM*, Vol. 6, 1963, pp. 313-314.

[21] C. T. Kelley, *Iterative Methods of Optimization*, SIAM, 1999.