Design of Observer-Based 2-d Control Systems with Delays Satisfying Asymptotic Stablitiy Condition

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Abstract: This paper is concerned with the asymptotic stability of 2-d linear discrete systems with delays by means of observer-based controllers. The key point to accomplish it is the introduction of a Lyapunov function, which evaluates the energy of these 2-d systems and allows us to adopt the linear matrix inequality (LMI) approach. Finally, we emphasize that work on the asymptotic stability of these kinds of systems via 2-d observers with delays is still a novelty, to the best of author's knowledge.

Key–Words: 2-d linear discrete systems, delayed systems, 2-d asymptotic stability, observer design

1 Introduction

The asymptotic stability of 2-d systems has been a subject of research for many decades. In particular, the investigations on the grounds of the state space model have been intensively carried out since the introduction of the 2-d linear discrete system model, which was suggested by Roesser in the middle of 1970's [1]. From the practical standpoint, the importance of these studies resides in the fact that the discretisation of equations like the partial differential equation expressing the variation of temperature along a pipe in chemical systems are translated into 2-d state space model form [2], which are very suitable for computer simulations. In addition, the last decade has witnessed tremendous advances in the device technology and computational methods, leading to means to handle numerically with control systems problems. In this context, the linear matrix inequality (LMI) approach, which grew as an outcome of efforts to apply the optimisation methods onto the 1-d control design problems [3], has emerged as one of the most attractive and fruitful computational tools for dealing also with 2-d control systems issues ([4] and references therein).

The interesting point regarding these reports on 2-d control systems is that they have primarily concerned with finding a way to apply the LMI framework on the systems without delays, which have no terms with past information in the dynamics and/or outputs. On the other hand, the control problems for 2-d systems with delays have been addressed only recently. The design of memory and memoryless state feedback controller for 2-d systems with delays on the grounds of the LMI formalism is handled in the works by Izuta [5, 6, 7]. Also, he has focused on the problem of obtaining output feedback controller for 2-d systems with delays in the dynamics and outputs [8, 9].

In this paper, motivated by Izuta's report on the design of 2-d observers for 2-d systems with delays [10], we consider systems with delays in the dynamics and outputs and adopt an approach that lead to the controller in one step; i.e. by solving a single LMI. It is worth mentioning that there are some related works on 2-d observers for systems without delays. In the works done by Hinamoto [11], a design method based on the state transition matrix has been suggested; and in the investigations by Yaz [12], a computational scheme based on the matrix inequality is pursued. Nevertheless they are interesting in their own rights, the involving structure of the systems with delays prevents us from sharing the advantages of these procedures.

More specifically, the aim of this paper is to establish an observer controller for 2-d linear discrete system with delays (LDSwD) such that the composite feedback system, which is composed by the given system, an observer controller and feedback law, is asymptotically stable. Furthermore, to accomplish it, a Lyapunov function, which expresses the system's energy and makes it possible the to deal with the problem in the LMI settings, will be introduced.

The remainder of this paper is organised as follows. In section 2, the 2-d LDSwD, the observer and the basic definitions are presented. The results are given in section 3, which begins with the introduction of the Lyapunov function for 2-d LDSwD. Finally, a few remarks are given in the last section, 4.

2 Preliminaries

In this section, the definitions of the systems, the controller and the concept of asymptotic stability concerning 2-d systems are stated. And to make it clear what will be done in the sequel, we re-enunciate the problem to be solved in formal terms.

Definition 1 *A state space model for 2-d linear discrete systems with delays is given by*

$$\begin{aligned}\begin{bmatrix} x_1(i+1,j) \\ x_2(i,j+1) \end{bmatrix} &= \begin{bmatrix} A_{11} & A_{12} \\ A_{21} & A_{22} \end{bmatrix} \begin{bmatrix} x_1(i,j) \\ x_2(i,j) \end{bmatrix} \\ &+ \begin{bmatrix} A^{\theta}_{11} & A^{\theta}_{12} \\ A^{\theta}_{21} & A^{\theta}_{22} \end{bmatrix} \begin{bmatrix} x_1(i-\theta_1,j) \\ x_2(i,j-\theta_2) \end{bmatrix} \\ &+ \begin{bmatrix} B_{11} & B_{12} \\ B_{21} & B_{22} \end{bmatrix} \begin{bmatrix} u_1(i,j) \\ u_2(i,j) \end{bmatrix} \\ &+ \begin{bmatrix} B^{w}_{11} & B^{w}_{12} \\ B^{w}_{21} & B^{w}_{22} \end{bmatrix} \begin{bmatrix} w_1(i,j) \\ w_2(i,j) \end{bmatrix} \end{aligned} \tag{1}$$

$$\begin{aligned}\begin{bmatrix} y_1(i,j) \\ y_2(i,j) \end{bmatrix} &= \begin{bmatrix} C_{11} & C_{12} \\ C_{21} & C_{22} \end{bmatrix} \begin{bmatrix} x_1(i,j) \\ x_2(i,j) \end{bmatrix} \\ &+ \begin{bmatrix} C^{\phi}_{11} & C^{\phi}_{12} \\ C^{\phi}_{21} & C^{\phi}_{22} \end{bmatrix} \begin{bmatrix} x_1(i-\phi_1,j) \\ x_2(i,j-\phi_2) \end{bmatrix} \\ &+ \begin{bmatrix} D^{w}_{11} & D^{w}_{12} \\ D^{w}_{21} & D^{w}_{22} \end{bmatrix} \begin{bmatrix} w_1(i,j) \\ w_2(i,j) \end{bmatrix} \end{aligned}$$

where, for $k = 1, 2$ *and* $i, j \in Z_+$, $x_k(i,j) \in \Re^{m_k \times 1}$ *mean the current state variables.* $u_k(i,j) \in \Re^{p_k \times 1}$ *and* $y_k(i,j) \in \Re^{q_k \times 1}$ *are the inputs and outputs, respectively. Also, for* $l = 1, 2$, $w_k(i,j) \in \mathcal{L}_2$ *stand for the disturbances;* A_{kl}, A^{θ}_{kl}, B^{w}_{k}, B_{kl}, C_{kl}, C^{ϕ}_{kl}, *and* D^{w}_{kl} *are real valued matrices of appropriate dimension.*

Remark 2 *The term "delays" originates from the components* $\theta_k \in Z_+$ *and* $\phi_k \in Z_+$ *(*$k = 1, 2$*), which bear the influences of the past state information on the system's dynamics and the outputs. Note that the model with both* θ_k *and* ϕ_k *singled out is the Roesser model [1].*

In order to simplify the notations and compactly write the expressions, we shall denote the above and further equations according to the following the rules.

Definition 3 *A vector be simply written as*

$$v(i+\dagger, j+\ddagger) \overset{\text{def}}{=} \begin{bmatrix} v_1(i+\dagger, j) \\ v_2(i, j+\ddagger) \end{bmatrix}, \qquad \dagger,\ \ddagger\ \in Z. \tag{2}$$

and a real-valued matrix as

$$M_* \overset{\text{def}}{=} \begin{bmatrix} M^*_{11} & M^*_{12} \\ M^*_{21} & M^*_{22} \end{bmatrix}. \tag{3}$$

Thus, definition (1) translates into

Definition 4 *System (1) in compact form is denoted as*

$$\begin{aligned} x(i+1,j+1) &= Ax(i,j) \\ &\quad +A_{\theta}x(i-\theta_1, j-\theta_2) \\ &\quad +Bu(i,j) + B_w w(i,j), \\ y(i,j) &= Cx(i,j) \\ &\quad +C_{\phi}x(i-\phi_1, j-\phi_2) + Dw(i,j). \end{aligned} \tag{4}$$

Remark 5 *In the sequel, when it is clear from the context, the newly appearing vectors and matrices will not be formally defined. Moreover, for the sake of compactness of the notations, when no confusions and ambiguities arise, we may abuse notations and write the vectors and matrices without any distinction between the full description as in definition 1 and the compact form as in definition 4.*

Now the control systems is composed by

Definition 6 *An observer for (1) is*

$$\begin{aligned} z(i+1,j+1) &= Az(i,j) \\ &\quad +A_{\theta}z(i-\theta_1, j-\theta_2) \\ &\quad +Bu(i,j) + L[Cz(i,j) \\ &\quad +C_{\phi}z(i-\phi_1, j-\phi_2) - y(i,j)], \end{aligned} \tag{5}$$

and

Definition 7 *A feedback law for the control system is given by*

$$\begin{aligned} u(i,j) &= Kz(i,j) + K_{\theta}z(i-\theta_1, j-\theta_2) \\ &\quad +K_{\phi}z(i-\phi_1, j-\phi_2). \end{aligned} \tag{6}$$

Taking into consideration these definitions we define the composite system as

Definition 8 *Let* $\hat{x}(i,j)$ *mean*

$$\hat{x}(i,j)^T = \begin{bmatrix} x^T(i,j) & z^T(i,j) \end{bmatrix}. \tag{7}$$

Then the augmented system is expressed as

$$\begin{aligned} \hat{x}(i+1,j+1) = {} & \hat{A}\hat{x}(i,j) \\ & +\hat{A}_\theta \hat{x}(i-\theta_1, j-\theta_2) \\ & +\hat{A}_\phi \hat{x}(i-\phi_1, j-\phi_2) \\ & +\hat{B_w} w(i,j), \\ y(i,j) = {} & \hat{C}\hat{x}(i,j) \\ & +\hat{C}_\phi \hat{x}(i-\phi_1, j-\phi_2) + Dw(i,j), \end{aligned} \tag{8}$$

where

$$\begin{aligned} \hat{A} &= \begin{bmatrix} A & BK \\ -LC & A+LC+BK \end{bmatrix}, \\ \hat{A}_\theta &= \begin{bmatrix} A_\theta & -BK_\theta \\ 0 & A_\theta + BK_\theta \end{bmatrix}, \\ \hat{A}_\phi &= \begin{bmatrix} 0 & -BK_\phi \\ -LC_\phi & LC_\phi + BK_\phi \end{bmatrix}, \\ \hat{B}_w &= \begin{bmatrix} B_w \\ -LD \end{bmatrix}, \quad \hat{C} = \begin{bmatrix} C & 0 \end{bmatrix}, \\ \hat{C}_\phi &= \begin{bmatrix} C_\phi & 0 \end{bmatrix}. \end{aligned} \tag{9}$$

Remark 9 *In the sequel, we assume that the initial values of the dynamics are expressed as*

$$\begin{cases} \hat{x}_1(i,j) = \hat{x}^1_{(i,j)}, \\ \quad -\max\{\theta_1, \phi_1\} \le i \le 0, \forall j \ge 0, \\ \hat{x}_2(i,j) = \hat{x}^2_{(i,j)}, \\ \quad -\max\{\theta_2, \phi_2\} \le j \le 0, \forall i \ge 0, \\ \hat{x}^1_{(0,0)} = \hat{x}^2_{(0,0)}. \end{cases} \tag{10}$$

Throughout this paper we are concerned with the asymptotic stability of 2-d systems, which is defined as

Definition 10 *[2] Let $\bar{x}(i,j)$ be the dynamics of any unforced 2-d systems with delays. Then the system is said to be asymptotically stable if*

$$\lim_{(i+j)\to\infty} \| \bar{x}(i,j) \| \to 0. \tag{11}$$

Thus, the problem to be solved hereafter is

Problem 11 *to establish an observer controller on the basis of the LMI formalism such that the augmented system (8) is asymptotically stable.*

3 Results

In this section, we provide a solution to the proposed problem in two steps. Firstly, we require the observer error $e(i,j) = x(i,j) - z(i,j)$ to vanish as the indices increase; and complete the design operation by imposing that the composite system must fulfil the asymptotic stability condition. Moreover, since the reasonings are on the grounds of the LMI approach, we are compelled to introduce a suitable Lyapunov function at the very beginning of the design procedure.

Theorem 12 *Consider the dynamics of a generic 2-d system with delays given by*

$$\begin{aligned} \bar{x}(i+1,j+1) = {} & \bar{A}\bar{x}(i,j) \\ & +\bar{A}_\theta \bar{x}_\theta(i-\theta_1, j-\theta_2) \\ & +\bar{A}_\phi \bar{x}_\phi(i-\phi_1, j-\phi_2) \end{aligned} \tag{12}$$

and let the Lyapunov function be

$$\begin{aligned} V_\Sigma[\bar{x}(i,j)] \overset{\text{def}}{=} {} & V[\bar{x}(i,j)] \\ & +V_\theta[\bar{x}(i,j)] + V_\phi[\bar{x}(i,j)], \end{aligned} \tag{13}$$

where

$$\begin{aligned} & V[\bar{x}(i,j)] = \bar{x}^T(i,j)\Pi\bar{x}(i,j), \\ & V_\theta[\bar{x}(i,j)] = \textstyle\sum_{m=i-\theta_1}^{i-1} \bar{x}_1^T(m,j)\Theta_m \bar{x}_1(m,j) \\ & \qquad + \textstyle\sum_{n=j-\theta_2}^{j-1} \bar{x}_2^T(i,n)\Theta_n \bar{x}_2(i,n), \\ & V_\phi[\bar{x}(i,j)] = \textstyle\sum_{p=i-\phi_1}^{i-1} \bar{x}_1^T(p,j)\Phi_p \bar{x}_1(p,j) \\ & \qquad + \textstyle\sum_{q=j-\phi_2}^{j-1} \bar{x}_2^T(i,q)\Phi_q \bar{x}_2(i,q), \\ & \Theta_\star = (\Theta_\star)^T > 0, \quad \forall \star, \\ & \Pi = \operatorname{diag}\{\Pi_1, \Pi_2\} \\ & \Pi_k = \Pi_k^T > 0, \quad k = 1,2; \\ & \Phi_{\star\star} = (\Phi_{\star\star})^T > 0, \quad \forall \star\star. \end{aligned} \tag{14}$$

then the variation of energy ΔV_Σ given by

$$\Delta V_\Sigma \overset{\text{def}}{=} V_\Sigma[\bar{x}(i+1,j+1)] - V_\Sigma[\bar{x}(i,j)] < 0 \tag{15}$$

if

$$\Upsilon = \begin{bmatrix} \bar{A}^T\Pi\bar{A} - \begin{pmatrix} \Pi \\ +\Xi \end{pmatrix} & * & * \\ \bar{A}_\theta^T \Pi \bar{A} & \begin{pmatrix} \bar{A}_\theta^T \Pi \bar{A}_\theta \\ -\Theta \end{pmatrix} & * \\ \bar{A}_\phi^T \Pi \bar{A} & \bar{A}_\phi^T \Pi \bar{A}_\theta & \bar{A}_\phi^T \Pi \bar{A}_\phi - \Phi \end{bmatrix} < 0, \tag{16}$$

where $*$ stands for the symmetric entry of the matrix and

$$\begin{aligned} \Xi &= \Xi^T = \Theta_i \oplus \Theta_j + \Psi_i \oplus \Psi_j > 0, \\ \Theta &= \Theta^T = \Theta_{i-\theta_1} \oplus \Theta_{j-\theta_2} > 0, \\ \Psi &= \Psi^T = \Psi_{i-\phi_1} \oplus \Psi_{j-\phi_2} > 0, \end{aligned} \tag{17}$$

with $\oplus$ meaning the direct product of the operands.

Proof: Firstly, note that

$$\begin{aligned} V_\Sigma[\bar{x}(i+1,j+1)] = \xi^T \Gamma \xi \\ +\zeta_1^T(i-1,j-1)\Theta_1\zeta_1(i-1,j-1) \\ +\zeta_2^T(i-1,j-1)\Theta_2\zeta_2(i-1,j-1) \\ +\eta_1^T(i-1,j-1)\Phi_1\eta_1(i-1,j-1) \\ +\eta_2^T(i-1,j-1)\Phi_2\eta_2(i-1,j-1), \end{aligned} \tag{18}$$

where

$$\Gamma = \begin{bmatrix} \bar{A}^T\Pi\bar{A} & * & * \\ \bar{A}_\theta^T\Pi\bar{A} & \bar{A}_\theta^T\Pi\bar{A}_\theta & * \\ \bar{A}_\phi^T\Pi\bar{A} & \bar{A}_\phi^T\Pi\bar{A}_\theta & \bar{A}_\phi^T\Pi\bar{A}_\phi \end{bmatrix}, \tag{19}$$

and

$$\begin{aligned} \xi &= \begin{bmatrix} \bar{x}(i,j) \\ \bar{x}(i-\theta_1,j-\theta_2) \\ \bar{x}(i-\phi_1,j-\phi_2) \end{bmatrix}, \\ \star_1(i,j) &= \begin{bmatrix} \bar{x}_1(i,j) \\ \vdots \\ \bar{x}_1(i-\star\star_1+1,j) \end{bmatrix}, \\ &\begin{cases} \star=\zeta, \star\star=\theta \\ \quad \text{or} \\ \star=\eta,\ \star\star=\phi \end{cases} \\ \bullet_2(i,j) &= \begin{bmatrix} \bar{x}_2(i,j) \\ \vdots \\ \bar{x}_2(i,j-\bullet\bullet_2+1) \end{bmatrix}, \\ &\begin{cases} \bullet=\zeta, \bullet\bullet=\theta \\ \quad \text{or} \\ \bullet=\eta,\ \bullet\bullet=\phi \end{cases} \end{aligned} \tag{20}$$

and

$$\begin{aligned} \Theta_1 &= \Theta_i \oplus \cdots \oplus \Theta_{i-\theta_1+1}, \\ \Theta_2 &= \Theta_j \oplus \cdots \oplus \Theta_{j-\theta_2+1}, \\ \Phi_1 &= \Phi_i \oplus \cdots \oplus \Phi_{i-\phi_1+1}, \\ \Phi_2 &= \Phi_j \oplus \cdots \oplus \Phi_{j-\phi_2+1}. \end{aligned} \tag{21}$$

Now since $V_\Sigma[\bar{x}(i,j)]$ is given by

$$\begin{aligned} V_\Sigma[\bar{x}(i,j)] = \bar{x}(i,j)^T\Pi\bar{x}(i,j) \\ +\zeta_1^T(i-1,j-1)\Theta_1\zeta_1(i-1,j-1) \\ +\zeta_2^T(i-1,j-1)\Theta_2\zeta_2(i-1,j-1) \\ +\eta_1^T(i-1,j-1)\Phi_1\eta_1(i-1,j-1) \\ +\eta_2^T(i-1,j-1)\Phi_2\eta_2(i-1,j-1), \end{aligned} \tag{22}$$

some algebraic calculations yield

$$\Delta V_\Sigma[\bar{x}(i+1,j+1)] = \xi^T\ \Upsilon\ \xi \tag{23}$$

and the claim follows. □

The above result guarantees the asymptotic stability of the system as shown next.

Theorem 13 *Consider the system as given in theorem 12 and assume that $\Upsilon < O$. Then the system is asymptotically stable.*

Proof: Consider the diagonal lines

$$D_r = \{(i,j) : i+j = r\} \tag{24}$$

and

$$D_{\bar{r}} = \{(\bar{i},\bar{j}) : \bar{r} = \bar{i}+\bar{j} > r\} \tag{25}$$

with $\bar{r} >$ maximum delay. Since by hypothesis $\Upsilon < 0$; i.e. $\Delta V_\Sigma < 0$, it follows that

$$\sum_{\text{along} D_{\bar{r}}} V_\Sigma[\bar{x}(\bar{i},\bar{j})] < \sum_{\text{along} D_r} V_\Sigma[\bar{x}(i,j)]. \tag{26}$$

holds. In other words, $V_\Sigma[\bar{x}(\hat{i},\hat{j})]$ will vanish asymptotically for all $D_{\hat{r}}$, such that

$$\hat{r} = \hat{i}+\hat{j} > \bar{r} > r, \tag{27}$$

which is the claim of the theorem. □

The next result provides us with a procedure for establishing an observer, when it exists.

Theorem 14 *Consider the observer error $e(i,j)$, which reads*

$$\begin{aligned} e(i+1,j+1) = (A+LC)e(i,j) \\ +A_\theta e(i-\theta_1,j-\theta_2) \\ +LC_\phi e(i-\phi_1,j-\phi_2). \end{aligned} \tag{28}$$

Then (28) vanishes in the sense of definition 10 if there exist a matrix V and positive definite matrices X, Y, Z, W such that the LMI

$$\begin{bmatrix} E_1 & E_2 \\ E_2^T & E_3 \end{bmatrix} < 0 \tag{29}$$

is feasible, where

$$E_1 = \begin{bmatrix} -X+Y & 0 & 0 \\ 0 & -Z & 0 \\ 0 & 0 & -W \end{bmatrix}, \tag{30}$$

$$E_2 = \begin{bmatrix} A^TX + C^TV & 0 & 0 \\ A_\theta^T X & & 0 \\ C_\phi^T V & 0 & 0 \end{bmatrix}, \tag{31}$$

$$E_3 = \begin{bmatrix} -X & 0 & 0 \\ 0 & -I & 0 \\ 0 & 0 & -I \end{bmatrix}. \tag{32}$$

Furthermore, from the solution of (29), (5) is completely determined with

$$L = V^TX^{-1} \tag{33}$$

Proof: Since the hypothesis allows us to make use of the previous theorems, (16) applied to (28) can be written as

$$\begin{aligned} \Upsilon &= \mathrm{diag}\{-\Pi + \Xi, -\Theta, -\Phi\} \\ &+ \Upsilon_1^T \Upsilon_2^{-1} \Upsilon_1 < 0 \end{aligned} \quad (34)$$

where

$$\Upsilon_1 = \begin{bmatrix} (A+LC)\Pi & 0 & 0 \\ A_\theta \Pi & 0 & 0 \\ (LC_\phi)\Pi & 0 & 0 \end{bmatrix} \quad (35)$$
$$\Upsilon_2 = \begin{bmatrix} \Pi^{-1} & 0 & 0 \\ 0 & I & 0 \\ 0 & 0 & I \end{bmatrix}.$$

Now, on recalling here the Schur Complement [3], (34) is translated into the LMI

$$\begin{bmatrix} \bar{E}_1 & \bar{E}_2 \\ \bar{E}_2^T & \bar{E}_3 \end{bmatrix} < 0 \quad (36)$$

where

$$\bar{E}_1 = \begin{bmatrix} -\Phi + \Xi & 0 & 0 \\ 0 & -\Theta & 0 \\ 0 & 0 & -\Phi \end{bmatrix},$$
$$\bar{E}_2 = \begin{bmatrix} \Pi A^T + \Pi C^T L^T & 0 & 0 \\ \Pi A_\theta^T & 0 & \\ \Pi C_\phi^T L^T & 0 & 0 \end{bmatrix}, \quad (37)$$
$$\bar{E}_3 = \begin{bmatrix} -\Pi & 0 & 0 \\ 0 & -I & 0 \\ 0 & 0 & -I \end{bmatrix}.$$

Next, we apply the congruence transformation, i.e. pre and post multiply the LMI (36) by $\mathrm{diag}\{\Pi^{-1}, \Pi^{-1}, \Pi^{-1}, \Pi^{-1}, I, I\}$ and redefine the variables of the resulting LMI as

$$\begin{aligned} X = \Pi^{-1}, \quad Y = \Xi, \quad Z = \Theta, \\ W = \Phi, \quad V = XL^T. \end{aligned} \quad (38)$$

As a consequence we obtain (29). □

Once the matrix L is determined, it remains to establish (6) to complete the feedback control system. This is accomplished with the following result.

Theorem 15 *Consider system (8) and the hypotheses as in theorems (12) and (13). Then a controller (6) that asymptotically stabilises the unforced system (8) is given by*

$$\begin{aligned} K &= B^\dagger R^T X_2^{-1} \\ K_\theta &= B^\dagger S^T X_2^{-1} \\ K_\phi &= B^\dagger U^T X_2^{-1} \end{aligned} \quad (39)$$

where † means for the pseudo inverse and the matrices R, S and U are a solution of the LMI

$$\begin{bmatrix} \hat{E}_1 & \hat{E}_2 \\ \hat{E}_2^T & \hat{E}_3 \end{bmatrix} < 0 \quad (40)$$

in which the positive definite matrices $X = X_1 \oplus X_2$, $Y = Y_1 \oplus Y_2$, $Z = Z_1 \oplus Z_2$, $W = W_1 \oplus W_2$ are also variables of the LMI and

$$\hat{E}_1 = \begin{bmatrix} -X + Y & 0 & 0 \\ 0 & -Z & 0 \\ 0 & 0 & -W \end{bmatrix},$$
$$\hat{E}_2 = \begin{bmatrix} X_1 A^T & -X_1 C^T L^T & 0 \\ R & \begin{pmatrix} X_2 A^T X_2 \\ +X_2 C^T L^T \\ +R \end{pmatrix} & 0 \\ X_1 A_\theta^T & 0 & 0 \\ -S & \begin{pmatrix} X_2 A_\theta^T \\ +R \end{pmatrix} & 0 \\ 0 & -X_1 C_\phi^T L^T & 0 \\ -U & \begin{pmatrix} X_2 C_\phi^T L^T \\ +U \end{pmatrix} & 0 \end{bmatrix}, \quad (41)$$
$$\hat{E}_3 = \begin{bmatrix} -X & 0 & 0 \\ 0 & -I & 0 \\ 0 & 0 & -I \end{bmatrix}.$$

Proof: The proof parallels the one presented for theorem 14. The only difference between them is that here we do not need the congruence transformation to pursue the desired LMI. □

4 Final Remarks

In this paper, we provided a solution to the problem of designing a observer-based controller for 2-d linear discrete systems with delays. Basically, the procedure is composed by two LMIs, which are computed separately. The first step requires that the observer must estimate the dynamics of the system in the sense that the error between the system's and observer's dynamics signals vanishes as the "time" increases, whereas the second one imposes the asymptotic stability condition on the augmented system, which is composed by the original system, the observer and the feedback loop. In doing so, from the first LMI we obtain the observer controller and we complete the design of control system by computing the second one. It is worth noting that the approach presented here is useful in cases where the dynamics of the system is not available for feed backing, which happens in most of practical situations.

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