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Soft Computing Alternatives to Modeling and Predicting Economic Dynamics when Dealing with Forward-Looking Rational Competitors

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Abstract: - Economics and engineering are currently concerned with modeling and predicting complex dynamics. However, economy is part of an anthropic reality, whereas a non-anthropic reality is actually the field of engineering. Furthermore, these two fields seem to have irreconcilable epistemological foundations. This paper is about whether new flexible computational paradigms, such as Soft Computing, allow dealing with dynamics induced by forward-looking human behaviors, and how different are these approaches from the adaptive backward-looking mechanisms, currently implemented in engineering. While standard hypothesis in macroeconomics is rational expectations, some new modeling frameworks provide insides in real-word markets. These evolve through the interaction between competitors who typically exhibit heterogeneous beliefs and bounded rationally. To model the complex dynamics emerging from such behaviors, more versatile (neural, genetic, fuzzy, or hybrid) methods are required. Subsequently, the problem this paper raises is to which extent computational methods spanning different fields of reality are compliant with the attempt to unifying the science that WSEAS has explicitly assumed.

Key-Words: - Soft computing; Economic dynamics; Rational expectations; Perfect vs. bounded rationality; Heterogeneous interacting agents.

1 Epistemological considerations

An essential requirement for a prediction to be reliable is its assumptions to remain unaltered during the prediction horizon. This is typically the case when modeling the physical reality or even the non-anthropic bio-physical reality. The critical case is when attempting to predict the forward-looking human behavior. This implies a reverberating rationality phenomenon (similar to the reflection in parallel glasses), which is commonly known from strategic games with complete information, where two players could mutually annihilate their actions as long as they are able to rationally anticipate their intentions.

The same phenomenon has to be considered when modeling macroeconomic dynamics. The modeler acts as a policy-maker at the macroeconomic level. The rational agents are anticipative with respect to macroeconomic policies regarding themselves, evaluate their effects on individual businesses and change accordingly their behavior, thus altering the initial settings: changes in policies induce changes in behavioral parameters of the macroeconomic model, which will start to drift.

Therefore, purely adaptive backward-looking policies could be inefficient. For reliable predictions, rational expectations need to be integrated in the model, in order to capture the forward-looking behavior of the agents. The rational expectations revolution, promoted by Lucas' critique in 1976 ([8]), originated from such kind of epistemological considerations.

2 The rational expectations revolution

The main motivation behind the development of rational expectations models was to provide reliable policy evaluation procedures. In his seminal paper "Econometric Policy Evaluation: A Critique" (1976), Robert Lucas Jr. argued that the parameters of the models conventionally used for policy evaluation would shift when policy changed. The main reason for this shift is that expectations mechanisms are adaptive or backward-looking in conventional models and thereby unresponsive to those changes in policy that would be expected to change expectations of future events. Hence, the policy evaluation results using conventional models would be misleading.

An example of a backward-looking macroeconomic model is the following nonlinear dynamic simultaneous equation system:

$$f(y_t, y_{t-1}, x_t, \beta) = \varepsilon_t \sim iid(0, \Sigma) \qquad (1)$$

where f denotes a vector function. The arguments y_t, y_{t-1}, x_t and β are vectors of current and lagged endogenous variables, exogenous variables and parameters. ε_t is a vector of errors or shocks, assumed to be inter-temporally independent and identically distributed (*iid*), with zero means and a contemporaneous covariance matrix Σ.

By contrary, the *sine qua non* of a forward-looking model is the appearance of forecasts of events based on information available before the events take place, i.e., conditional expectations of future-dated variables (leads). An example of a nonlinear forward-looking model is as follows:

$$f(y_t, y_{t-1}, E(y_{t+1} \mid I_t), x_t, \beta) = \varepsilon_t \sim iid(0, \Sigma) \qquad (2)$$

where $E(y_{t+1} \mid I_t)$ is the conditional expectation based on all information through period t and $I_t = \{f, y_0, \ldots, y_{t-1}, x_1, \ldots, x_t, \beta, \Sigma\}$ denotes the information set at the start of the period t. With $\bar{y}_{t+1} = E(y_{t+1} \mid I_t)$, one can define an implicit solution:

$$y_t = g(y_{t-1}, \bar{y}_{t+1}, x_t, \varepsilon_t, \beta) \qquad (3)$$

While the backward-looking model can be solved recursively, the solution of the forward-looking nonlinear model cannot be generally computed by recursion. Dependence on future expectations in addition to past realizations has implications in the uniqueness of the solution and the method of its approximation. Generally, the solution for all periods has to be found simultaneously.

The forecast generated by this process will be equal to the expectations that appear in the model. In this sense, expectations are consistent with the model, or equivalently, expectations are rational.

Several partial solutions have tried to mitigate the criticism on the use of classical control theory in economics, which has been proved to not be suitable for dealing with rational expectations. A possible solution is to formulate macroeconomic policy as a game between policy-makers and economic agents. Another solution could be to observe and estimate the parameters of the agents' behavior in response to policy changes. This describes a process by which the policy-maker uses stochastic control methods to learn about changes in the behavior of the agents. Thus he is always one step behind the economic agents. In a dynamic setting, the policy-maker announces a policy, the agent responds and the policy-maker observes the changes in behavior and use Kalman filter methods to update parameters over time and to provide updated mean and covariance estimates in each time period. The covariance matrix of parameter estimates can then be used in deciding on policy levels for the next time period.

An example of linear quadratic optimization for models with rational expectations and learning could be formulated as follows: find the set of admissible instruments $U = \{u_0, u_1, \ldots, u_{T-1}\}$ that minimizes the welfare loss function

$$J_T = E_0\left[\sum_{t=0}^{T-1} \beta^t L_t(x_t, u_t) + \beta^T L_T(x_T)\right] \qquad (4)$$

subject to the model

$$\begin{aligned} x_{t+1} &= A(\theta_t)x_t + B(\theta_t)u_t + \\ &\quad + C(\theta_t)z_t + \sum_{\tau=1}^{k} D_\tau(\theta_t)E_t x_{t+\tau} + \varepsilon_t \end{aligned} \qquad (5)$$

where L_t and L_T are quadratic forms, $D_\tau(\theta_t)$ is a parameter matrix, $E_t x_{t+\tau}$ is the expected state for time $t+\tau$ as seen from time t, k is the maximum lead in the expectations functions and ε_t is a white noise. As a principle, the rational expectations have to be eliminated from the model, in order to compute the admissible set of instruments.

In general, solving stochastic control problems that embed rational expectations is a difficult task, when preserving the strong hypothesis of perfectly rational agents. This clearly legitimates the interest in considering weaker hypotheses and looking at more versatile methods for achieving tractability and robustness. Soft computing (neural, genetic, fuzzy or hybrid) technologies may help agents to face limited knowledge and information, by involving them in a learning framework that tolerates bounded rationality.

3 Bounded rationality and adaptive learning

3.1 Homogeneous vs. heterogeneous agents

A reductionism has to be implicitly assumed in standard macroeconomics for conforming to the Rational Expectations Hypothesis: the abstract concept of a representative agent. This implies that all agents are homogeneous and do not interact. Under such assumption, the dynamics of the aggregate replicate the dynamics of elements, which are in equilibrium and exhibit only non systematic differences (noises).

However, real-word markets incorporate agents who are heterogeneous in their decision behavior, and typically do *not* exhibit perfect rationality. This suggests relaxing the strong hypothesis of rational expectations and adopting a methodological approach based on heterogeneous interacting agents. Heterogeneity implies bounded rationality and may be motivated by various reasons: incomplete information, limited capacity or significant costs of processing it; changes in technology and institutions; political events, rumors, disturbing news; diversity of agent typology; different agent capabilities of learning and evolving.

The heterogeneity of economic agents and the interaction between them are captured by the occurrence of scaling phenomena and the skewed distribution of several variables, such as firms' size, growth rates etc. This affects the concept of macroeconomic equilibrium, which does not require any more that every agent is in equilibrium (i.e., does not depend on "microscopic" details), but stats that the stability is rather an emergent property of the aggregate as a whole. A state of macroeconomic equilibrium can be maintained by a large number of transitions in opposite directions. If the system is far from equilibrium, self-organizing phenomena may also occur. On the other hand, the imperfect information and the systematic interactions among agents may produce output fluctuations.

3.2 Using neural networks for adaptive learning of rational expectations

The potential of soft computing methods in general, and of neural networks in particular, to deal with nonlinear process modeling and prediction derives from two important characteristics:

- their capability to be universal approximators (i.e., to estimate almost any computable function on a compact set, provided that enough experimental data and enough computing resources are available);
- their tolerance for model misspecification, which allows difficult problems, such as the *ex ante* specification of functional form in econometric model building, to be tractable in a less stringent and versatile manner: the specification of neural network architecture (i.e., number of layers, number of neurons in each layer, type of activation functions, and so on).

In order to be competitive on market, agents have to be able of forming rational expectations. When facing model misspecification (i.e., the true nonlinear functional form of the model is unknown) they must approximate rational expectations as a result of a learning process. An auxiliary model is required to accomplish this process, based on the assumption that it is flexible enough to represent various kinds of possible relationships between the relevant variables. Neural networks might be well suited for this task, due to their two characteristics mentioned above. Using the inductive capabilities of neural networks, the agents may be able to learn the formation of rational expectations, without the requirement of specifying *ex ante* the true nonlinear functional form of the model.

To exemplify the way agents learn to form expectations, one can consider a cobweb-like model ([6]), where the values of an endogenous variable y_t depends on a k-dimensional vector of observable exogenous bounded variables $x_t \in \Omega_x \subset \Re^k, \forall t$, and an unobservable bounded error $\varepsilon_t \in \Omega_\varepsilon \subset \Re, \forall t$. The reduced form of the model is given by

$$y_t = \alpha\, y_t^e + g(x_t) + \varepsilon_t \qquad (6)$$

where y_t^e denotes the agents' expectation of the endogenous variable y in period t, and $g(x)$ is a continuous function for all $x \in \Omega_x$. The exogenous variables x_t can be observed before the expectation y_t^e is formed. The standard assumptions hold: $\varepsilon_t \sim iid$, $E[\varepsilon_t]=0$, $E[\varepsilon_t^2]=\sigma^2$, $E[\varepsilon_t \mid x_t]=0$.

Given the reduced form (6) and the previous assumptions, rational expectations are given by:

$$\bar{y}_t^e = E[y_t \mid x_t] = E\left[\frac{g(x_t)+\varepsilon_t}{1-\alpha}\right] = \frac{g(x_t)}{1-\alpha} = \phi(x_t) \qquad (7)$$

where $\phi(x_t)$ denotes the rational expectation function and defines uniquely the rational expectations of the endogenous variable for all $x \in \Omega_x$, as long as $\alpha \neq 1$.

Without an *ex ante* specification of the functional form of $\phi(x_t)$, agents could use neural networks as an auxiliary model for learning expectations. A neural network with k input units, *one* hidden layer consisting of m units and *one* output unit is well suited for accomplishing this task.

Each of the hidden units $i=1,\ldots,m$ receives a signal that is the weighted sum of all inputs x_j, $j=1,\ldots,k$, i.e., $\tilde{h}_i = \sum_{j=1}^{k} w_{i,j}\, x_j + w_{i,0}$, where $w_{i,0}$ denotes a threshold value. In each hidden unit, the signal received is transformed by an activation function $S:\Re \to [0,1]$, $S(z) = 1/\left(1+\exp(-z)\right)$, such that $h_i = S(\tilde{h}_i)$ is the output signal of the hidden unit i. Finally, the output unit receives the weighted sum of all these output signals: $y = \sum_{i=1}^{m} h_i + q_0$, where

q_0 denotes a threshold value. The neural network defines a mapping from inputs x_j to the output y, as follows:

$$y = \sum_{i=1}^{m} q_i S\left(\sum_{j=1}^{k} w_{i,j}\, x_j + w_{i,0}\right) + q_0 = f(x,\theta) \qquad (8)$$

where $x \in \Re^k$, $\theta \in \Re^q$, $q = 1 + m(k+2)$, and $\theta' = (q_0, q_1, w_{1,0}, \ldots, w_{1,k}, q_2, \ldots, w_{m,k})$.

Given that neural networks are universal approximators, one can assume that a well configured architecture provides, at least theoretically, enough predictive accuracy for agents to learn expectations.

3.3 Robust control when dealing with misspecification

Robust control is a promising tool for a policy-maker who regards this model as an approximation, that is, an unknown member of a set of unspecified models near his approximating model. Important steps have been made in the direction of applying robust control to evaluating economic policies by T. Sargent and L.P. Hanssen ([5]). Instead of assuming that policy makers know the model in the form of a transition low that links the motion of state variables to controls (such as in the ordinary control theory), robust control theory alters the mapping from shock temporal properties to policy rules. It seeks one rule to use for a set of models that might also govern the data. The currently used methods (H_∞ and entropy criteria in the frequency domain, or robust filtering) need some adaptations to incorporate discounting into the objective functionals.

3.3 Capturing and tuning nonlinear characteristics by fuzzy control

Presumably, the interest in applying fuzzy control to economic processes consists of at least two advantages: on the one hand, of prescribing control actions by linguistic descriptions, and on the other hand, of the capability of transition from linear to nonlinear modes of control, conjugated with fine-tuning procedures. We addressed the latter opportunity in our previous paper ([4]), by providing a fuzzy extension to the Phillips' stabilization model in two variants: for a closed economy (using a fuzzy PID controller) as well as for an open economy (using a fuzzy state-feedback controller). Since fuzzy control can be described as a non-linear mapping, the corresponding fuzzy controller acts as a non-linear controller and hence provides an increasing flexibility. In the first stage, we focused on the emulation of a conventional controller (either a PID, or a state-feedback one) through a linear fuzzy controller as a start point for further exploitations of the full capabilities of the non-linear fuzzy controller. Given that a fuzzy controller contains a linear controller as a special case, it is true to say that it performs at least as well as the latter. We also briefly suggested how to make it non-linear and how to use fine-tuning procedures for achieving the validation objective of the controller. The potential for performing better depends on the designer capability to exploit the non-linear options in the fuzzy controller to his advantage.

4 Using soft computing methods in a multi-agent modeling framework

4.1 Complex dynamics in financial markets induced by heterogeneous expectations

The Efficient Market Hypothesis (EMH) assumes identical investors who share rational expectations of an asset's future price, and who instantaneously and rationally discount all market information into this price. However, it is widely known that in real-word markets the traders may exhibit heterogeneous beliefs about future prices of a risky asset that may considerably deviate from fully rational expectations. Financial markets can be viewed as complex evolutionary systems having internal dynamics induced by two competing trading agents: "fundamentalists", believing that prices will move towards their fundamental rational expectations value, as given by the expected discounted sum of future dividends; "trend-followers", believing that asset prices are not completely determined by fundamentals, but that they may be predicted by simple technical trading rules, extrapolation of trends and other patterns observed in past prices.

Numerical experiments and some empirical evidences ([3]) have emphasized that heterogeneity in beliefs may lead to market instability and complicated dynamics, such as cycles or even chaotic fluctuations, in financial markets. Asset price fluctuations are caused by an endogenous mechanism relating the fraction of fundamentalists and trend-followers to the distance between the fundamental and the actual price. A large fraction or weight of the fundamentalists tends to stabilize prices, whereas a large fraction of trend-followers tends to destabilize prices.

Asset price fluctuations are caused by the interaction between these stabilizing and destabilizing competitors. Experimental evidences show that, under the hypothesis of heterogeneous expectations among traders the emerging dynamics of asset price changes dramatically, with bifurcation routes to strange attractors, especially if switching to more successful strategies becomes more rapid.

4.2 Neural network based agents dealing with econometric forecast models

Neural network based econometric forecast models are frequently embedded in a multi-agent modeling framework. They assume connectionist relationships between the input signals and the target values. Both feed-forward and recurrent neural networks can be used for multi-agent modeling, depending upon the dynamic behavior of the market. They allow capturing nonlinear characteristics and are able of fitting a wide range of forecast structures. Agents can build a price forecast at time t, $E_t p_{t+1,j}$, using a network training with several inputs including several lagged prices, and trade prices averaged over all agents from earlier periods. Agents can be randomly matched and trade occurs when agent pairs have different expected future prices. They then split the difference and trade at the price in between their two expected values.

In the model in [1], the agents use feed-forward neural networks to predict upcoming stock price movements. Basically, the authors introduce two kinds of market participants: smart and naive agents. Smart agents predict stock price movements on the basis of 3-layer feed-forward neural network with four input signals. Inputs to the 3-layer network are the most recent market prices π_{t-1}, π_{t-2} and the former transaction prices $P_{ij,t-1}$, $P_{ij,t-2}$. In contrast, the forecast models of naive agents are simplified: Naive agents only rely on the most recent market price π_{t-1} in order to forecast the future development of the stock price. The underlying neural network architecture consists only of a single input neuron containing the most recent market price and one output neuron computing the price forecast.

A multi-agent model based on recurrent neural networks is presented in [10]. The model includes three types of agents, value traders, momentum traders, and noise traders. The agents place there funds either in a risky stock paying a stochastic dividend or in a riskless bond. Value investors believe that the actual stock price reflects the discounted stream of all future dividends. Momentum traders and noise traders are technicians who only consider historical price patterns. While momentum and noise traders are modeled as rule-based agents who refer to technical trading rules, value traders form their expectations on the basis of Elman's recurrent neural networks. These recurrent neural networks incorporate so-called context units, which feed the information of previous activation values back into the network. More precisely, value investors use recurrent neural networks to predict the dividend growth of the risky asset. Afterwards, the market price of the risky asset is estimated on the basis of the dividend growth by using Gordon's constant dividend growth model.

Another neural network based approach can be found in [7]. In this artificial stock market, the agents invest their funds either in a riskless bond or in a risky asset paying a stochastic dividend. As a specialty, the agents have different decision making horizons: some agents are long-term investors while others rely on short-term planning horizons. According to their planning horizon, the agents choose from a broad spectrum of forecast models that are fitted to historical data. Forecast models and agents are therefore separated. Each forecast model is composed of a simple feed-forward network incorporating one hidden neuron and a limited number of input signals. The input signals consist of technical and fundamental indicators. The neural networks are evolved by using a genetic algorithm.

As it can be seen from these examples, the decision making schemes of econometric agents do not incorporate semantic specifications of the underlying cognitive processes in terms of e.g. perception, internal processing and action. In other words, the econometric forecast models of the agents merely assume connectionist relationships between the input signals and the target values.

4.3 Cognitive system based models, learning, and evolution

Cognitive system based agent models are an approach of capturing the semantic specifications of the agents' decision schemes in a structural framework. More precisely, the decision making of an agent is modeled by a basic cognitive system. The cognitive system incorporates three properties: perception, internal processing and action. These properties constitute necessary conditions for a cognitive system and may include various other features. As a structural representation of such a cognitive system, one may refer to time-delay recurrent neural networks. The cognitive process generates not only expectations of the market price, but also concrete trading decisions. Since a cognitive agent has a specific objective function (e.g. utility maximization), the resulting actions are always goal-oriented. As it can be seen from this outline, the buying and selling decisions of a cognitive agent are *not* deduced from a specific econometric forecast model which only presumes a connectionist or, possibly, an ad-hoc functional relationship between external influences and the market development. Rather, the decisions of the agent are formulated in terms of the underlying cognitive system. This is truly a more realistic approach of modeling the agents' behavior.

Heterogeneous behavior may also originate from the learning and evolution of the agents.

Multi-agent models that refer this source of heterogeneity typically incorporate agents who are endowed with set of dynamic trading rules. The rule sets are evolved by genetic algorithms. In such a model, agents are very homogeneous at the start in their abilities and strategy structures. During the market process, the agents learn and develop more sophisticated trading strategies. In other words, differences in behavior and strategy evolve endogenously as the market runs, and thus agent heterogeneity becomes a changing feature of the market. Examples including genetic algorithms as a source of heterogeneity are given in [2] and [7].

Besides genetic algorithms, one may also refer to gradient based learning techniques in order to generate heterogeneous decision behavior. First, the agents may simply differ in their underlying learning techniques. Different ways of learning should lead to heterogeneous agents.

An interesting question is how the agents evolve their trading strategies, adapt to changing market conditions and learn how to improve their behavior. Already mentioned, learning is also an important source of heterogeneous decision behavior. Generally speaking, 'learning' or 'adaptive behavior' means that the agents are able to modify parts of their decision making schemes in a specific manner. By this, the agents adapt their behavior to changing market conditions.

The learning and adaptive behavior has different purposes. For example, rule-based agents may either develop completely new trading strategies or change a few parameters of already existing ones. Another possibility is that rule-based agents switch between different trading strategies as the market evolves. This corresponds to contagious decision making or so-called herding behavior. Moreover, forecasting agents learn by fitting the free parameters of their forecast models to historical data. During the learning, the agents generate a structural hypothesis of the underlying market dynamics, i.e. they try to identify invariant structures out of varying time series. The generated structures allow the agents to predict the future development of the market price.

5 Conclusion

In this paper, we mainly intended to confront the area of Economic Dynamics and Control with its roots and some connected fields, and to anchor the formal results in epistemological considerations, as a *sine qua non* condition for deeply understanding the underlying mechanisms and the present and future major trends.

Subsequently, the paper also attempted to evaluate the potential of some flexible trans-disciplinary paradigms for the always actual Aristotle’s propensity to *unifying the science*. When taking a look at these modern tools, we are rather optimistic. However, when exploring the epistemological foundations of the two qualitatively different fields (the anthropic one and the non-anthropic one), we find essential reasons to remain temperately pessimistic.

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