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Topography Reconstruction by Interferometric SAR Look Vector's Orthogonal Decomposition

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Abstract: - Topography reconstruction by interferometric synthetic aperture radar (InSAR) refers to a method used to determine target's three- dimensional (3D) position. This position is obtained by principal measurements and the imaging geometry of system. By coherently combining signals from the conventional and additional SAR antennas, the interferometric phase difference between the received signals can be formed for each imaged point. In this scenario, the phase difference is essentially related to the geometric path length difference to the image point, which depends on the topography. The range equation, the Doppler equation and the interferometric phase equation of InSAR provide the relation between the fundamental measurements and the point's position on topography. So the point's position may be obtained by solving three interferometric equations. The InSAR equations contain the unknown point's position in a highly nonlinear manner and are difficult to be solved directly. For this problem, researchers have resorted to the orthogonal decomposition of the look vector of radar and have derived several reconstruction algorithms.

In this paper, we analyze the topography reconstruction algorithm based on Madsen's orthogonal decomposition (MOD). We show that the plane wave model has been introduced in the derivation of this algorithm which causes a significant reconstruction error especially for airborne case.

Key-Words: Look vector's orthogonal decomposition, Topography reconstruction, InSAR.

1 Introduction

Interferometric radar has been proposed and successfully demonstrated as a topographic mapping technique by Graham [1], Zebker and Goldstein [2], and Gabriel and Goldstein [3]. A radar interferometer is formed by relating the signals from two spatially separated antennas; the separation of the two antennas is called the baseline. The spatial extent of the baseline is one of the major performance drivers in an interferometric radar system: if the baseline is too short the sensitivity to signal phase differences will be so small to be undetectable, while if the baseline is too long additional noise due to spatial decorrelation corrupts the signal.

Two distinct implementation approaches have been developed for topographic radar interferometers; they differ in how the interferometric baseline is formed. In the first case the baseline is formed by two physical antennas which illuminate a given area on the ground simultaneously: this is the usual approach for aircraft implementations where the physical mounting structures may be spaced for sufficient baseline. This approach is used by [2] for the NASA CV-990 radar, and is also used in the TOPSAR topographic mapping radar mounted on the NASA DC-8 aircraft [4]. The second type of implementation utilizes a single satellite antenna in a nearly-exact repeating orbit. Then it forms an interferometer baseline by relating radar signals on the repeat passes over the same site. Topographic maps using this technique have been demonstrated by Goldstein et al [2] and Gabriel et al [3], [5].

Although, interferometric topographic mapping is possible using each approach, a significant advantage accrues from using the single-pass aircraft implementation over the repeat-orbit spacecraft method. Specifically in the first approach, we avoid problems associated with temporal decorrelation of the surface, that is, the result that changes on the wavelength scale of the surface lead to additional decorrelation noise in the interferogram. If the change on the surface is large, as could happen if, precipitation occurred, the phases of the received signals may be wholly unrelated. On the other hand, spaceborne platforms provide views of inaccessible regions of the Earth.

The Across-track interferometric technique exploits the phase differences of at least two complex-valued SAR images acquired from slightly different positions and/or at different times to extract height information of topography. It can measure the slant range difference at the order of sub-wavelength, thus it has the ability to extract accurately topography height information [6]-[8]. The airborne InSAR imaging geometry is depicted in Fig.1.

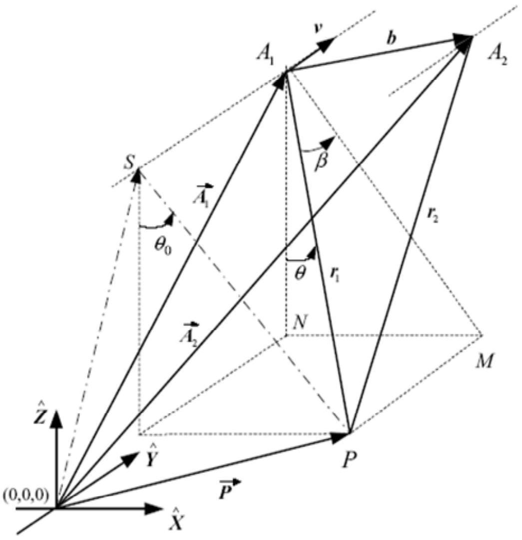

Fig.1 Airborne InSAR imaging geometry.

The coordinate system denoted by $\hat{X}\hat{Y}\hat{Z}$ is the ECR (Earth Centered Rotating) coordinate system with the origin located at the earth's gravitational center, with $\hat{Y}$ parallel to the nominal track A_i, the slant range vector from antenna A_i to target P is given by $\vec{r}_i$, where the subscript i refers to the antenna number or track number. The angle θ, β respectively represents the look angle and squint angle of antenna A_1. The target location vector is given by vector $\vec{P}$, the baseline vector is defined as $\vec{b} = \vec{A}_2 - \vec{A}_1$.

The slant range r_i, the Doppler frequency f_D and the interferometric phase φ are fundamental measurements of InSAR, the relation among them can be described by three basic interferometric equations : the equation of range sphere, the Doppler equation and the phase equation [9]. These equations are respectively given by:

$$\vec{P} - \vec{A}_i = \vec{r}_i \tag{1}$$

$$\frac{\lambda f_D}{2} = < \hat{r}_i, \vec{v} > \tag{2}$$

$$\varphi = \frac{2\pi Q}{\lambda}\left(\vec{r}_2 - \vec{r}_1\right) \tag{3}$$

where $Q = 1$ for the standard mode InSAR system, $Q = 2$ for the ping-pong mode and $\hat{r}_i$ is the unit vector in the line-of-sight direction of $\vec{r}_i$.

Topography reconstruction by InSAR refers to determine target's 3D location using principal measurements and the imaging geometry of system; it may be achieved by solving the equation set (1)-(3). However, the equation set is difficult to be solved directly, that due to its highly nonlinear characteristics. To solve this problem, several reconstruction algorithms have been proposed and they are generally divided into two classes: numerical algorithms [10] and analytical algorithms [11]-[12]. In numerical algorithms, the linearization of the nonlinear equation set may be achieved by Taylor expanding InSAR equations (1)-(3) to the first order and a linear equations system is obtained. Then the unknown position coordinates may be easily achieved by solving the new equation set. The main drawback of numerical algorithms is their inaccuracy due to using only the constant and the linear term of Taylor series. However, analytical algorithms may give more accurate solution than numerical ones because they determine the position coordinates by solving the equation set formed by (1)-(3) without introducing any approximation. In analytical algorithms, two research trends are resorted to solve the highly nonlinear equation set. Thus, analytical algorithms fall into two classes: the direct analytical method [11] and the look vector's orthogonal decomposition method [13]. In this paper, we will focus on the study of topography reconstruction based on the look vector's orthogonal decomposition.

According to (1), the target location can be expressed as

$$\vec{P} = \vec{A}_1 + \vec{r}_1 = \vec{A}_1 + r_1 . \hat{r}_1 \tag{4}$$

where the antenna vector $\vec{A}_1$ may be provided by navigation equipments and the slant range r_1 may be determined according to the principle of radar ranging. Therefore, the determination of the target location is reduced to the determination of the unit vector. Then, look vector's orthogonal decomposition method is introduced just from this idea. The look vector is projected onto an orthogonal basis that is defined as local antenna coordinate system, and then transformed into the general coordinate system. Thus, substituting the unit vector in the general coordinate system into (4) may achieve topography reconstruction.

According to the idea of the orthogonal decomposition, Madsen has introduced a local coordinate system defined on the aircraft and has given a method for the orthogonal decomposition of the look vector [13]. However, the look vectors for the two acquisitions have been assumed to be parallel, and thus the range difference approximately equals to the projection of the baseline vector on the look direction. That is to say, a plane wave model of the electromagnetic wave front has been introduced. The simplified model may be valid only for spaceborne InSAR geometries with small swaths and small ratios of the baseline to the slant range [14]. However, the airborne InSAR has a relatively larger baseline-range ratio than the spaceborne one. Thus, the plane wave model will introduce a significant height reconstruction error.

2 Plane wave model

According to (3), the interferometric phase of InSAR is proportional to the range difference and may be expressed by the baseline vector and range vector as [9]

$$\varphi=\frac{2\pi Q}{\lambda}(r_2-r_1)=\frac{2\pi Q}{\lambda}r_1\left[\left(1-\frac{2b<\hat{r}_1,\hat{b}>}{r_1}+\left(\frac{b}{r_1}\right)^2\right)^{1/2}-1\right] \tag{5}$$

The unit vectors of $\vec{r}_1$ and $\vec{b}$ respectively denoted by $\hat{r}_1$ and $\hat{b}$. $x=\frac{b}{r_1}$, $\gamma=\frac{2\pi Q r_1}{\lambda}$, $\kappa=2<\hat{r}_1,\hat{b}>$. The Taylor expanding of (5) at $x=0$ is given by

$$\varphi\approx-\frac{\gamma\kappa}{2}x+\frac{\gamma^2}{\gamma+1}x^2+\frac{\gamma^3\kappa}{2(\gamma+1)^2}x^3+.... \tag{6}$$

if $b<<r_1$, the following relation can be obtained only by using the constant and linear term of Taylor series

$$\varphi_{pw}\approx-\frac{\gamma\kappa}{2}x=-\frac{2\pi Q}{\lambda}<\hat{r}_1,\vec{b}> \tag{7}$$

where φ_{pw} is the plane wave model of the interferometric phase [2], [9], [13], [15]-[16].

As discussed above, the target position is determined by the intersection of three surfaces described by (1)-(3) [9]. Fig.2 depicts the range sphere, the Doppler cone and the phase cone relevant to antenna A_1. The range sphere is centered at antenna A_1 and has a radius equal to slant range r_1. The Doppler cone has a generating axis along the velocity vector and the cone angle is proportional to the Doppler frequency. From (5), the phase surface is a hyperboloid with focuses located at A_i and a symmetrical axis along the baseline vector. Thus, the target position is given by the intersection locus by the three surfaces. Under the plane wave model, the interferometric phase hyperboloid reduces to a phase cone, and the target position is determined by the intersection locus of range sphere, the Doppler cone and the phase cone, illustrated by P in Fig.2. According to this interpretation, the plane wave model may introduce an unavoidable reconstruction error.

We note that a conventional SAR system resolves targets in the range direction by measuring the time it takes a radar pulse to propagate to the target and return to radar. The along-tack location is determined from the Doppler frequency shift that results wherever the relative velocity between the radar and target is not zero. A target in the radar image could be located anywhere on the intersection locus, which is a circle in the plane formed by the radar line of sight to the target and vector pointing from the aircraft to nadir.

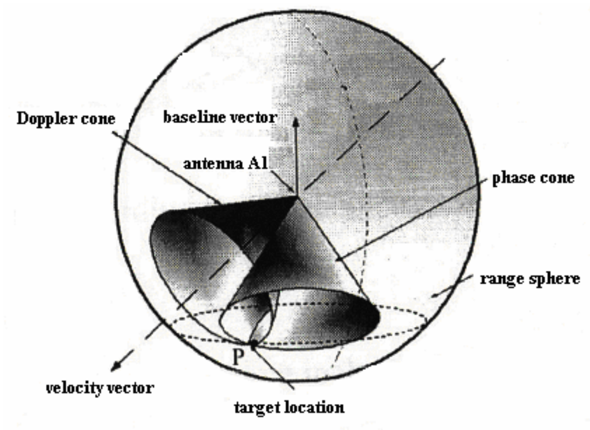

Fig.2 Geometric interpretation of topography 3D reconstruction by InSAR.

Physically, the so called plane wave model is that when $b<<r_1$ is valid. The signals scattered by the same point P on the surface of topography arrive at the antennas in a parallel manner, thus, the wave front may be approximated as a plane. So, the range difference from antennas to point P approximately equals the projection of the baseline vector onto the look direction, where $\Delta r=\overline{A_1C}=<\hat{r}_1,\vec{b}>$ as illustrated in Fig.3. However, the wave front is actually spherical, the range difference is $\overline{A_1B}$. Thus, an interferometric phase analytic error will be unavoidably introduced. From (5) and (6), this error can be expressed as

$$\Delta\varphi=\varphi-\varphi_{pw}=\frac{2\pi Q}{\lambda}(r_2-r_1)=\frac{2\pi Q}{\lambda}r_1\cdot$$

$$\left[\left(1-\frac{2b<\hat{r}_1,\hat{b}>}{r_1}+\left(\frac{b}{r_1}\right)^2\right)^{1/2}-1+\frac{b}{r_1}<\hat{r}_1,\hat{b}>\right] \tag{8}$$

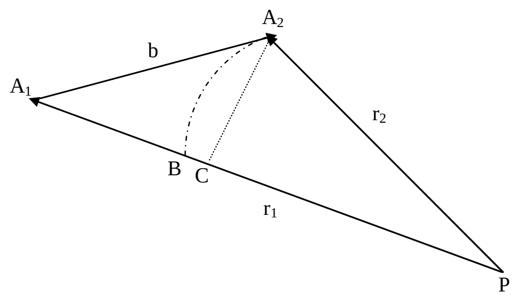

Fig.3 The plane wave model.

As illustrated in Fig.4, for height measurement by InSAR, the universal geometry may be the two-dimensional (2D) InSAR geometry, under which the baseline component lies in the plane of the look vector and the nadir direction, normal to the flight direction. According to this geometry, we have

$$<\hat{r}_1,\hat{b}>=\sin(\theta-\alpha) \tag{9}$$

where α is the angle the baseline makes with respect to a reference horizontal plane. From (8), we obtain the height reconstruction error introduced by the plane wave model

$$\Delta P_z = -\frac{\lambda r_1}{2\pi Q b\cos(\theta-\alpha)}\left[1+\frac{\lambda\varphi}{2\pi Q r_1}\right]\sin(\theta)\Delta\varphi \qquad (10)$$

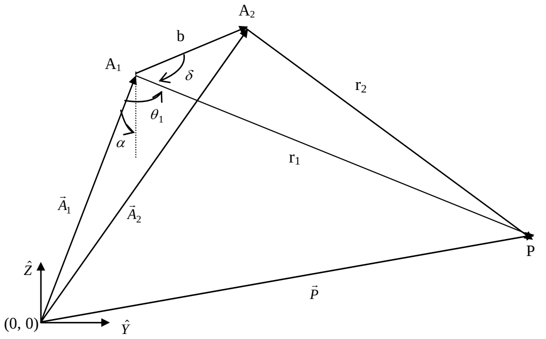

Fig.4 2D InSAR geometry.

As mentioned above, the plane wave model was originally applied to the spaceborne InSAR system with a small swath [17]. The model allows simplified estimation of the baseline based on tie points (points of known elevation on the ground) distributed in range and in azimuth if there is divergence of the baseline. However, one should be aware that significant systematic reconstruction errors are introduced. The plane wave model is inaccurate for satellite geometries with large swaths, and will introduce a systematic height reconstruction error. For cases with parallel tracks, the ERS results show that the plane wave model causes systematic height errors on the order of 10’s of meter. Moreover, under 2D InSAR, the systematic errors increase in cases with larger track divergences. Although these errors may be mitigated through tie point baseline tweaking [14], [18], they can't be eliminated completely. Therefore, the model must be carefully treated to get an accurate topography reconstruction by spaceborne InSAR.

For airborne InSAR geometries, the phase error and the height reconstruction error introduced by the plane wave model are approximately of the same order as the accuracies of the phase measurement and the height reconstruction by InSAR [19]. On the other hand, even for the airborne InSAR with dual antennas constructed on the rigid platform, the attitude of the aircraft may be dynamic changed. So, track divergences also exist in airborne InSAR as that in spaceborne InSAR, which in turn increases the height reconstruction error introduced by the plane wave model.

3 Madsen's orthogonal decomposition

In 3D InSAR imaging geometry, the baseline comprises a component along track and component across track. These two components correspond to two different types of applications: the differential interferometry and the topography measurement. On this decomposition basis, Madsen has introduced an orthogonal decomposition that defines a local coordinate system. The MMC (Madsen Moving Coordinate) system $\hat{v}\hat{n}\hat{w}$ is illustrated in Fig.5, with $\hat{X}$ parallel to the antenna velocity vector $\vec{v}$. Its three orthogonal bases are

$$\begin{aligned} \hat{v} &= \frac{\vec{v}}{v} \\ \hat{n} &= \frac{\vec{b} - \ll \vec{b}, \hat{v} >, \hat{v} >}{\left|\vec{b} - \ll \vec{b}, \hat{v} >, \hat{v} >\right|} \\ \hat{w} &= \hat{n}\times\hat{v} \end{aligned} \qquad (11)$$

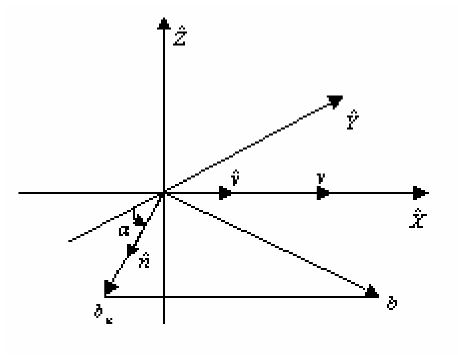

Fig.5 MMC and ECR systems

Then, the unit look vector $\hat{r}_1$ may be expressed by the orthogonal basis as

$$\begin{aligned} \hat{r}_1 &= \mu_1\hat{v} + \eta_1\hat{n} + \zeta_1\hat{w}, \\ \mu_1 &= <\hat{r}_1, \hat{v}> \\ \eta_1 &= <\hat{r}_1, \hat{n}> \\ \zeta_1 &= <\hat{r}_1, \hat{w}> \end{aligned} \qquad (12)$$

Under the plane wave model, the interferometric phase now reads [9]

$$\varphi = -\frac{2\pi Q}{\lambda} < \hat{r}_1, \vec{b} > \qquad (13)$$

Substituting (2) and (13) into (12) derives the unit vector in the local system

$$\hat{r}_{1vnw} = \begin{bmatrix} \dfrac{\lambda f_D}{2v} \\ -\dfrac{\lambda\varphi}{2\pi Q b_n} - \dfrac{b_v}{b_n}\dfrac{\lambda f_D}{2v} \\ \pm\sqrt{1-\mu_1^2-\eta_1^2} \end{bmatrix} \qquad (14)$$

where $b_v = <\vec{b}, \hat{v}>$, $b_n = <\vec{b}, \hat{n}>, \vec{b} = b_v\hat{v} + b_n\hat{n}$. The sign of ζ_1 is given by "+" for the left looking of radar and "-" for the right.

4 Topography reconstruction algorithm

The transformation from the local coordinate system to the general one can be achieved by equation (15)

$$\Gamma = \begin{bmatrix} 1 & 0 & 0 \\ 0 & \cos\alpha & -\sin\alpha \\ 0 & \sin\alpha & \cos\alpha \end{bmatrix} \tag{15}$$

Theoretically, after transformation by (15), we have fulfilled the transformation. However, in airborne InSAR, the variation of aircraft attitude (depicted by yaw, pitch and roll) causes the unit look vector in the general coordinates to be transformed into a new coordinate. Thus, in order to achieve the topography 3D reconstruction, the unit look vector must be transformed back into the general coordinate system from the new one again. The transformation can be fulfilled by Euler rotation matrices corresponding to three attitude angles. The above transformation yields the unit look vector in the general coordinate system including the attitude of the aircraft.

$$\hat{r}_1 = YPR\,\hat{r}_{1vnw} \tag{16}$$

where

$$R = \begin{bmatrix} 1 & 0 & 0 \\ 0 & \cos\theta_r & -\sin\theta_r \\ 0 & \sin\theta_r & \cos\theta_r \end{bmatrix},$$

$$P = \begin{bmatrix} \cos\theta_p & 0 & -\sin\theta_p \\ 0 & 1 & 0 \\ \sin\theta_p & 0 & \cos\theta_p \end{bmatrix}, \tag{17}$$

$$Y = \begin{bmatrix} \cos\theta_y & \sin\theta_y & 0 \\ -\sin\theta_y & \cos\theta_y & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

where Y, P, R are Euler rotation matrices and $\theta_r, \theta_p, \theta_y$ are the attitude angles respectively for roll, pitch and yaw.

Finally, we calculate the (x, y, z) coordinates of the targets by using (14) - (17) with (12) and (4).

5 Conclusion

In this paper, the topography reconstruction algorithm based on Madsen's orthogonal decomposition has analyzed. Madsen has given an orthogonal decomposition of the look vector under the plane wave model of the electromagnetic wave front. However, InSAR systems require extremely accurate knowledge of the baseline length and orientation angle-millimeter or better knowledge for the baseline length and 10's of arc second for the baseline orientation angle. These requirements necessitate an extremely rigid and controlled baseline, a precise baseline metrology system and rigorous calibration procedures.

Also, we note that the phase accuracy requirements for interferometric systems typically range from 0.1°-10°. This imposes rather strict monitoring of phase changes, which are not related to the imaging geometry in order to produce accurate topographic maps.

From our analysis, we may finally conclude that the plane wave model introduce a significant reconstruction error especially for the airborne InSAR, and must be discarded to accurately reconstruct the topography.

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