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A new position controller: Pascal's Cartesian Controllers

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Abstract: The paper's main objective is to propose a new controller for robot manipulators on Cartesian Coordinates with formal stability proof. To verify the proposed controller's behavior we need to compare it against the Cartesian PD controller, this comparison is accomplish by means of the Performance Index method that is an advantage to obtain a Scalar value of the sum of the error.

Key–Words: Cartesian Controller, Jacobian Transposed Controller, DRILL-BOT, Energy Shaping, Artificial Potential Energy, Performance Index.

1 Introduction

This work is focused in the Position Control for robots manipulators using Cartesian Controllers, because the robot manipulators move freely in their work space which is interpreted by the user like Cartesian Space, the goal of position control is to move the manipulator's end-effector from initial position q_0 to a fixed desired target q_d (constant in time). The Joint Control is used to determine the characteristics of the Cartesian Control using the Jacobian Transposed Matrix $J(q)^T$, contribution made by S. Arimoto in 1981, eliminating the possible singularity [1, 2]. The Robot Manipulators offer interesting theoretical and practical challenges to control researchers due to nonlinear and multivariable nature of their dynamical behavior. From a practical point of view, the real time implementation of robot controllers can be an expensive project and a time consuming activity if an adequate test system is not available [3]. A great amount of works in Cartesian control algorithms for robot manipulators illustrate their results by simulations and only a few have been accomplish whit experimental results [3]. In this work we describe a prototype for research and development of robot Cartesian control algorithms with open architecture which allows the development and easily experimental test of Cartesian control strategies on a servomotor Cartesian robot manipulator with three degrees of freedom. Beside experimental system, we present a theoretical result, we propose a particular case of nonlinear Cartesian controller for position control. This controller preserves global asymptotic stability of the closed loop system, it's supported by a rigorous stability analysis including the full Lagrangian analysis. This paper is organized as follows: Section 2 describes the dynamics of rigid robots and its main property. In Section 3 we describe a proposed controller, the control problem formulation and the main stability analysis. The experimental system description and experimental results on a three degrees of freedom in Section 4. Finally, we offer some concluding remarks in Section 5.

2 Robot Dynamics

For Cartesian Control design purposes, and to design better controllers, it is necessary to reveal the dynamic behavior of the robot via a mathematical model obtained from basic physical laws. We use Lagrangian Dynamics [4] to obtain the describing mathematical equations. We begin our development with the general Lagrange equation of motion [1, 5, 6, 7, 8]. Consider then Lagrange's equations for a conservative system as given by:

$$\frac{d}{dt}\left[\frac{\partial \mathcal{L}(q,\dot{q})}{\partial \dot{q}}\right] - \frac{\partial \mathcal{L}(q,\dot{q})}{\partial q} = \tau - f(\tau,\dot{q}) \qquad (1)$$

where $q, \dot{q} \in \mathrm{R}^{n\times 1}$ are vectors of joint displacements and velocities respectively, $f(\tau_x, \dot{x}) \in \mathrm{R}^{n\times 1}$ is the friction vector and the Lagrangian $\mathcal{L}(q,\dot{q})$ is the difference between the kinetic and potential energies [1, 5, 6, 7, 8, 9],

$$\mathcal{L}(q,\dot{q}) = \mathcal{K}(q,\dot{q}) - \mathcal{U}(q). \qquad (2)$$

It is well known that in the absence of friction and other disturbances, the dynamics of a serial n-link rigid robot can be written as [10, 11, 12]:

$$M(q)\ddot{q} + C(q,\dot{q})\dot{q} + g(q) = \tau \qquad (3)$$

where $q, \dot{q}, \ddot{q} \in \mathrm{R}^{n\times 1}$ are vectors of joint displacements, velocities and acceleration respectively, $M(q) \in \mathrm{R}^{n\times n}$ is the symmetric positive definite manipulator inertial matrix, $C(q,\dot{q}) \in \mathrm{R}^{n\times n}$ is the matrix of Centripetal and Coriolis torques and $g(q) \in \mathrm{R}^{n\times 1}$ is the vector of gravitational torques obtained as the gradient of the robot potential energy. Inverse kinematics is one of the functions basic to robot manipulator control systems. Cartesian position and orientation x of the end-effector is described as a function f of the joint variable q [13], $x = f(q)$, and various approaches to solve the inverse problem have been introduced, either by determining f^{-1} symbolically, q = $f^{-1}(x)$, or by utilizing the partial derivation of $\dot{x} = J(q)\dot{q}$, we obtained to the inverse Jacobian matrix, $\dot{q} = J(q)^{-1}\dot{x}$. After some operations we can relate the Joint space with the Cartesian space, obtaining the table 1.

Table 1: Joint Coordinated to Cartesian

Joint Coordinated	Cartesian Coordinated
$\dot{q} = J(q)^{-1}\dot{x}$	$\dot{x} = J(q)\dot{q}$
$\ddot{q} = J^{-1}\ddot{x} - J^{-1}\dot{J}J(q)^{-1}\dot{x}$	$\ddot{x} = J(q)\ddot{q} + \dot{J}(q)\dot{q}$

Whereas the Hamilton system and the vector of generalized momenta $\rho = [\rho_1, \ldots, \rho_k]^T$, defined for any Langrangian $\mathcal{L}(q,\dot{q})$ as $\rho = \dfrac{\partial \mathcal{L}(q,\dot{q})}{\partial \dot{q}}$ [14], is simply given by:

$$\rho = M(q)\dot{q}, \qquad (4)$$

and by defining the state vector, $[q_1, \ldots, q_k, \rho_1, \ldots, \rho_k]^T$, the k second order equation (1) transform into $2k$ first-order equations:

$$\left[\frac{\partial \mathcal{H}(q,\rho)}{\partial \rho}\right] = \frac{\partial \mathcal{K}(q,\dot{q})}{\partial \rho} + \frac{\partial \mathcal{U}(q)}{\partial \rho} = M(q)^{-1}\rho = \dot{q}$$
$$\dot{\rho} = -\frac{\partial \mathcal{H}(q,\rho)}{\partial q} + \tau \qquad (5)$$

where $\mathcal{H}(q,\rho)$ is the total energy of the system. The equations (5) are called the *Hamiltonian equations of Motion* [14]. The following energy balance immediately follows from (5):

$$\frac{d\mathcal{H}(q,\rho)}{dt} = \left(\frac{\partial \mathcal{H}(q,\rho)}{\partial q}\right)^T \dot{q} + \left(\frac{\partial \mathcal{H}(q,\rho)}{\partial \rho}\right)^T \dot{\rho}. \qquad (6)$$

given the result [14]:

$$\mathcal{W} = \frac{d\mathcal{H}(q,\rho)}{dt} = \dot{q}^T\tau. \qquad (7)$$

Expressing that the increase in energy of the system is equal to the supplied work (conservation of energy). When forces act on a mechanism, work (in the technical sense) is done if the mechanism moves through a displacement [15]. Work is defined as a force acting through a distance and is a scalar with units of energy. Since work has units of energy it must be the same measured in any set of generalized coordinates. Specifically, we can equate the work done in Cartesian terms with the work done in joint space terms [15]. In the multidimensional case, work is the dot product of a vector force or torque and a vector displacement [15].

$$\mathcal{W} = \mathcal{F}\dot{x} \qquad (8)$$

to relate equation $\dot{x} = J(q)\dot{q}$, (7) and (8) we obtained:

$$\tau = J(q)^T\mathcal{F} \qquad (9)$$

where τ is the vector of applied torques, $J(q)$ is the Jacobian Matrix and $\mathcal{F}$ is the Force applied at the end-effector. The equation (9) is called Jacobian Transposed Controller [2]. Replacement the Jacobian Transposed Controller, equation (9), on the Dynamic Model, equation (3), and using the equations described in the Table 1, we obtain:

$$M(x)\ddot{x} + C(x,\dot{x})\dot{x} + g(x) = \tau_x, \qquad (10)$$

where:

$$\begin{aligned} M(x) &= J(q)^{-T}M(q)J(q)^{-1} && (11)\\ C(x,\dot{x}) &= J^{-T}[CJ^{-1} - MJ^{-1}\dot{J}J^{-1}] && (12)\\ g(x) &= J(q)^{-T}g(q) && (13)\\ \tau_x &= \mathcal{F} && (14) \end{aligned}$$

we obtained a Dynamic Model representation on Jacobian Transposed terms. It is important to keep in mind that we assume that the manipulator's end-effector interacts with an infinitely stiff environment hence, its motion is constrained to a smooth $(n-m)$ dimensional submanifold Φ, defined by $\phi(q) = 0$ where the function $\phi : \mathrm{R}^n \to \mathrm{R}^m$ is at least twice continuously differentiable this way we assume that there exists an operating region $\Omega \subset \mathrm{R}^n$ defined as $\Omega = \Omega_1 \times \Omega_2$, where Ω_1 is a convex subset of R^{n-m}, Ω_2 is an open subset of R^m. We also assume the existence of a function $k_1 : \Omega_1 \to \mathrm{R}^m$ twice continuosly differentiable, $k_1 \in \mathcal{C}^2$, such as $\phi(q^1, k(q^1)) = 0$ for all $q^1 \in \Omega_1$. Under these conditions, the vector q^2 can be uniquely defined by the vector q^1 such that $q^2 = k(q^1)$ for all $q^1 \in \Omega_1$. Notice that under this assumption the Jacobian $J(q)$ is non singular only $\forall\ q \ \in \Omega$ that is to say $J(q)^{-1} \exists \forall\ q \ \in \Omega$ [16]. Although the equation of motion (10) is complex, it has several fundamental properties which can be exploited to facilitate control system design. We use the following important properties:

Property 1 *Considering all revolute joints, the inertial matrix $M(x)$ is lower and upper bounded by [14]:*

$$\mu_1(x)I \leq M(x) \leq \mu_2(x)I \tag{15}$$

where I stands for the $m \times n$ Identity matrix. We should consider that $M(x)$ it is symmetric positive definite inertial matrix because this defined in the way $Q^T AQ$ where $Q = J(q)^{-1}$ and A is symmetrical matrix [17].

Property 2 *The matrix $\dot{x}^T[\dot{M}(x) - 2C_m(x, \dot{x})]\dot{x} \equiv 0$ is skew-symmetric, that is [14],*

$$\dot{M}(x) = C(x, \dot{x}) + C(x, \dot{x})^T. \tag{16}$$

Furthermore, the matrix $C(x, \dot{x})$ is linear on $\dot{x}$ and bounded on x, hence for some $k_c \in \mathrm{R}_+$ [14]:

$$\|C(x, \dot{x})\| \leq k_c(x)\|\dot{x}\|. \tag{17}$$

Property 3 *The generalized gravitational forces vector*

$$g(x) = \frac{\partial \mathcal{U}(x)}{\partial x} \tag{18}$$

satisfies [14]:

$$\left\| \frac{\partial g(x)}{\partial x} \right\| \leq k_g \tag{19}$$

for some $k_g \in \mathrm{R}_+$, where $\mathcal{U}(x)$ is the potential energy expressed in the cartesian space and is supposed to be bounded from below [14].

3 Cartesian Controllers

In this section we present our main result concerning the stability analysis of the proposed Cartesian controllers. Now we are in position to formulate the Cartesian control problem. Typically we propose controllers using the *Energy Shaping* on Joint Coordinates [3, 10, 11, 12, 14, 18, 19, 20], now we use this methodology on Cartesian Space. The *Energy Shaping* is a controller method design, this method considerate the Dynamic Model without friction and others disturbances [9, 10, 11, 12, 18, 19, 20, 21]. We use the following Cartesian control scheme:

$$\tau_x = \nabla \mathcal{U}(k_p, \tilde{x}) - f_v(k_v, \dot{x}) + g(x) + f(\tau_x, \dot{x}) \tag{20}$$

where $\tilde{x}$ is the position error in Cartesian coordinates, $\mathcal{U}(k_p, \tilde{x})$ is the *Artificial Potential Energy* described by:

$$\mathcal{U}(k_p, \tilde{x}) = \frac{f(\tilde{x})^T k_p f(\tilde{x})}{2} \tag{21}$$

and the term $f_v(k_v, \dot{x})$ is the Derivative Action. We use the following Lyapunov scheme:

$$V(\dot{x}, \tilde{x}) = \frac{\dot{x}^T M(x)\dot{x}}{2} + \mathcal{U}(k_x, \tilde{x}). \tag{22}$$

The Energy Shaping Methodology consist in found a $\mathcal{U}(k_x, \tilde{x})$ function to fulfill the next Lyapunov's conditions:

$$\begin{array}{ll} V(0,0) = 0 & \forall\ \dot{x}, \tilde{x} = 0 \\ V(\dot{x}, \tilde{x}) > 0 & \forall\ \dot{x}, \tilde{x} \neq 0 \end{array} \tag{23}$$

and to do the derivation of the Lyapunov equation [20] we obtain,

$$\dot{V}(\dot{x}, \tilde{x}) = \dot{x}^T M(x)\ddot{x} + \frac{\dot{x}^T \dot{M}(x)\dot{x}}{2} - \frac{\partial \mathcal{U}(k_p, \tilde{x})^T}{\partial \tilde{x}}\dot{x}, \tag{24}$$

fulfill the condition:

$$\dot{V}(\dot{x}, \tilde{x}) \leq 0, \tag{25}$$

verify asymptotical stability with LaSalle theorem:

$$\dot{V}(\dot{x}, \tilde{x}) < 0. \tag{26}$$

Consider the next cartesian controllers schemes.

3.1 Cartesian PD Controller

$$\tau_x = J^T[K_p\tilde{x} - K_v\dot{x}] + g(x) + f(\tau_x, \dot{x}) \tag{27}$$

where $\tilde{x}$ denotes the position error on Cartesian Coordinates, K_p, K_v are the proportional and derivative gains. The control problem can be stated as that of selecting the design matrices K_p and K_v such that the position error $\tilde{x}$ vanishes asymptotically, i.e. $lim_{t\to\infty}\tilde{x}(t) = 0 \in \mathrm{R}^n$.

The closed-loop system equation obtained by combining the Cartesian robot model, equation (10), and control scheme, equation (27), can be written as:

$$\frac{d}{dt}\begin{bmatrix} \tilde{x} \\ \dot{x} \end{bmatrix} = \begin{bmatrix} -\dot{x} \\ M(x)^{-1}\left[K_p\tilde{x} - K_v\dot{x} - C(x, \dot{x})\dot{x}\right] \end{bmatrix} \tag{28}$$

which is an autonomous differential equation and the origin of the state space is its unique equilibrium point. To carry out the stability analysis of equation (28), we proposed the following Lyapunov function candidate based on the Energy Shaping Methodology [12, 20] oriented on Cartesian space:

$$V(\dot{x},\tilde{x}) = \frac{\dot{x}^T M(x)\dot{x}}{2} + \frac{\tilde{x}^T K_p \tilde{x}}{2}. \tag{29}$$

The first term of $V(\dot{x},\tilde{x})$ is a positive definite function with respect to $\dot{x}$ because $M(x)$ is a positive definite matrix. The second one of Lyapunov function candidate (29) is a positive definite function with respect to position error $\tilde{x}$, because K_p is a positive definite matrix. Therefore $V(\dot{x},\tilde{x})$ is a globally positive definite and radially unbounded function.

The time derivative of Lyapunov function candidate (29) along the trajectories of the closed-loop (28),

$$\dot{V}(\dot{x},\tilde{x}) = \dot{x}^T M(x)\ddot{x} + \frac{\dot{x}^T \dot{M}(x)\dot{x}}{2} + \tilde{x}^T K_p \dot{\tilde{x}} \tag{30}$$

and after some algebra and using the property 2 it can be written as:

$$\dot{V}(\dot{x},\tilde{x}) = -\dot{x}^T K_v \dot{x} \leq 0, \tag{31}$$

which is a globally negative semidefinite function and therefore we conclude stability of the equilibrium point. In order to prove asymptotic stability we exploit the autonomous nature of closed-loop (28) to apply the *LaSalle Invariance Principle*:

$$\dot{V}(\dot{x},\tilde{x}) < 0. \tag{32}$$

In the region:

$$\Omega = \left\{ \begin{bmatrix} \tilde{x} \\ \dot{x} \end{bmatrix} \in \mathrm{R}^n : V(\tilde{x},\dot{x}) = 0 \right\} \tag{33}$$

the unique invariant is $\begin{bmatrix} \tilde{x}^T & \dot{x}^T \end{bmatrix}^T = 0 \in \mathrm{R}^{2n}$.

3.2 Pascal's Cartesian Controller

$$\tau_x = K_p \psi_{\tilde{x}} - K_v \psi_{\dot{x}} + g(x) + f(\tau_x, \dot{x}) \tag{34}$$

where $\tilde{x}$ denotes the position error on Cartesian Coordinates, K_p, K_v are the proportional and derivative gains, and $\psi_{\tilde{x}} = \tanh(\tilde{x}) \sqrt[2j]{1+\tanh^{2j}(\tilde{x})}$, $\psi_{\dot{x}} = \tanh(\dot{x}) \sqrt[2j]{1+\tanh^{2j}(\dot{x})}$.

The closed-loop system equation obtained by combining the cartesian robot model, equation (10), and control scheme, equation (34), can be written as:

$$\frac{d}{dt}\begin{bmatrix} \tilde{x} \\ \dot{x} \end{bmatrix} = \begin{bmatrix} -\dot{x} \\ M(x)^{-1}\left[K_p \psi_{\tilde{x}} - K_v \psi_{\dot{x}} - C(x,\dot{x})\dot{x}\right] \end{bmatrix} \tag{35}$$

which is an autonomous differential equation and the origin of the state space is its unique equilibrium point. Basing us on the **Pascal's triangle** and the next trigonometrical hyperbolic function,

$$\cosh^2(x) + \mathrm{senh}^2(x) = 2\cosh^2(x) - 1 \tag{36}$$

we solve the terms inside the radical, giving the following triangle:

$$\begin{array}{ccccccccccc}
 & & & & 2 & -1 & & & & & \\
 & & & 2 & 1 & & -2 & & & & \\
 & & 2 & -1 & & 3 & & -3 & & & \\
2 & 1 & & & -4 & & 6 & & -4 & &
\end{array} \tag{37}$$

Inside the radical we have:

$$\begin{array}{c}
2\cosh^2(x) - 1 \\
2\cosh^4(x) + 1 - 2\cosh^2(x) \\
2\cosh^6(x) - 1 + 3\cosh^2(x) - 3\cosh^4(x)
\end{array} \tag{38}$$

To carry out the stability analysis of equation (35), we proposed the following Lyapunov function candidate based in the Energy Shaping Methodology [12, 20] oriented on cartesian space:

$$V(\dot{x},\tilde{x}) = \frac{\dot{x}^T M(x)\dot{x}}{2} + \begin{bmatrix} \sqrt{\ln(\cosh(\tilde{x}_1))} \\ \sqrt{\ln(\cosh(\tilde{x}_2))} \\ \vdots \\ \sqrt{\ln(\cosh(\tilde{x}_n))} \end{bmatrix}^T , \quad K_p \begin{bmatrix} \sqrt{\ln(\cosh(\tilde{x}_1))} \\ \sqrt{\ln(\cosh(\tilde{x}_2))} \\ \vdots \\ \sqrt{\ln(\cosh(\tilde{x}_n))} \end{bmatrix} \tag{39}$$

the first term of $V(\dot{x},\tilde{x})$ is a positive define function with respect to $\dot{x}$ because $M(x)$ is a positive definite matrix. The second one of Lyapunov function candidate (39) is a positive definite function with respect to position error $\tilde{x}$, because K_p is a positive define matrix. Therefore $V(\dot{x},\tilde{x})$ is a globally positive definite and radially unbounded function. The time derivative of Lyapunov function candidate (39) along the trajectories of the closed-loop (35),

$$\dot{V}(\dot{x},\tilde{x}) = \dot{x}^T M(x)\ddot{x} + \frac{\dot{x}^T \dot{M}(x)\dot{x}}{2}$$
$$+ \begin{bmatrix} \sqrt{\ln(\cosh(\tilde{x}_1))} \\ \sqrt{\ln(\cosh(\tilde{x}_2))} \\ \vdots \\ \sqrt{\ln(\cosh(\tilde{x}_n))} \end{bmatrix}^T K_p \left[\frac{\tanh\tilde{x}}{\sqrt{\ln(\cosh(\tilde{x}))}} \right] \dot{\tilde{x}} \tag{40}$$

and after some algebra and using the property 2 it can be written as:

$$-\dot{x}^T K_v \begin{bmatrix} \tanh(\dot{x}_1)\sqrt[2j]{1+\tanh^{2j}(\dot{x}_1)} \\ \tanh(\dot{x}_2)\sqrt[2j]{1+\tanh^{2j}(\dot{x}_2)} \\ \vdots \\ \tanh(\dot{x}_n)\sqrt[2j]{1+\tanh^{2j}(\dot{x}_n)} \end{bmatrix} \leq 0. \tag{41}$$

which is a globally negative semidefinite function and therefore we conclude stability of the equilibrium point. In order to prove asymptotic stability we exploit the autonomous nature of closed-loop (35) to apply the *LaSalle invariance principle*:

$$\dot{V}(\dot{x},\tilde{x}) < 0. \tag{42}$$

In the region

$$\Omega = \left\{ \begin{bmatrix} \tilde{x} \\ \dot{x} \end{bmatrix} \in \mathrm{R}^n : V(\tilde{x},\dot{x}) = 0 \right\} \tag{43}$$

the unique invariant is $\begin{bmatrix} \tilde{x}^T & \dot{x}^T \end{bmatrix}^T = 0 \in \mathrm{R}^{2n}$.

4 Experimental System Description

We have designed and build an experimental system for research of Cartesian robot control algorithms and currently it is a turn key research system for developing and validation of Cartesian control algorithms for robot manipulators. The experimental system is a servomotor robot manipulator with three degrees of freedom moving in the three dimensional space as it is shown in the figure 3.

The structure are made of stainless iron, direct drive shaft with servomotors from Reliance Electronics. Advantages of this type of drive shaft include high torque. The servomotor has an Incremental Encoder from Hewlett Packard.

Plotting the terms of the radical has:

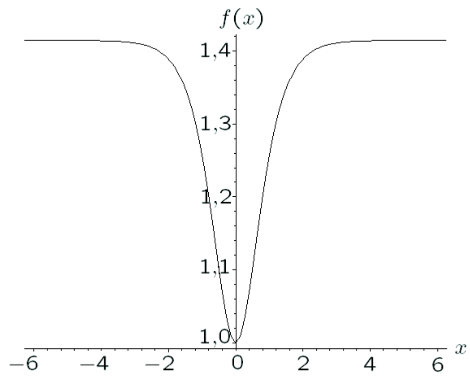

Figure 1: Radical terms Graphic.

When multiplying for the function tanh we modify the graph in the following way:

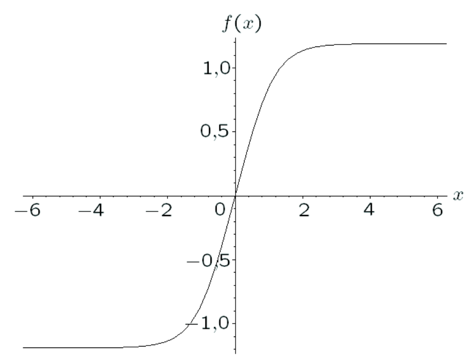

Figure 2: Complete behavior.

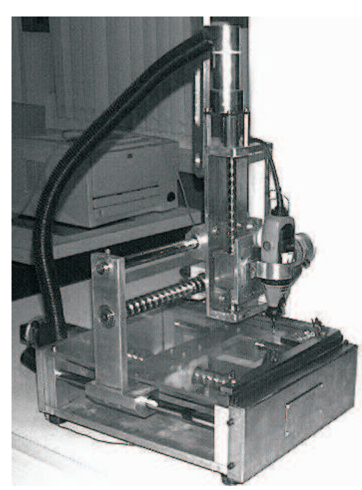

Figure 3: Experimental Prototype. ”DRILL-BOT’

The motors used in the experimental cartesian robot are the model $E450$ $[450oz - in.]$. The servos are operated in torque mode, so the motors acts a reference if torque signal. Position information is obtained from incremental encoders located on the motors, which have a resolution of $1024000\ p \setminus rev$.

4.1 Experimental Results

To support our theoretical developments, this section presents an experimental comparison of two position controllers on Cartesian Coordinates on three degrees of freedom Cartesian robot manipulator. To investigate the performance among controllers, they have been classified as τ_{PD} for the simple PD controller and τ_{Pascal} represent our propose controller, both on Cartesian space. The experimental comparison consists in finding which is the better performance among evaluated controllers by using the scalar-valued $\mathcal{L}^2$ norm. A smaller $\mathcal{L}^2$ represents lesser position error and thus is the better performance [22, 23, 24]. A position control experiment has been designed to compare the performances of the controllers on a Cartesian robot. The experimental consist on moving the manipulator's end-effector from its initial position to desired position. For the present application the desired cartesian positions were chosen as: $[x_{d_1}, x_{d_2}, x_{d_3}]^T = [0.785, 0.615, 0.349]^T$ [meters], where x_{d_1}, x_{d_2} and x_{d_3} represent the x, y and z axes of the prototype.

4.2 Performance Index

Robot manipulators are very complex mechanical systems, due to the nonlinear and multivariable nature of the dynamic behavior. For this reason, in the robotics community there are not well-established criterias for proper evaluation of controllers for robots. However, it is accepted in practice to compare the performance of controllers by using the scalar-valued $\mathcal{L}^2$ norm as an objective numerical measure for an entire error curve. The $\mathcal{L}^2[\tilde{x}]$ norm measures the root-mean-square average of the $\tilde{x}$ position error, which is given by:

$$\mathcal{L}^2 [\tilde{x}] = \sqrt{\frac{1}{t - t_0} \int_{t_0}^{t} \|\tilde{x}\|^2 \, dt} \qquad (44)$$

where $t_0, t \in \mathbb{R}_+$ are the initial and final times, respectively. A smaller $\mathcal{L}^2[\tilde{x}]$ represents lesser position error and it indicates the best performance of the evaluated controller. The overall results are summarized in Figure 6 which includes the performance indexes for the analyzed controllers.

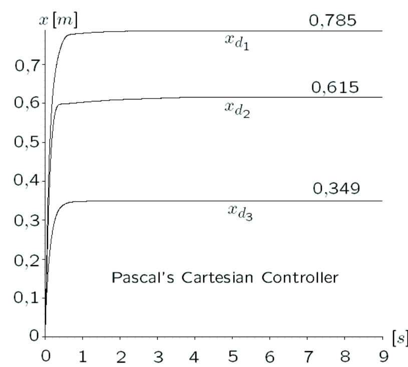

Figure 4: Pascal's Cartesian Controller Position

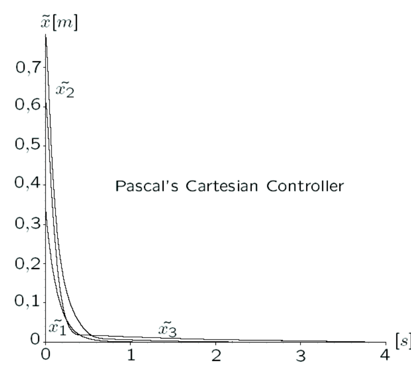

Figure 5: Pascal's Cartesian Controller Error Position

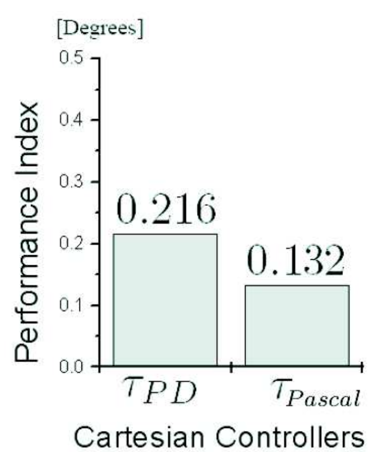

Figure 6: Performance Index.

5 Conclusion

In this paper we have described a new Cartesian Controller with formal stability proof and obtained experimental results using a Cartesian robot. The goal of the test system is to support the research as well as to develop new Cartesian control algorithms for robot manipulators. Our theoretical results are the propose of Cartesian controllers. We have shown global asymptotic stability for Lyapounov functions. Experiments on Cartesian robot manipulator have been accomplish to show the stability and performance for the Cartesian controllers. We can conclude that our controller is faster than the Cartesian PD Controller.

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