

# Modifications of Intensifiers and Fuzzy Neuronal Receptive Fields: Algorithmic Developments and Applications-MIMO Case

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**Abstract:** - In this paper modifications of intensifiers have been done for the injection into the receptive field of the fuzzy neural networks. Algorithmic developments for these modifiers are carried out for single-input single-output (SISO) as well as for multi-input multi-output (MIMO) fuzzy neural networks. This work can be beneficial for applications in different fields such as image processing, pattern recognition, control engineering, etc. The effects of the modified intensifiers on the localized fuzzy receptive field strengths as well as the overall performances of the fuzzy neural networks have been studied. Simulation results have been presented using complex nonlinear dynamical system (MIMO Case study) suffering from uncertainties. Also, comparative studies with previous works have been given, exhibit improved performances using the proposed technique.

**Key Words:**-Intensifiers, Fuzzy neurons, Receptive fields, Algorithmic developments.

## 1 Introduction

Contrast intensification function have been proposed originally by Zadeh [1-4]. It is known that, at the extremes of fuzzy sets the membership function is less ambiguous. The fuzziness associated with the extreme values is minimized, so we have less difficulty in saying that the input value ( $x$ ) is a member (or not a member) of the fuzzy set. On the other hand, in case of overlapped fuzzy regions, for values that centered around the midpoint in the fuzzy truth function, it is difficult to decide to which a fuzzy set, a given input vector belongs to. This area is of maximum fuzziness. This is a region around [0.5] membership area that can belong to either fuzzy set simultaneously. It is known as "ambiguity" or "decidability". The localized receptive fields [5] of the fuzzy neural networks faces this difficulty. The receptive field is provided in sec. 2.

In this research, modifications of the intensifier have been done for single-input single-output (SISO) fuzzy neural networks. This effort has been extended to be suitable for multi-input multi-output (MIMO) case sec3. This work can be beneficial for applications in different fields such as image processing, pattern recognition, control engineering, etc. The localized fuzzy receptive field is of overlapping fuzzy region in nature. This is due to nonlinear dynamical variations of the environment. The modified intensifier is injected into the receptive field. Simulation results are given in sec.6

## 2 Fuzzy neuronal receptive Fields

The structure of FNN has three layers of compact multidimensional fuzzy neurons; each one is a T-norm of nonlinear continuous function.

The neurons in the input and hidden layers are of nonlinear types where the neurons at the output layer are of nonlinear type.

Each neuron has its center vector and width vector and the dimensions of these vectors are the same as the dimensions of the input vectors.

The average field strength of the network  $\bar{S}_i(x)$  is given by,

$$\bar{S}_i(x) = \frac{\prod_i \mu_{h_i}(x)}{\sum_H \prod_i [\mu_{h_i}(x)]} \quad (1)$$

## 3 Modifications of Contrast Intensifier – SISO Case

Modifications of the intensifier have been carried out. It has its effects on the localized receptive field of the fuzzy neural networks. Let us give the following definition first.

### Definition 1:

The bandwidth ( $N\beta$ ) is defined as:

$$N\beta = \|TP - TP'\| \quad (2)$$

iff  $\mu(TP) = \mu(TP') = 0.5$

Otherwise (i.e.  $\mu(TP) \neq \mu(TP') \neq 0.5$ ) the distance is defined as  $P\beta$  as Where, TP is defined as the point on the surface of the fuzzy membership. TP' is defined as another point on the surface of the adjacent intersected fuzzy membership,  $\|\cdot\| \equiv$  is Euclidean norm. Assume initial uniform distribution of the input space points. (See Fig. 1)

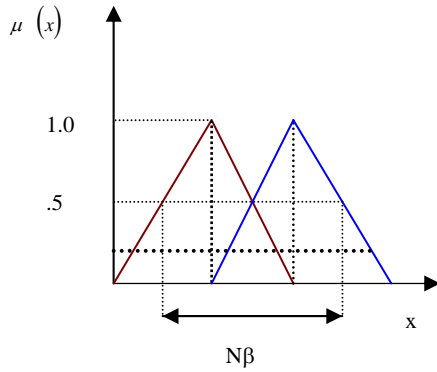


Fig.1

### Proposition 1:

Let  $U$  be a compact set where  $U = [a, b] \subset R$ . Consider a fuzzy neural network, where each FNN unit has the centroid vector and width vector  $(\beta_i, c_i) (i = 1, 2, \dots, m) \in U \subset R$ ; where  $\beta_i$  is the neuronal width  $c_i$  is the FNN neuronal centroid.

The average field strength of the network  $\bar{S}_i(x)$  is given by,

$$\bar{S}_i(x) = \frac{\prod_i \mu_{H_i}(x)}{\sum_H \prod_i [\mu_{H_i}(x)]} \quad (3)$$

Then, The FNN network has the following properties:

The average pseudo field strength  $P\bar{S}_i$  is given by

$$P\bar{S}_i(TP) = \begin{cases} \frac{1}{\sqrt{2}} [\mu_{INT(H_i)}(TP)]^{1/2} \text{ iff } N\beta \leq P\beta \\ 1 - 0.5(1 - \mu_{INT(H_i)}(TP))^{1/2} \text{ iff } N\beta > P\beta \end{cases}$$

$$P\bar{S}_i(x) = \begin{cases} \frac{1}{\sqrt{2}} [\mu_{INT(H_i)}(x)]^{1/2}; \text{ iff } N\beta \leq P\beta \\ 1 - 0.5(1 - \mu_{INT(H_i)}(x))^{1/2}; \text{ iff } N\beta > P\beta \end{cases} \quad (4)$$

### Proof:

$$\text{Since } \bar{S}_i(x) = \frac{\prod_i \mu_{H_i}(x)}{\sum_H \prod_i [\mu_{H_i}(x)]} \quad (5)$$

( $x \in U ; i = 1, 2, \dots, m$ )

and the conditions are satisfied, then from definition 1, one can obtain,

$$\begin{aligned} \mu_{H_i}(TP) = 0.5 & \text{ iff } N\beta = P\beta \\ 1 \geq \mu_{H_i}(TP) > 0.5 & \text{ iff } N\beta > P\beta \\ 0 < \mu_{H_i}(TP) < 0.5 & \text{ iff } N\beta < P\beta \end{aligned} \quad (6)$$

Since,

$$P\bar{S}_i(x) = \prod_i \mu_{H_i}(x) \quad (i = 1, 2, \dots, m), x \in U$$

Thus, one can obtain

$$P\bar{S}_i(TP) = \begin{cases} 0.5 & \text{ iff } N\beta = P\beta \\ 0.5 < P\bar{S}_i(TP) \leq 1 & \text{ iff } N\beta > P\beta \\ 0 < P\bar{S}_i(TP) < 0.5 & \text{ iff } N\beta < P\beta \end{cases}$$

Where,

$$TP \text{ is } TP_i \text{ or } TP'_i (i = 1, 2, m) \quad (7)$$

Since the function of contrast intensification is represented by

$$\mu_{INT(H_i)}(x) = \begin{cases} 2(\mu_{H_i}(x))^2 \text{ for } 0 \leq \mu_{H_i}(x) \leq 0.5 \\ 1 - 2[1 - \mu_{H_i}(x)]^2 \\ \text{for } 0.5 \leq \mu_{H_i}(x) \leq 1 \\ \forall x \in U \end{cases} \quad (8)$$

Then,

$$\mu_{H_i}(x) = \frac{1}{\sqrt{2}} \cdot [\mu_{INT(H_i)}(x)]^{1/2} \text{ for } 0 \leq \mu_{H_i}(x) \leq 0.5$$

Thus,

$$P\bar{S}_i(TP) = \frac{1}{\sqrt{2}} [\mu_{INT(H_i)}(TP)]^{1/2} \text{ iff } N\beta \leq P\beta$$

Also,

$$\mu_{H_i}(x) = 1 - [0.5(1 - \mu_{INT(H_i)}(x))]^{1/2} \text{ for } 0.5 \leq \mu_{H_i}(x) \leq 1$$

$$P\bar{S}_i(TP) = 1 - [0.5(1 - \mu_{INT(H_i)}(x))]^{1/2}, N\beta > P\beta$$

Thus, (9)

$$\begin{aligned}
 P\bar{S}_i(TP) &= \begin{cases} \frac{1}{\sqrt{2}} [\mu_{INT(H_i)}(TP)]^{1/2} & \text{iff } N\beta \leq P\beta \\ 1 - 0.5(1 - \mu_{INT(H_i)}(TP))^{1/2} & \text{iff } N\beta > P\beta \end{cases} \\
 P\bar{S}_i(x) &= \begin{cases} \frac{1}{\sqrt{2}} [\mu_{INT(H_i)}(x)]^{1/2}; \forall x \in U - [TP_i, TP_i'] \\ \text{iff } N\beta \leq P\beta \\ 1 - 0.5(1 - \mu_{INT(H_i)}(x))^{1/2}; \forall x \in [TP_i, TP_i'] \\ \text{iff } N\beta > P\beta \end{cases}
 \end{aligned} \quad (10)$$

#### 4 Modifications of the contrast intensifier-MIMO Case

##### Proposition 2:

Let  $U$  be a compact set where  $U = [a_1, b_1] \times [a_2, b_2] \times \dots \times [a_n, b_n] \subset R^n$ . Consider our fuzzy neural network, where each FNN unit is characterized by the n-dimensional vectors:

$(\bar{\beta}, \bar{c})$  Where  $\beta_i$  is the bandwidth,  $\beta = \sum_i \beta_i$   $c_i$  is the fuzzy neuronal centroid. The average field strength of the network  $\bar{S}_{j_1, j_2, \dots, j_n}(\bar{x})$  is given by,

$$\phi_j = \bar{S}_j(\bar{x}) = \frac{\prod_{k=1}^n \mu_{H_{jk}}(\bar{x})}{\sum_{j_1=1}^{m_1} \dots \sum_{j_n=1}^{m_n} \dots \prod_{k=1}^n \mu_{H_{jk}}(\bar{x})} \quad (11)$$

Then, Our FNN network has the following properties:

1. The pseudo of the individual field strength

$P S_{jk}^k(x_k)$  is given by

$$\begin{aligned}
 P S_{jk}^k(x_k) &= \\
 &\begin{cases} 0.5 & ; \text{iff } N\beta = P\beta \\ 0.5 < P S_{jk}^k(x_k) \leq 1 & \text{iff } N\beta > P\beta \\ 0 < P S_{jk}^k(x_k) \leq 0.5 & \text{iff } N\beta < P\beta \end{cases}
 \end{aligned} \quad (12)$$

The pseudo of the average field strength  $\bar{P}\bar{S}_{j_1, \dots, j_n}$  is given by

$$P\bar{S}_j(TP) = \begin{cases} (0.5^{n/2}) \prod_{k=1}^n [\mu_{INT(H_{jk})}(x_k)]^{1/2} \\ \text{iff } N\beta \leq P\beta \\ \prod_{k=1}^n [1 - 0.5(1 - \mu_{INT(H_{jk})}(x_k))^{1/2}] \\ \text{iff } N\beta > P\beta \end{cases} \quad (13)$$

**Proof:**

$$\text{Since } \phi_j = \bar{S}_j(x) = \frac{\prod_{k=1}^n \mu_{H_{jk}}(\bar{x})}{\sum_{j_1=1}^{m_1} \dots \sum_{j_n=1}^{m_n} \dots \prod_{k=1}^n \mu_{H_{jk}}(\bar{x})} \quad (14)$$

From definition 1 and proposition 1, one can obtain,

$$\begin{aligned}
 \mu_{H_{jk}}(x_k) &= 0.5 & \text{iff } N\beta = P\beta \\
 1 \geq \mu_{H_{jk}}(x_k) &> 0.5 & \text{iff } N\beta > P\beta \\
 0 < \mu_{H_{jk}}(x_k) &< 0.5 & \text{iff } N\beta < P\beta
 \end{aligned} \quad (15)$$

We can prove that,

$$\bar{S}_{j_1, \dots, j_n}(\bar{x}) = \prod_{k=1}^n S_{j_k}^k(x_k) \quad (16)$$

Then,

$$\begin{aligned}
 P S_{j_k}^k(x_k) &= \prod_{k=1}^n \mu_{H_{jk}}(\bar{x}), \\
 (j_k &= 1, 2, \dots, m_k; k = 1, 2, \dots, n)
 \end{aligned}$$

Thus, one can obtain

$$P S_{j_k}^k(x_k) = \begin{cases} 0.5 & \text{iff } N\beta = P\beta \\ 0.5 < P S_{j_k}^k(x_k) \leq 1 & \text{iff } N\beta > P\beta \\ 0 < P S_{j_k}^k(x_k) < 0.5 & \text{iff } N\beta < P\beta \end{cases}$$

2. Since the function of contrast intensification is represented by

$$\mu_{INT(H_{jk})}^k(x_k) = \begin{cases} 2\left(\mu_{H_{jk}}^k(x_k)\right)^2 \\ \text{for } 0 \leq \mu_{H_{jk}}^k(x_k) \leq 0.5 \\ 1 - 2\left(\mu_{H_{jk}}^k(x_k)\right)^2 \\ \text{for } 0.5 \leq \mu_{H_{jk}}^k(x_k) \leq 1 \end{cases} \quad (17)$$

$\forall \bar{x} \in U$

Then,

$$\mu_{H_{jk}}^k(x_k) = \frac{1}{\sqrt{2}} \cdot \left[ \mu_{INT(H_{jk})}^k(x_k) \right]^{1/2} \quad \text{for } 0 \leq \mu_{H_{jk}}^k(x_k) \leq 0.5 \quad (18)$$

Thus,

$$PS_{j_1 \dots j_n}^{\bar{}}(x_k) = (0.5^{n/2}) \prod_{k=1}^n \left[ \mu_{INT(H_{jk})}^k(x_k) \right]^{1/2} \quad \text{iff } N\beta \leq P\beta \quad (19)$$

Also,

$$\mu_{INT(H_{jk})}^k(x_k) = 1 - \left[ 0.5 \left( 1 - \mu_{INT(H_{jk})}^k(x_k) \right) \right]^{1/2} \quad \text{for } 0.5 \leq \mu_{INT(H_{jk})}^k(x_k) \leq 1$$

$$PS_{j_1 \dots j_n}^{\bar{}}(x_k) = \prod_{k=1}^n \left\{ 1 - \left[ 0.5 \left( 1 - PS_{j_1 \dots j_n}^{\bar{}}(x_k) \right) \right]^{1/2} \right\}, \quad \text{iff } N\beta > P\beta$$

Thus,

$$PS_j^{\bar{}}(TP) = \begin{cases} (0.5^{n/2}) \prod_{k=1}^n \left[ \mu_{INT(H_{jk})}^k(x_k) \right]^{1/2} \\ N\beta \leq P\beta \\ \prod_{k=1}^n \left[ 1 - 0.5 \left( 1 - \mu_{INT(H_{jk})}^k(x_k) \right) \right]^{1/2} \\ N\beta > P\beta \end{cases} \quad \text{iff} \quad (20)$$

## 5 The Proposed Learning Algorithm

The proposed on-line self organizing algorithm is used in this paper. The modified intensifier is injected in the receptive field during training using the concepts described before. The algorithm is:

1. Determine the winner neuron using the minimum distance

$$\|x_i - c^j\| \leq d_o$$

$$\|d_o\| = \sqrt{x^T x} \quad (21)$$

Then, adjust the weights using

$$\Delta w^j = \alpha_1 [x_i(t) - w^j(t-1)] \quad (22)$$

where  $x_i(t)$  is the input vector and  $c^j$  is the centroid of the  $j$ th neuron.

2. Compute the outputs of the neurons in the output layer after incorporating the modified intensifier.
3. The global learning rule can be derived as

$$\Delta w_{oh}(t) = \frac{\alpha_2 (u_k^* - u_k) \varphi^j}{\sum_{j=1}^h \varphi^j} \quad (23)$$

This rule adjust the weights between the output and the hidden neurons, where  $u_k^*$  is the desired control signal and computed at the beginning of each iteration  $k$ , and  $u$  is the response of each neuron at the output layer.  $T$  is the t-norm. The iterative learning algorithm [5] is given by,

$$u_{k+1}^* = u_k^* + P.e + Q.ce \quad (24)$$

Where  $P, Q$  are constant learning gain matrices. The error and change of errors are  $e = y_d - y$  and  $ce$ . The desired and actual responses are  $y_d$  and  $y$  respectively.

## 6 Simulation Results

The performance of the proposed technique is tested for Controlling two link robotic arm. In the simulation 60% mass uncertainties are considered. The dynamics of such robotic system are strongly nonlinear and also suffer from Uncertainties. The equations used in the simulation can be obtained from LaGrange Euler formulation [6]. Where  $u_1$  and  $u_2$  are the controlled torques at the joints of links 1 and 2 respectively and,

$$\begin{aligned} u_1 &= H_{11} \ddot{\theta}_1 + H_{12} \ddot{\theta}_2 - 2h \dot{\theta}_1 \dot{\theta}_2 - h \dot{\theta}_2^2 \\ u_2 &= H_{22} \ddot{\theta}_2 + H_{12} \ddot{\theta}_1 + h \dot{\theta}_1^2 \end{aligned} \quad (25)$$

$$H_{11} = I_1 + I_2 + m_1 l_1^2 + m_2 (L_1^2 + l_2^2 + 2L_1 l_2 \cos \theta_2)$$

$$H_{12} = I_2 + m_2 l_2^2 + m_2 L_1 l_2 \cos \theta_2$$

$$H_{22} = I_2 + m_2 l_2^2$$

$$h = m_2 L_1 l_2 \sin \theta_2$$

The modified intensifier is injected into the receptive field of the proposed fuzzy neural network. The results are summarized in Figs. (2-6) and Tables [1-3]. Also, comparative results are given to show the benefits of the proposed methodology. A comparison between the case of

injecting and non-injecting the intensifier into the receptive field is shown in Fig.2. The response of link one

(From 60 deg. to 80 deg. with increasing the link mass with 60%) in the case of presence of intensifier (series 2 in Fig.2) is better than that of the other case (series 1). The time required to achieve the desired goal is 0.8 sec versus 1.4 sec in the other case. The control signals are shown in Fig.3. The performance in case of incorporating the intensifier is better and much smoother than that of the other case see Fig.4. Comparative studies with previous works are shown in Table 1. The author [6] uses the same robotic system for testing his proposed feed forward neural network with back propagation and reinforcement learning. In our work, the number of generated hidden neurons is 5 while in the other work is 20. Also, the number of iterations in the present work is 50 while in the other is 100 iterations. More details are given in Table 1. The receptive fields are shown in Fig.5 where the fuzzy neurons are generated initially with membership values =1.0, represented by circles. After processing the values of the field strengths are modified and the circles are varied. In Fig.5, the number of neurons is 7, and the number of iterations is 110 for achieving the desired goals. Table 2 gives more details. The effects of the intensifier are depicted in Fig. 6. The number of neurons is 5 and the number of iterations is 50. It shows smaller size and faster responses than the works shown in Tables 1-2.

## 7 Conclusions

### 7.1 summary and extensions of the current work

In this work, modifications of the intensifier have been done for SISO and MIMO cases. The modified intensifier is injected into the localized receptive field. Thus; the fuzzy neural networks have the feature and property of working as lenses with certain field strengths for achieving the desired objectives. Using the proposed technique, the fuzzy neural networks exhibit good performances. It has simpler structure with automatic generated receptive field of 5 neurons than the adaptive neuro-fuzzy approach with 56 neurons [7]. Injecting the modified intensifier into the localized receptive field affects its strengths and shapes (as shown in Figs.4 and 5) which in turn increases the learning process speed (the number of iterations for convergence is 50, in case of injecting the intensifier while it is 110 in case of absence of it as shown in Tables 1, 2 and 3).

As an extension of the work in this paper, visualization of the activity of the localized receptive field has to be carried out for analyzing its behavior.

### 7.2 Applications to Other Fields

The modifications of the contrast intensifiers with MIMO cases are beneficial for the applications with other fields such as image processing [8] and pattern recognition [9]. An image is represented mathematically by a spatial brightness function  $f(m, n)$  where  $(m, n)$  denotes the spatial co-ordination of a pixel in the image. Each pixel can be considered as a fuzzy singleton with a value of membership defining the degree of brightness level. Another application is the pattern recognition. For effective recognition of images we need to preprocess the polluted image with noise to achieve the best image possible for the recognition process.

### References

- [1] Zadeh, L. "A fuzzy set-theoretic interpretation of linguistic hedges," *J. Cybern.*, vol.2, no.2, pp4-34., 1972.
- [2] Zadeh, L. "Outline of a new approach to the analysis of complex systems and decision processes," *IEEE Trans. Sys., Man, Cyber.* vol.SMC-3, pp. 28-44, 1973.
- [3] Zadeh, L. "The concepts of a linguistic variable and its application to approximate reasoning-I," *Inf. Sci.*, vol.8, pp. 199-249, 1975a.
- [4] Zadeh, L. "The concepts of a linguistic variable and its application to approximate reasoning-II" *Inf. Sci.*, vol.8, pp. 301-357, 1975b.
- [5] Roger, J.-S., Sun, C., -T., "Functional equivalence between radial basis function networks and fuzzy inference systems," *IEEE Trans. on Neural networks*, vol.4, no.1, Jan.1993.
- [6] Albet Y. Zomaya, "Reinforcement learning for adaptive control of nonlinear systems," *IEEE Trans.on.sys.man,cyber.* vol.24,no.2,pp.357363,1994
- [7] Funchun Sun, Zengqi sun, hanxiong Li, "Neuro-fuzzy adaptive control based on dynamic inversion for robotic manipulators," *Fuzzy sets and systems*, pp.1-17, 2002.
- [8] Pal S., King R., "Image enhancement using smoothing with fuzzy sets," *IEEE Trans. Syst.,Man ,Cyber.* Vol. SMC.-11, PP.494-501, 1981.
- [9] Bezdek J., "Pattern recognition with fuzzy objective function algorithms," Plenum press, New York, 1981.

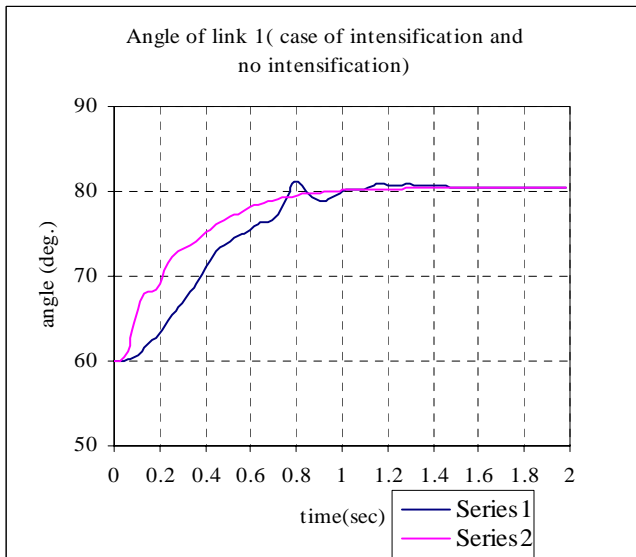


Fig.2

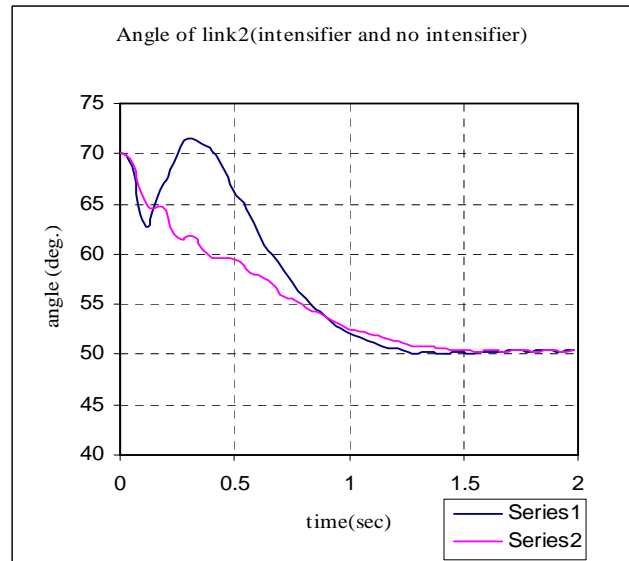


Fig.4

Table 1 : Comparison with previous works

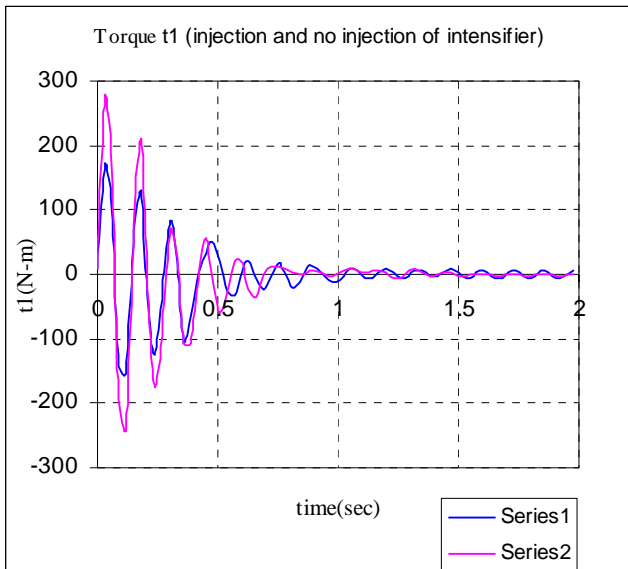


Fig.3

Works	Neurons	Learning	Iterations	APE (%)
MLP[6] 1 <sup>st</sup> layer 2 <sup>nd</sup> layer 3 <sup>rd</sup> layer	Not created 5 20 2	Bp+ reinforcement Learning	100	Not reported
FNN(with intensifier This work 1 <sup>st</sup> layer 2 <sup>nd</sup> layer 3 <sup>rd</sup> layer	Created 4 5 2	Modified SOM+ Injecting modified fuzzy intensifier	50	.0025
FNN (without intensifier 1 <sup>st</sup> layer 2 <sup>nd</sup> layer 3 <sup>rd</sup> layer	Created 4 7 2	Modified SOM Without Injecting fuzzy intensifier	110	2.9

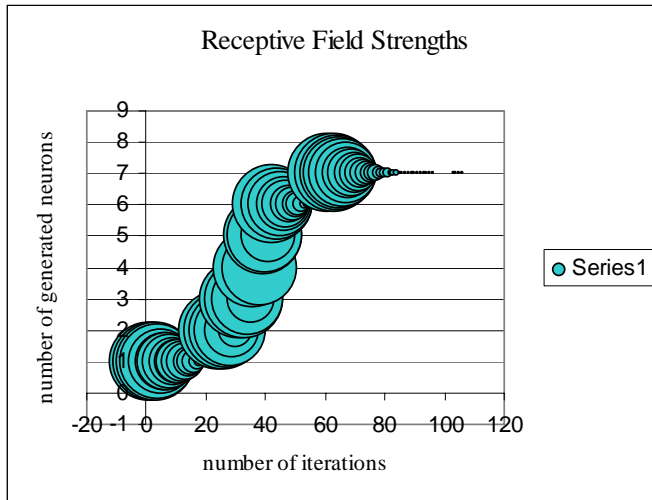


Fig.5

Table 2: Effects of absence of intensifier in the receptive field

Iterations			Created neurons
1	to	23	1 <sup>st</sup>
24	to	30	2 <sup>nd</sup>
31	to	35	3 <sup>rd</sup>
56	to	38	4 <sup>th</sup>
39	to	41	5 <sup>th</sup>
42	to	60	6 <sup>th</sup>
61	to	110	7 <sup>th</sup>
Total iterations =110			Neurons=7

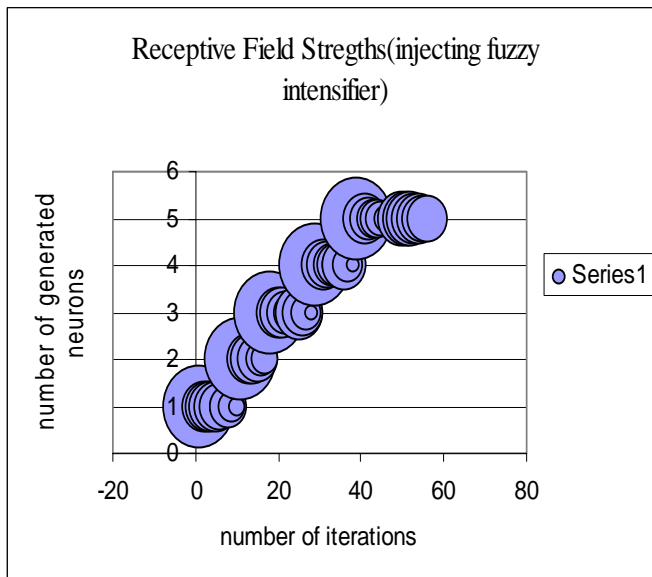


Fig.6

Table 3: Effects of injecting intensifier into the receptive field

Iterations			Created neurons
1	to	10	1 <sup>st</sup>
11	to	17	2 <sup>nd</sup>
18	to	28	3 <sup>rd</sup>
29	to	38	4 <sup>th</sup>
39	to	50	5 <sup>th</sup>
Total iterations =50			Total number of Neurons=5