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On stability and instability in acoustic flows

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Abstract

The standard method of handling the linear acoustic problem of alternating material layers is using a Floquet/Bloch form for solutions, assuming that time-dependence of solutions is periodic (the mono-frequency condition). However, an important question as to whether the solution is bounded under mono-frequency conditions has remained out of the scope. Even for finite domains of alternating layers, where a solution cannot grow to infinity, this is an important issue: if a solution becomes big enough, then the linear approximation can no longer be valid.

In this paper we study the 1-dimensional flow-acoustic problem for periodically arranged materials where a fluid flow in general persists in both constituent media. It is proven that the flow-acoustic equations are linear Hamiltonian equations with periodic coefficients. Exact conditions are found for the stability of a zero solution to the acoustic equations under the standard assumption that the background density and fluid velocity are constants in each constituent media.

Keywords: flow acoustic, periodic domains, stability

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1 Introduction

In recent years, acoustics in periodic structures has been widely studied: it has been shown that, due to special arrangement of the two materials, the speed of sound in the mixed structure can drop drastically [1], allowing for several possible applications, see, e.g., Refs. [2, 3, 4], and also Ref. [5], where the flow acoustics in periodic structures has been studied. The question of stability for flow acoustics has been addressed in [6], where we have assumed that the solutions may have general time-dependence (not necessarily being of mono-frequent form).

The standard method of handling the (linear) acoustic problem is using the Bloch form for solutions, assuming that time-dependence of solutions is periodic (the mono-frequency condition). However, an important question of whether the solution is bounded under the mono-frequency condition, has remained out of the scope. Although, as the material domain is itself bounded, no solution can grow to infinity (due to linearity of the problem), one should be aware if it becomes big enough – in this case the linear approximation can no longer be valid.

In this paper we study the acoustic problem for periodically arranged materials (materials A and B) in the 1-dimensional case, i.e., periodicity being extended only in one direction. We assume that a fluid flows in both constituent media (In Ref [5], the fluid flows only in one material, but this has no influence on the acoustic equations).

We first show that the acoustic equations in question are linear Hamiltonian equations with periodic coefficients. Thus, the question of whether the solution for acoustic equations is bounded (in the infinite domain) reduces to the well-known problem of stability of the zero solution of linear Hamiltonian equations: if the zero solution is stable, then there cannot be unbounded solutions. The zero solution is stable, if the absolute value of the trace of the monodromy matrix is less than 2, and we calculate this matrix explicitly for our system. Notice that stability and growth of solutions have nothing to do with the *method* one uses to solve the system.

2 Setting up equations of flow acoustics

Under the mono-frequency condition, the linearized equations for the acoustic flow are (see, e.g., Ref. [5]):

$$\begin{aligned}\frac{i\omega}{c^2}p+\frac{v_0}{c^2}\frac{\partial p}{\partial x}+\rho_0\Big(\frac{\partial v_x}{\partial x}+\frac{\partial v_y}{\partial y}+\frac{\partial v_z}{\partial z}\Big)&=0,\\ i\omega v_x+v_0\frac{\partial v_x}{\partial x}+v_y\frac{\partial v_0}{\partial y}+v_z\frac{\partial v_0}{\partial z}&=-\frac{1}{\rho_0}\frac{\partial p}{\partial x},\\ i\omega v_y+v_0\frac{\partial v_y}{\partial x}&=-\frac{1}{\rho_0}\frac{\partial p}{\partial y},\\ i\omega v_z+v_0\frac{\partial v_z}{\partial x}&=-\frac{1}{\rho_0}\frac{\partial p}{\partial z}.\end{aligned}\qquad(1)$$

Here $\mathbf{v}_0 = (v_0(y,z),0,0)$ is the background flow, $\rho_0(y,z)$ is the pressure of the background flow, p is the acoustic pressure, and $\mathbf{v}=(v_x,v_y,v_z)$ is the acoustic flow velocity.

We are looking for the solution of the following form:

$$\frac{\partial p}{\partial x}=i\beta p,\quad \frac{\partial \mathbf{v}}{\partial x}=i\beta\mathbf{v}.\qquad(2)$$

Substituting relations (2) into equations (1), we get an equation for the pressure p, cf. Ref. [5]:

$$\begin{aligned}&\frac{1}{\rho_0(\omega+\beta v_0)^2}\frac{\partial^2 p}{\partial y^2}-\frac{1}{\rho_0^2(\omega+\beta v_0)^2}\frac{\partial\rho_0}{\partial y}\frac{\partial p}{\partial y}\\ &-\frac{2\beta}{\rho_0(\omega+\beta v_0)^3}\frac{\partial v_0}{\partial y}\frac{\partial p}{\partial y}++\frac{1}{\rho_0(\omega+\beta v_0)^2}\frac{\partial^2 p}{\partial z^2}\\ &-\frac{1}{\rho_0^2(\omega+\beta v_0)^2}\frac{\partial\rho_0}{\partial z}\frac{\partial p}{\partial z}-\frac{2\beta}{\rho_0(\omega+\beta v_0)^3}\frac{\partial v_0}{\partial z}\frac{\partial p}{\partial z}\\ &+\Big(\frac{1}{c^2\rho_0}-\frac{\beta^2}{\rho_0(\omega+\beta v_0)^2}\Big)p=0.\end{aligned}\qquad(3)$$

We consider now the 1-dimensional case, when the functions ρ_0, c and v_0 do not depend on y, and we assume that the pressure p does not depend on y either. We have also assumed periodicity in z, i.e., ρ_0, c and v_0 are 2π-periodic functions of z. Then Equation (3) can be written as

$$\frac{\partial}{\partial z}\Big(\frac{1}{\rho_0(\omega+\beta v_0)^2}\frac{\partial p}{\partial z}\Big)+\Big(\frac{1}{c^2\rho_0}-\frac{\beta^2}{\rho_0(\omega+\beta v_0)^2}\Big)p=0.\qquad(4)$$

This is an Euler-Lagrange equation. We can put Equation (4) into the Hamiltonian form by introducing the canonical conjugate variable

$$q=\frac{1}{\rho_0(\omega+\beta v_0)^2}\frac{\partial p}{\partial z}.$$

Equation (4) now becomes

$$q'=-\frac{\partial H}{\partial p},\quad p'=\frac{\partial H}{\partial q},\qquad(5)$$

where $(\cdot)'=\partial/\partial z$, and the Hamiltonian equals

$$H(p,q)=\frac{1}{2}\Big(\rho_0(\omega+\beta v_0)^2q^2+\Big(\frac{1}{c^2\rho_0}-\frac{\beta^2}{\rho_0(\omega+\beta v_0)^2}\Big)p^2\Big)$$

Thus the spacial z-coordinate plays now the role of time in Hamilton's equations (5).

Remark. The Hamiltonian system (5) is integrable: this is a consequence of the Lyapunov-Floquet theorem that states that there exists a linear periodic in z transformation $p,q\to P,Q$, that transforms the linear system (5) with the coefficients 2π-periodic in

z, to a linear system with the constant coefficients. However, for arbitrary functions ρ_0 and v_0, one cannot find this variable transformation *explicitly*, cf. Ref. [7].

3 Periodic structure and monodromy matrix

Let n denote the number of the layers of materials A and B. We assume here that the width of each $A-B$-layer is 2π, thus when n grows, we "build" the layers one above the over.

We denote by ρ_0^A, v_0^A and c_A the values of the background density, fluid velocity and the velocity of sound in the material A, and by ρ_0^B, v_0^B, c_B the corresponding values in the material B:

$$\rho_0 = \rho_0^A, \quad v_0 = v_0^A, \quad c = c_A \quad \text{for} \quad z \in [0, z_1),$$
$$\rho_0 = \rho_0^B, \quad v_0 = v_0^B, \quad c = c_B \quad \text{for} \quad z \in [z_1, 2\pi)$$

Both ρ_0^A, v_0^A, c_A, ρ_0^B, v_0^B and c_B are the functions of z, and we assume that they are smooth. However, the functions ρ_0, v_0 and c need not even to be continuous. Equation (5) reads

$$p' = F_1^A q, \quad q' = -F_2^A p, \quad z \in [0, z_1),$$

$$p' = F_1^B q, \quad q' = -F_2^B p, \quad z \in [z_1, 2\pi), \tag{6}$$

where the functions $F_{1,2}^{A,B}$ are

$$F_1^A = \rho_0^A(\omega + \beta v_0^A)^2, \quad F_2^A = \frac{1}{c_A^2 \rho_0^A} - \frac{\beta^2}{\rho_0^A(\omega + \beta v_0^A)^2}, \tag{7}$$

$$F_1^B = \rho_0^B(\omega + \beta v_0^B)^2, \quad F_2^B = \frac{1}{c_a^2 \rho_0^B} - \frac{\beta^2}{\rho_0^B(\omega + \beta v_0^B)^2} \tag{8}$$

When we glue the solutions in water and in air together, we demand that *the canonical coordinates q, p should be continuous*, rather than p, p' (cf., e.g., [8]). This is important for the derivative

$$\frac{\partial}{\partial z}\Big(\frac{1}{\rho_0(\omega + \beta v_0)^2}\frac{\partial p}{\partial z}\Big)$$

to be defined (in the generalized sense). To this extent, our model closely resembles the Ben Daniel-Duke model [9].

For applications, one should solve Equation (4) with the boundary conditions: the flow region is bounded by the interval $[z_L, z_R]$ (its width is of order n), and we demand that $q = 0$ for $z = z_L$, $z = z_R$ (the normal of the pressure gradient is zero on the boundary), and $p \neq 0$ for $z = z_L$ (to get nontrivial solutions). Solutions then exist for a sequence of discrete values of the wavenumber β.

It is important to find out how solutions to Equation (4) behave in the phase plane p, p' (or p, q): if a typical solution increases exponentially, then for big values of n the linear model cannot be applied. Thus, we have to investigate the stability of the zero solution to Equation (4). Note that stability and growth of solutions have nothing to do with the *method* one uses to solve the system: for example, the same problems arise when one uses the Fourier expansions and the Bloch form to find the solution.

The stability of the zero solution to Equation (4) is determined by the trace of a *monodromy matrix*, that is defined in the following way. Consider a mapping $p, q|_{z=0} \to p, q|_{z=2\pi}$. This mapping is linear (as the equation itself is linear), and thus can be written as

$$\begin{pmatrix} p(2\pi) \\ \\ q(2\pi) \end{pmatrix} = \mathrm{A} \begin{pmatrix} p(0) \\ \\ q(0) \end{pmatrix},$$

where A is a constant matrix (i.e., $\partial \mathrm{A}/\partial z = 0$). This matrix is called the monodromy matrix, see, e.g., Ref. [10].

As our system is Hamiltonian, the determinant $det(\mathrm{A}) = 1$ (as the phase flow of a Hamiltonian system preserves area $dp \wedge dq$). If the trace $|Tr(\mathrm{A})| > 2$, then there is an eigenvalue, that has an absolute value greater than 1, thus a typical solution grows exponentially fast in space along the z-direction. If the trace $|Tr(\mathrm{A})| < 2$, then both eigenvalues lie on the unit circle on the complex plane (and are complex conjugated), and are different, thus every solution is bounded.

Suppose that both in water and in air the zero solution is stable. However, this may not guarantee the stability of complete solution (cf. Ref. [10]). Together with the other parameters, the stability of the solution in composite materials depends on the ratio of these two layers – i.e., materials A and B.

4 Stability condition

In applications, the monodromy matrix should be calculated numerically – cf. Remark, Section 2, but one needs not necessarily have the explicit expression for the monodromy matrix to examine stability of the system. There are numerous results on stability of zero solutions to linear Hamiltonian systems with periodic coefficients and the structure of the stability domains. We will here only refer to the classical result of Lyapunov [11] on sufficient conditions for stability, and to Ref. [12] for the topology of the set of stable systems.

We will assume here that the *functions* ρ_0*,* v_0*, and* c *are constants in each media*, which is a reasonable approximation, especially for time-averaged turbulent flow profiles, cf. Ref. [5].

As we have assumed above, in each media, the zero solution is stable. Thus,

$$F_2^A = \frac{1}{c_A^2\rho_0^A} - \frac{\beta^2}{\rho_0^A(\omega+\beta v_0^A)^2} > 0,$$
$$F_2^B = \frac{1}{c_a^2\rho_0^B} - \frac{\beta^2}{\rho_0^B(\omega+\beta v_0^B)^2} > 0,$$

as $F_1^{a,w} > 0$ and $H = 1/2(F_1q^2 + F_2p^2)$.

We first solve the system (6). Let $p(0) = p_0$, $q(0) = q_0$. Then, since $H = \frac{1}{2}(F_1q^2 + F_2p^2)$, for $z = z_1$,

$$p(z_1) = p_0\cos\sqrt{F_1^AF_2^A}z_1 + q_0\sqrt{\frac{F_1^A}{F_2^A}}\sin\sqrt{F_1^AF_2^A}z_1,$$
$$q(z_1) = -p_0\sqrt{\frac{F_2^A}{F_1^A}}\sin\sqrt{F_1^AF_2^A}z_1 + q_o\cos\sqrt{F_1^AF_2^A}z_1,$$

thus the mapping $p(0), q(0) \to p(z_1), q(z_1)$ is given by the matrix

$$\mathrm{A}_A = \begin{pmatrix} \cos\sqrt{F_1^AF_2^A}z_1 & \sqrt{\frac{F_1^A}{F_2^A}}\sin\sqrt{F_1^AF_2^A}z_1 \\ -\sqrt{\frac{F_2^A}{F_1^A}}\sin\sqrt{F_1^AF_2^A}z_1 & \cos\sqrt{F_1^AF_2^A}z_1 \end{pmatrix}$$

Similarly, the mapping $p(z_1), q(z_1) \to p(2\pi), q(2\pi)$ is given by the matrix

$$\mathrm{A}_B = \begin{pmatrix} \cos\sqrt{F_1^BF_2^B}(2\pi-z_1) & \sqrt{\frac{F_1^B}{F_2^B}}\sin\sqrt{F_1^BF_2^B}(2\pi-z_1) \\ -\sqrt{\frac{F_2^B}{F_1^B}}\sin\sqrt{F_1^BF_2^B}(2\pi-z_1) & \cos\sqrt{F_1^BF_2^B}(2\pi-z_1) \end{pmatrix},$$

(the mapping $p(z_1), q(z_1) \to p(2\pi), q(2\pi)$ is the same as the mapping $p(0), q(0) \to p(2\pi - z_1), q(2\pi - z_1)$).

Now, the monodromy matrix is $\mathrm{A} = \mathrm{A}_A\mathrm{A}_B$. To find the trace, we are only interested in the diagonal elements of this matrix:

$$(\mathrm{A}_A\mathrm{A}_B)_{11} = \cos\sqrt{F_1^AF_2^A}z_1\cos\sqrt{F_1^BF_2^B}(2\pi-z_1) - -\sqrt{\frac{F_1^A}{F_2^A}}\sqrt{\frac{F_2^B}{F_1^B}}\sin\sqrt{F_1^AF_2^A}z_1\sin\sqrt{F_1^BF_2^B}(2\pi-z_1),$$

$$(\mathrm{A}_A\mathrm{A}_B)_{22} = -\sqrt{\frac{F_2^A}{F_1^A}}\sqrt{\frac{F_1^B}{F_2^B}}\sin\sqrt{F_1^AF_2^A}z_1\sin\sqrt{F_1^BF_2^B}(2\pi-z_1) + \cos\sqrt{F_1^AF_2^A}z_1\cos\sqrt{F_1^BF_2^B}(2\pi-z_1). \quad (9)$$

Thus, *the stability region is determined by condition*

$$|Tr(\mathrm{A})| = |2\cos\sqrt{F_1^AF_2^A}z_1\cos\sqrt{F_1^BF_2^B}(2\pi-z_1) - \left(\sqrt{\frac{F_1^A}{F_2^A}}\sqrt{\frac{F_2^B}{F_1^B}} + +\sqrt{\frac{F_2^A}{F_1^A}}\sqrt{\frac{F_1^B}{F_2^B}}\right)\times \sin\sqrt{F_1^AF_2^A}z_1\sin\sqrt{F_1^BF_2^B}(2\pi-z_1)| < 2, \quad (10)$$

where

$$F_1^A = \rho_0^A(\omega+\beta v_0^A)^2, \quad F_2^A = \frac{1}{c_A^2\rho_0^A} - \frac{\beta^2}{\rho_0^A(\omega+\beta v_0^A)^2},$$
$$F_1^B = \rho_0^B(\omega+\beta v_0^B)^2, \quad F_2^B = \frac{1}{c_a^2\rho_0^B} - \frac{\beta^2}{\rho_0^B(\omega+\beta v_0^B)^2}.$$

If the trace $|Tr(\mathrm{A})| = 2$, then solutions are bounded if and only if the matrix A is diagonalizable. In other cases – namely, if the trace $|Tr(\mathrm{A})| > 2$, or if $|Tr(\mathrm{A})| = 2$, but if the matrix A is not diagonalizable, almost all solutions are unbounded.

5 Discussion

In typical applications, as for example, for the water-air mixture (material A (B) is water (air)), the value F_1^A is large, F_2^A is small, and all other values (including the product $F_1^A F_2^A$) are of order 1 for all relevant frequencies, i.e., 1 kHz or higher. Indeed, for water (A) and air (B), we have

$$\rho^A = 1000\,kg/m^3, \qquad \rho^B = 1.28\,kg/m^3,$$
$$c_A = 1500\,m/s, \qquad c_B = 343\,m/s$$

Typical values of β are $\beta \simeq \omega/c$, and we assume that the background flow velocity v_0 is of order $1m/s$. One can show that in this case

$$O(F_1^A) \gg O(F_1^A F_2^A) \gg O(F_2^A).$$

In this case the estimate for z_1 can be found in the following way.

First, from (10) we notice, that for almost all values either $\sin\sqrt{F_1^A F_2^A}z_1$ or $\sin\sqrt{F_1^B F_2^B}(2\pi - z_1)$ should be close to zero. This provides the zero-order estimate in $\sqrt{F_2^A/F_1^A}$:

$$z_1 = \frac{1}{\sqrt{F_1^A F_2^A}}\pi k, \quad \text{or} \quad z_1 = \frac{1}{\sqrt{F_1^B F_2^B}}\pi k + 2\pi, \quad k \in \mathbb{Z},$$

which, together with the condition $0 \le z_1 \le 2\pi$ gives only a *finite* number of such values of z_1.

Suppose now that $2\sqrt{F_1^A F_2^A} \notin \mathbb{N}$, and consider the case when $|\zeta \equiv 2\pi - z_1|$ is small (zero in the zero-order approximation). Formally, from (10), for $2\sqrt{F_1^A F_2^A} \in \mathbb{N}$ and $\zeta = 0$, we get $|Tr(\mathrm{A})| = 2$.

The first-order approximation for $z_1 = 2\pi - \zeta$ is determined by

$$\begin{aligned}&|2(\cos 2\pi\sqrt{F_1^A F_2^A} + \sqrt{F_1^A F_2^A}\zeta \sin 2\pi\sqrt{F_1^A F_2^A}\,) \\ &-\left(\sqrt{\frac{F_1^A}{F_2^A}}\sqrt{\frac{F_2^B}{F_1^B}} + \sqrt{\frac{F_2^A}{F_1^A}}\sqrt{\frac{F_1^B}{F_2^B}}\right)\sqrt{F_1^B F_2^B}\zeta \sin 2\pi\sqrt{F_1^A F_2^A}| \\ &< 2, \quad (11)\end{aligned}$$

which gives two inequalities:

$$0 \le \zeta < \frac{2 + 2\cos 2\pi\sqrt{F_1^A F_2^A}}{\left(F_1^B\sqrt{\frac{F_2^A}{F_1^A}} + F_2^B\sqrt{\frac{F_1^A}{F_2^A}} - \sqrt{F_1^A F_2^A}\right)\sin 2\pi\sqrt{F_1^A F_2^A}}, \quad (12)$$

for $\sin 2\pi\sqrt{F_1^A F_2^A} > 0$, and

$$0 \le \zeta < \frac{2 - 2\cos 2\pi\sqrt{F_1^A F_2^A}}{\left(F_1^B\sqrt{\frac{F_2^A}{F_1^A}} + F_2^B\sqrt{\frac{F_1^A}{F_2^A}} - \sqrt{F_1^A F_2^A}\right)\sin 2\pi\sqrt{F_1^A F_2^A}}, \quad (13)$$

for $\sin 2\pi\sqrt{F_1^A F_2^A} < 0$.

From inequality (10) we can see how stability region looks like for $2\sqrt{F_1^A F_2^A}$ being close to a natural number.

Let us assume that $2\sqrt{F_1^A F_2^A} = 2 - \lambda$. Consider the expansion of (10) in ζ up to quadratic terms. One can show that for small values of $|\lambda|$,

$$\zeta < c_1\lambda + O(\lambda^2) \quad \text{for} \quad \lambda > 0,$$

$$\zeta < -c_2\lambda + O(\lambda^2) \quad \text{for} \quad \lambda < 0; \quad c_1, c_2 > 0.$$

Thus, for the water-air mixture, the simplest recommendation – to choose $2\pi - z_1$ small enough (i.e., reduce the layer of air) – does not necessarily lead to boundedness of solutions. For example, for most values of the parameters, the solutions become unbounded if inequalities (12-13) are not satisfied. The effect we have described is called "parametric resonance" and it has been studied in details, see, e.g., Ref [8].

6 Variations in number of layers for a given flow domain

In the previous sections we assumed that, as the number of layers n grows, so does the size of the flow domain. Instead, we can consider the case when the domain is fixed, and as n grows, the width of each layer becomes smaller.

The equations of motion remain the same, however the period of the periodic structure changes: in sin's and cos's, the coefficient 2π should be replaced with $2\pi/n$, and $0 < z_1 < 2\pi/n$.

From (10) we get

$$|Tr(\mathrm{A})| < 2 - \frac{c}{n^2} < 2$$

for some constant $c > 0$, provided $n \gg 1$, for all values of z_1 and all other parameters. Thus, for big values of the layer number n, we always get bounded solutions.

7 Conclusion

In this paper, we studied the question as to whether the solution to the linear flow acoustic equations in domains with alternating material layers can grow in space under mono-frequency conditions. For finite domains, where a solution cannot grow to infinity, this is an important issue: if a solution becomes big enough, then the linear approximation can no longer be valid.

We prove that the 1-dimensional acoustic problem for periodically arranged materials (where a fluid may flow in both constituent media) is described by a linear Hamiltonian system with periodic coefficients. We find the exact conditions for stability of a zero solution to acoustic equations under the standard assumption that the background density and fluid velocity are constants in each constituent media.

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