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Optimal control related to effects of microwave heating on thermal states of biological bodies

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Abstract: This paper considers the nonlinear optimal control governed by a Pennes transient bioheat transfer type model with Robin boundary conditions. The control estimate the blood perfusion rate and the heat transfer parameter, which affect the effects of thermal physical properties on the transient temperature of biological tissues. The result can be very beneficial for thermal diagnostics in medical practices. Existence and the uniqueness of the solution is proved as well as stability under mild assumptions. Afterwards the optimal control problem is formulated. An optimal solution is proven to exist and, finally necessary optimality conditions are given.

Key words. Optimal criterion, microwave heating, bioheat, adjoint equation, boundary control, pointwise controllers.

1 Introduction

1.1 Statement of the problem

The goal of this contribution is the study of optimal control problems related to the effects of thermal physical properties on the transient temperature of biological tissues, using a Pennes transient bioheat transfer type model. It involves the temperature distribution u. The time evolution of u is governed by the following system

$$\begin{array}{l}\dfrac{\partial u}{\partial t} = div(\kappa(x)\nabla u) - p(u-u_a) \\ \qquad +F(x,t,u)+f, a.e.\ in\ \ \mathcal{Q}=\Omega\times(0,T), \\ \text{subjected to the boundary conditions} \\ \dfrac{\partial u}{\partial n} = q(u_{out}-u)\ \ in\ \ \Sigma=\partial\Omega\times(0,T), \\ \text{and the initial conditions} \\ u(0)=u_0\ \ in\ \ \Omega,\end{array} \tag{1}$$

under the pointwise constraints

$$\begin{array}{l} a \le p(x,t) \le b\ \ a.e.(t,x)\in\mathcal{Q}, \\ c \le q(x,t) \le d\ \ a.e.(t,x)\in\mathcal{Q}. \end{array} \tag{2}$$

where Ω is an open bounded domain in $\mathbb{R}^m$, $m \le 3$ with a smooth boundary $\partial\Omega$ of class C^∞, $\frac{\partial .}{\partial n} = (\kappa\nabla .).\mathbf{n}$, $\mathbf{n}$ being the outward normal to $\Gamma = \partial\Omega$, and a,b,c,d are given positive constants. p is the blood perfusion rate and q describes the heat transfer coefficient. The heat capacity is assumed to be constant and thermal conductivity of tissue κ is assumed to be variable and satisfies $\kappa = \sigma^2 \ge \mu > 0$. The second term in the right of equation (1) describes the heat transport between the tissue and microcirculatory blood perfusion, the third term F is the body heating fonction which describes the physical properties of material. The source term f describes the specific absorption rate, u_a is the blood temperature and u_{out} is the bolus temperature. The function u_0 is the initial value and is assumed to be in $\mathcal{C}^0(\overline{\Omega})$.

The use of microwave radiation orifor heating is now common in many industrial situations such smelting, sintering and drying; it also has many applications in interdisciplinary research areas, in joining mathematical, biological and medical fields, especially in clinical cancer therapy hyperthermia. Recently bioheat model have been the object of numerous studies (see e.g. [2, 4, 5, 6, 7, 9, 12, 13, 14, 15, 16] and the references therein). For the optimal control problems, we can mention [8] in which the authors have optimal results for bilinear elliptic bioheat transfer equation.

The new feature introduced in this works concerns the study of optimal control problems of nonlinear evolutive bioheat transfer systems, where the control acts in the state equation and in the boundary condition. The introduction of the nonlinear term F, in the bioheat system, is very important, because the physical properties of material have power law dependence on temperature (see [12]). The main result of the paper includes the existence of an optimal control and the first order necessary conditions of optimality.

1.2 Assumptions and notations

We suppose that the body heating coefficient F satisfies

(**H**) *F is a Carathéodory function from $\mathcal{Q} \times I\!R$ into $I\!R$. For almost all $(x,t) \in \mathcal{Q}$, $F(x,t,.)$ is Lipschitz and bounded function with*

(i) $| F(x,t,r) | \leq M_1,\ \forall\, r \in I\!R$ and a.e. in $\mathcal{Q}$,

(ii) F is differentiable. The partial derivative $F'_x(.,.,r)$ and $G = F'_r(.,.,r)$ are Lipschitz continuous in $\mathcal{Q}$ for all $r \in I\!R$, and are globally bounded in $\mathcal{Q} \times I\!R$.

Remark 1 *For u_l be a sequence converging toward u in $\mathcal{W}(\mathcal{Q})$ weakly and in $L^2(\mathcal{Q})$ strongly we prove easily that $F(.,.,u_l) \longrightarrow F(.,.,u)$ in $L^p(\mathcal{Q})$ strongly $\forall p \in [1,+\infty)$.*

For any pair of real numbers $r, s \geq 0$, we introduce the Sobolev space $H^{r,s}(\mathcal{Q})$ defined by $H^{r,s}(\mathcal{Q}) = L^2(0,T,H^r(\Omega)) \cap H^s(0,T,L^2(\Omega))$, which is a Hilbert space normed by

$$\left(\int_0^T \| v \|^2_{H^r(\Omega)}\, dt + \| v \|^2_{H^s(0,T,L^2(\Omega))}\right)^{1/2},$$

where $H^s(0,T,L^2(\Omega))$ denotes the Sobolev space of order s of functions defined on $(0,T)$ and taking values in $L^2(\Omega)$, and defined by $H^s(0,T,L^2(\Omega)) = [H^m(0,T,L^2(\Omega)), L^2(\mathcal{Q})]_\theta$, where $\theta \in (0,1)$, $s = (1-\theta)m$, m is an integer and $H^m(0,T,L^2(\Omega)) = \{v \in L^2(\mathcal{Q}) | \frac{\partial^j v}{\partial t^j} \in L^2(\mathcal{Q}), \forall j = 1, m\}$.

Remark 2 *Let $\Omega \subset I\!R^m$, $m \geq 1$, be an open and bounded set with a smooth boundary and q be a nonnegative integer. We have the following results (see e.g. [1])*

(i) $H^q(\Omega) \subset L^p(\Omega)$, $\forall p \in [1, \frac{2m}{m-2q}]$, with continuous embedding (with the exception that if $2q = m$, then $p \in [1,+\infty[$ and if $2q > m$, then $p \in [1,+\infty]$).

(ii) (Gagliardo-Nirenberg inequalities) There exists $C > 0$ such that

$$\| v \|_{L^p} \leq C \| v \|^\theta_{H^q} \| v \|^{1-\theta}_{L^2}, \forall v \in H^q(\Omega),$$

where $0 \leq \theta < 1$ and $p = \frac{2m}{m-2\theta q}$ (with the exception that if $q - m/2$ is a nonnegative integer, then θ is restricted to 0).

We can now introduce the following spaces: $\mathcal{H}(\mathcal{Q}) = L^\infty(0,T,L^2(\Omega)), \mathcal{V}(\mathcal{Q}) = L^2(0,T,H^1(\Omega))$, $\mathcal{W}(\mathcal{Q}) = L^2(0,T,H^1(\Omega)) \cap H^1(0,T,(H^1(\Omega))')$; it is well-known that $\mathcal{W}$ is continuously embedded in $\mathcal{C}([0,T],L^2(\Omega))$(see e.g. [10]).

The set of the admissible controls describing the constraints is

$$U_{ad} = \{(p,q) \in L^2(\mathcal{Q}) \times L^2(\Sigma) | a \leq p \leq b \text{ a.e. in } \mathcal{Q} \text{ and } c \leq q \leq d \text{ a.e. in } \Sigma\}.$$

Although U_{ad} is a subset of $L^\infty(\mathcal{Q}) \times L^\infty(\Sigma)$, we prefer to use the standard norms of the space $L^2(\mathcal{Q}) \times L^2(\Sigma)$. The reason is that we would like to take advantages of the differentiability of the latter norm away from the origin to perform our variational analysis.

Now we introduce the following objective functional

$$J(p,q) = \frac{1}{2} \| \gamma(u-u_{obs}) + \delta(p-p_{obs}) - m \|^2_{L^2(\mathcal{Q})} + \frac{\alpha}{2} \| p - p_r \|^2_{L^2(\mathcal{Q})} + \frac{\beta}{2} \| q - q_r \|^2_{L^2(\Sigma)}, \quad (3)$$

where the functions γ, δ are positives with space dependent entries and $L^\infty(\overline{\Omega})$, the function m is in $L^2(\mathcal{Q})$ and is corresponding to the online temperature control via radiometric temperature measurement system. The constants $\alpha \geq 0$ and $\beta \geq 0$ with $\alpha + \beta > 0$, are chosen as constants to establish the relative weight of the second and the third term in (3). The coefficients α and β may be interpreted as measures of the price of the control. The functions u_{obs} and p_{obs} are known offsets and are given by measurement results. The term (p_r, q_r) is a given reference data.

The paper is concerned with the following optimal control problem: Find an admissible control $(p^*, q^*) \in U_{ad}$ such that

$$(p^*,q^*) \text{ is a minimum of the cost functional } J \text{ subject to system } (1) \text{ and state constraints } (2). \quad (4)$$

Such a pair (p^*, q^*) is called an optimal solution to (or an optimal strategy pair for) the problem (4).

The rest of the paper is organized as follows. In the next section, we present the existence and the uniqueness of the solution of the problem (1) and obtain a stability result. In section 3, we study the optimal control problem corresponding to obtain the minimization of the objective functional J. The functional J is depending on the control (p, q) and the solution u in the domain Ω over the time interval under consideration $[0, T]$. We prove the existence of an optimal solution and give necessary optimality conditions. We derive the optimality system by differentiating the cost functional with respect to the control and evaluate the result at an optimal control.

2 Existence and stability results

Definition: A function $u \in \mathcal{W}$ is a weak solution of system (1) provided ($\forall v \in H^1(\Omega)$ and a.e. in $(0, T)$)

$$\begin{aligned} &< \frac{\partial u}{\partial t}, v > + \int_\Omega \kappa \nabla u . \nabla v dx \\ &\quad - \int_\Gamma q(u_{out} - u) v d\Gamma + \int_\Omega p(u - u_a) v dx \\ &\quad = \int_\Omega F(x, t, u) v dx + \int_\Omega f v dx, \\ &u(0) = u_0 \quad in \quad \Omega, \end{aligned} \tag{5}$$

here $< .,. >$ denotes the duality between $(H^1(\Omega))'$ and $H^1(\Omega)$.

Theorem 1 *(i) Let be given the initial conditions u_0 in $\mathcal{C}_0(\overline{\Omega})$ and source terms (p, q, f) in $U_{ad} \times L^2(\mathcal{Q})$. Then there exists a unique solution u in $\mathcal{W}(\mathcal{Q})$ of (1).*
(ii) Let (p_i, q_i), i=1,2 be two pairs of U_{ad}. If $u_i \in \mathcal{W}(\mathcal{Q})$ is the solution of (1) corresponding to data (p_i, q_i, u_0, f), i=1,2, then

$$\| u_1 - u_2 \|^2_{\mathcal{W}(\mathcal{Q})} \leq c(\| p_1 - p_2 \|^2_{L^2(\mathcal{Q})} + \| q_1 - q_2 \|^2_{L^2(\Sigma)}).$$

Proof. To obtain the existence of the solution we set $v = u$ and obtain some a priori estimates. The prove is completed by implementing the Galerkin method, taking advantage of the obtained estimates, using the hypothesis (H1) and using the continuous mapping from $H^{1/2+s}(\Omega)$ into $L^2(\Gamma)$, $0 < s < 1/2$, see e.g. [11] (to pass to the limit in the boundary term). To obtain the uniqueness result, we suppose that there exist two solutions u_1, u_2 of (1). Then $u = u_1 - u_2$ is solution of the following problem

$$\begin{aligned} &\frac{\partial u}{\partial t} - div(\kappa \nabla u) + pu = F(x, t, u_1) - F(x, t, u_2) \quad in \quad \mathcal{Q}, \\ &\frac{\partial u}{\partial n} = -qu \quad in \quad \Sigma, \\ &u(0) = 0 \quad in \quad \Omega. \end{aligned}$$

By using the hypothesis (H1) and the regularity of p, q we prove easily that $u_1 = u_2$ and then the uniqueness result.

To prove the estimate given in (ii), we set $p = p_1 - p_2$, $q = q_1 - q_2$ and $u = u_1 - u_2$. Then u is the solution of

$$\begin{aligned} &\frac{\partial u}{\partial t} - div(\kappa \nabla u) + p(u_2 - u_a) + p_1 u \\ &\qquad = F(x, t, u_1) - F(x, t, u_2) \quad in \quad \mathcal{Q}, \\ &\frac{\partial u}{\partial n} = q(u_{out} - u_2) - q_1 u \quad in \quad \Sigma, \\ &u(0) = 0 \quad in \quad \Omega. \end{aligned}$$

By multiplying the previous system by u and integrating over $\Omega \times (0, T)$ (by using Green's formula), this gives

$$\begin{aligned} &\frac{1}{2} \| u(.,t) \|^2_{L^2(\Omega)} + \int_0^t \| \sigma \nabla u \|^2_{L^2(\Omega)} + \int_0^t\int_\Omega p_1 u^2 dx ds \\ &\quad + \int_0^t\int_\Gamma q_1 u^2 dx ds = - \int_0^t\int_\Omega p(u_2 - u_a) u dx ds \\ &\quad + \int_0^t\int_\Omega (F(x, t, u_1) - F(x, t, u_2)) u dx ds \\ &\quad + \int_0^t\int_\Gamma q(u_{out} - u_2) u dx ds. \end{aligned}$$

According to the regularity of $u_i, i = 1, 2$, $p_i, i = 1, 2$, $q_i, i = 1, 2$, u_a and u_{out}, and to the hypothesis (H1) we have (by using Gronwall's formula)

$$\| u \|_{\mathcal{H}(\mathcal{Q}) \cap \mathcal{V}(\mathcal{Q})} \leq c(\| p \|^2_{L^2(\mathcal{Q})} + \| q \|^2_{L^2(\Sigma)}).$$

By using (5), Green's formula and the previous estimate, we can deduce the following estimate

$$\| \mathbf{u} \|^2_{\mathcal{W}(\mathcal{Q})} \leq c(\| p \|^2_{L^2(\mathcal{Q})} + \| q \|^2_{L^2(\Sigma)})$$

and then the result (ii).

3 Optimal control problem

Introduce now the following mapping $\mathcal{F} : U_{ad} \longrightarrow \mathcal{W}(\mathcal{Q})$, which maps the source term $(p, q) \in U_{ad}$ of (1) into the corresponding solution u in $\mathcal{W}(\mathcal{Q})$.

We first state and prove the existence theorem of the optimal solution.

Theorem 2 *There exists an optimal control $(p^*, q^*) \in U_{ad}$ and $u^* \in \mathcal{W}(\mathcal{Q})$ such that (p^*, q^*) is defined by (4) and $u^* = \mathcal{F}(p^*, q^*)$ is the solution of (1).*

Proof. Let (p_n, q_n, u_n) be a minimizing sequence such that: $\liminf_{n\rightharpoonup\infty} J(p_n, q_n) = \inf_{(\phi,\psi)\in U_{ad}} J(\phi, \psi)$ and u_n be the state corresponding to (p_k, q_k). Then the sequence (p_n, q_n) is uniformly bounded in U_{ad}. By using the estimate of theorem 1 we obtain u_n is uniformly bounded in $\mathcal{W}(\mathcal{Q})$. This result makes it possible to extract from (p_n, q_n, u_n) a subsequence also denoted by (p_n, q_n, u_n) and such that $(p_n, q_n) \rightharpoonup (p, q)$ weakly in U_{ad} and $u_n \rightharpoonup u$ weakly in $\mathcal{W}(\mathcal{Q})$. By using [10] we have that $u_n \longrightarrow u$ strongly in $L^2(\mathcal{Q})$.
By passing to the limit we obtain easily that u is the unique solution of (1) and that $J(p, q) \leq \liminf_{n\longrightarrow\infty} J(p_n, q_n)$. We can conclude that $J(p, q) = \inf_{(\phi,\psi)\in U_{ad}} J(\phi, \psi)$. This proves the existence of an optimal control (p, q, u).□

Now, let us derive the following optimality conditions for the solutions to Problem (4) by differentiating the cost functional J and the operator solution $\mathcal{F}$, with respect to the control (p, q) at an optimal control.

We are now going to show the differentiability result of the operator solution $\mathcal{F}$.

Proposition 1 *Given the initial conditions u_0 in $\mathcal{C}_0(\overline{\Omega})$, then the function $\mathcal{F}$ is continuously differentiable from U_{ad} to $\mathcal{W}(\mathcal{Q})$ in the following sense: $\dfrac{\mathcal{F}(p+\epsilon h, q+\epsilon k) - \mathcal{F}(p, q)}{\epsilon} \rightharpoonup w$ weakly in $\mathcal{W}(\mathcal{Q})$ as $\epsilon \longrightarrow 0$ for $(h, k) \in L^\infty(\mathcal{Q}) \times L^\infty(\Sigma)$ such that $(p+\epsilon h, g+\epsilon k) \in U_{ad}$. Furthermore, the derivative $\mathcal{F}'(p, q) : (h, k) \longrightarrow w$ is the unique solution of the following linear problem*

$$\begin{aligned}
&\frac{\partial w}{\partial t} - div(\kappa\nabla w) + h(u - u_a) \\
&\qquad +pw = G(x, t, u)w \;\; in \;\; \mathcal{Q}, \\
&\textit{subjected to the boundary conditions} \\
&\frac{\partial w}{\partial n} = k(u_{out} - u) - qw \;\; in \;\; \Sigma, \\
&\textit{and the initial conditions} \\
&w(0) = 0 \;\; in \;\; \Omega.
\end{aligned} \tag{6}$$

Proof. By using a similar argument as in the proof of theorem 1 and the regularity of (p, q, u), we can obtain the existence and uniqueness of w, the solution of (6).
Set $u = \mathcal{F}(p, q)$ and $u_\epsilon = \mathcal{F}(p + \epsilon h, q + \epsilon k) + \epsilon w_\epsilon$. According to the equations satisfied by u and u_ϵ, we have that w_ϵ satisfies the linear problem

$$\begin{aligned}
&\frac{\partial w_\epsilon}{\partial t} - div(\kappa\nabla w_\epsilon) + h(u - u_a) + (p + \epsilon h)w_\epsilon \\
&\qquad = \tfrac{F(x,t,u_\epsilon) - F(x,t,u)}{\epsilon} \;\; in \;\; \mathcal{Q}, \\
&\frac{\partial w_\epsilon}{\partial n} = k(u_{out} - u) - (q + \epsilon k)w_\epsilon \;\; in \;\; \Sigma, \\
&w_\epsilon(0) = 0 \;\; in \;\; \Omega.
\end{aligned} \tag{7}$$

By using a similar argument as in the proof of theorem 1 and the regularity of (p, q, u), we can obtain the following estimate

$$\| w_\epsilon \|^2_{\mathcal{W}(\mathcal{Q})} \leq C(\| h \|^2_{L^2(\mathcal{Q})} + \| k \|^2_{L^2(\Sigma)}),$$

for some constant independent of ϵ. It is easily follows that, by using the same arguments as the proof of theorem 2, $w_\epsilon \rightharpoonup w$ weakly in $\mathcal{W}(\mathcal{Q})$ as $\epsilon \longrightarrow 0$, where w is the unique solution of (6) and then the result of the theorem. □

Now, we give the characterization of the optimal control problem. For this, we introduce the following adjoint problem corresponding to the primal problem (1) (we denote by $u = \mathcal{F}(p, q)$).

$$\begin{aligned}
&-\frac{\partial \tilde{u}}{\partial t} - div(\kappa\nabla\tilde{u}) \\
&\qquad +\gamma(\gamma(\tilde{u} - u_{obs}) + \delta(h - p_{obs}) - m) \\
&\qquad\qquad +p\tilde{u} = G(x, t, u)\tilde{u} \; in \;\; \mathcal{Q}, \\
&\text{subjected to the boundary conditions} \\
&\frac{\partial \tilde{u}}{\partial n} = -q\tilde{u} \;\; in \;\; \Sigma, \\
&\text{and the final conditions} \\
&\tilde{u}(T) = 0 \;\; in \;\; \Omega.
\end{aligned} \tag{8}$$

Remark 3 *To prove the existence of a unique solution $\tilde{u} \in \mathcal{W}(\mathcal{Q})$, we change the variables of problem (8) by reversing the sense of time, i.e. $t := T - t$, and we apply a similar argument as in the proof of theorem 1.*

We can now give the first-order optimality conditions for the optimal control problem (4).

Theorem 3 *Let (p^*, q^*) be an optimal control and $u^* \in \mathcal{W}(\mathcal{Q})$ such that (p^*, q^*) is defined by (4) and $u^* = \mathcal{F}(p^*, q^*)$ solution of (1). Then (for $(h, k) \in L^\infty(\mathcal{Q}) \times L^\infty(\Sigma)$)*

$$\begin{aligned}
&\iint_{\mathcal{Q}} ((u^* - u_a)\tilde{u}^* + \alpha(p^* - p_r))h dx dt \\
&\qquad + \iint_{\mathcal{Q}} \delta(\gamma(u^* - u_{obs}) + \delta(p^* - p_{obs}) - m)h dx dt \geq 0, \\
&\iint_{\Sigma} (-(u_{out} - u^*)\tilde{u}^* + \beta(q^* - q_r))k dx dt \geq 0,
\end{aligned}$$

where $\tilde{u}^*$ is the solution of the adjoint problem (8), corresponding to the primal solution u^*.

Proof. Let $(p^*, q^*) \in U_{ad}$ be an optimal solution, i.e. the corresponding solution of problem (4) and $u^* = \mathcal{F}(p^*, q^*)$ the solution of problem (1). From Proposition 1 we know that $\mathcal{F}$ is differentiable. Therefore

$$J'(p,q).(h,k) = \frac{d}{d\lambda} J(p + \lambda h, q + \lambda k)|_{\lambda=0}$$
$$= \iint_Q (\gamma(u - u_{obs}) + \delta(p - p_{obs}) - m)(\gamma w + \delta h) dxdt$$
$$+\alpha \iint_Q (p - p_r) h dxdt + \beta \iint_Q (q - q_r) k dxdt,$$

where $w = \mathcal{F}'(p,q).(h,k)$ is the solution of problem (6).

Multiplying (6) by $\tilde{u}$, using Green's formula and integrating by time we obtain (according to the second and third parts of (6))

$$\iint_Q \left(-\frac{\partial \tilde{u}}{\partial t} - div(\kappa \nabla \tilde{u}) + p\tilde{u} - G(x,t,u)\tilde{u}\right) w dxdt$$
$$= -\iint_Q h(u - u_a)\tilde{u} dxdt + \iint_\Sigma k(u_{out} - u)\tilde{u} dxdt$$
$$- \int_\Omega \tilde{u}(T).w(T) dx.$$

Since $\tilde{u}$ is solution of (8) we have that

$$\iint_Q h(u - u_a)\tilde{u} dxdt - \iint_\Sigma k(u_{out} - u)\tilde{u} dxdt$$
$$= \iint_Q \gamma(\gamma(\tilde{u} - u_{obs}) + \delta(h - p_{obs}) - m) w dxdt.$$

According to the expression of $J'(p,q)$ we can deduce that

$$J'(p,q).(h,k) = \iint_Q ((u - u_a)\tilde{u} + \alpha(p - p_r)) h dxdt$$
$$+ \iint_Q \delta(\gamma(u - u_{obs}) + \delta(p - p_{obs}) - m) h dxdt$$
$$+ \iint_\Sigma (-(u_{out} - u)\tilde{u} + \beta(q - q_r)) k dxdt.$$

Since (p^*, q^*) is an optimal solution we have

$$\frac{\partial J}{\partial p}(p^*, q^*).h \geq 0 \; and \; \frac{\partial J}{\partial g}(f^*, g^*).k \geq 0,$$

and then

$$\iint_Q \delta(\gamma(u^* - u_{obs}) + \delta(p^* - p_{obs}) - m) h dxdt$$
$$+ \iint_Q ((u^* - u_a)\tilde{u}^* + \alpha(p^* - p_r)) h dxdt \geq 0, \quad (9)$$
$$\iint_\Sigma (-(u_{out} - u^*)\tilde{u}^* + \beta(q^* - q_r)) k dxdt \geq 0,$$

where $\tilde{u}^*$ is the solution of the adjoint problem (6) corresponding to the solution u^*.
This completes the proof.

4 Conclusion

In this article we have discussed the problem of estimate parameters of parabolic systems with Robin boundary conditions, which describe the bioheat equation in order to show the effects of the microwave heating on the thermal states of biological tissues.

The existence of a solution of the governing nonlinear system of equations is established and the Lipschitz continuity of the map solution is obtained. The differentiability and some properties of the map solution are proved. Afterwards, an optimal control problem has been formulated. Under suitable hypotheses, it is shown that one has existence of an optimal solution and the appropriate necessary optimality conditions for an optimal solution are derived. These conditions are obtained in a Lagrangian form.

Some numerical methods and other choices of control variables will be presented in a forthcoming paper [3].

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