# Canonical Structures of ARC Biquad Based on Single Transimpedance Operational Amplifiers 

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#### Abstract

The paper deals with simplest canonical structures of the second order ARC filters employing only single transimpedance operational amplifiers (also titled current feedback amplifiers) and passive components R and C . A systematic design procedure of this circuits based on the suitable autonomous networks is described. Several appropriate general autonomous circuits are presented and studied.


Key-Words: - Active RC filters, single amplifier biquads, transimpedance operational amplifiers, current feedback.

## 1. Introduction

Circuits which employ a single active commercially available functional block (SAB) and RC passive elements are really right choice to realize a secondorder transfer function. They usually offer a low cost, low DC power, low noise, low sensitivity and furthermore simple design.
Classical ARC filters based on standard operational amplifier are used only in low frequency applications, lower some hundred kHz . This limitation can be overcome replacing the conventional voltage amplifier by a transimpedance operational amplifier (TIOA) [1]. Owing to their current-mode operation, slew rate and bandwidth are higher. Note that usually the TIOA is also technically classified as a current feedback operational amplifier (CFA). Specially, a commercial type with external compensation pin Z (e.g. AD 844, [5]) gives much more suitable versatile structures, as is shown down. Note that in first approach the CFA is supposed as ideal, namely the parameters are:

$$
\begin{equation*}
Z_{T} \rightarrow \infty, Z_{X} \rightarrow 0, Z_{y} \rightarrow \infty, Z_{z} \rightarrow \infty, Z_{o} \rightarrow 0 . \tag{1}
\end{equation*}
$$

Then the real CFA and parasitic effects will be studied in detail, taking exact value of all impedances in (1).

Over the last decades, since the CFA was introduced, a several ARC filter structures based on this element have been developed, with different design approach in mind, see [1] - [4] and more others. Nevertheless a synthesis of this filter is still an active topic. In this paper the simplest structures of the second order filter (biquad) with only one single CFA, several resistors $R$ and maximally two capacitors $C$, what is qualification of the canonical structure, are discussed. The here used systematic design procedure, given firstly for the circuits with current conveyors, is based on general autonomous network with suitable characteristic equation.

## 2. General autonomous circuits

To design biquadratic ARC filters and or oscillators we firstly choose an appropriate general autonomous circuit with the characteristic equation (CE) in the second-order general form (2) or in the following standard one (3)


Fig. 1. General autonomous models of the biquads with one CFA and four single admittances.
a) Structure CFA1Y4A,
b) Structure CFA1 Y4B,
c) Structure CFA1Y4C,
d) Structure CFA1Y4D.

$$
\begin{equation*}
a_{2} s^{2}+a_{1} s+a_{0}=0 \tag{2}
\end{equation*}
$$

$$
\begin{equation*}
s^{2}+\frac{\omega_{0}}{Q} s+\omega_{0}^{2}=0 \tag{3}
\end{equation*}
$$

There $\omega_{0}$ is the natural (undamped) frequency and $Q$ is the quality factor.

The general autonomous circuit (Fig. 1) contains only one active CFA and certain number of admittances $Y_{\mathrm{i}}$ in feedback loop. An optimum is four passive admittances $Y_{\mathrm{i}}$.(Fig. 1), which are single $R$ or $C$ only.

The simplest structure CFA1Y4A is in Fig. 1a. There two admittances are in series $\left(Y_{1}, Y_{2}\right)$ and parallel ( $Y_{3}$, $Y_{4}$ ) combination. By routine symbolical nodal analysis (det $\mathrm{Y}=0$ ) the following CE is obtained

$$
\begin{equation*}
Y_{1} Y_{3}+Y_{2} Y_{3}+Y_{1} Y_{4}+Y_{2} Y_{4}-Y_{1} Y_{2}=0 \tag{4}
\end{equation*}
$$

The structure CFA1Y4B on Fig. 1b has the CE

$$
\begin{equation*}
Y_{2} Y_{3}+Y_{2} Y_{4}+Y_{3} Y_{4}-Y_{1} Y_{3}=0 \tag{5}
\end{equation*}
$$

The CFA1Y4C (Fig. 1c) has

$$
\begin{equation*}
Y_{1} Y_{4}+Y_{2} Y_{4}-Y_{2} Y_{3}=0 . \tag{6}
\end{equation*}
$$

The CFA1Y4D on Fig. 1d is corresponding to the CE

$$
\begin{equation*}
Y_{1} Y_{4}+Y_{2} Y_{3}+Y_{3} Y_{4}+Y_{2} Y_{4}=0 \tag{7}
\end{equation*}
$$

A suitable modification with five passive elements can be obtained decomposing some admittance $Y_{i}$ in the CFA1Y4 on two parallel connected. Then more filter architectures can be obtained. It is the reason for the modification of the structure OTA1Y4D (Fig. 1d) to the model with five admittances OTA1Y5, where $Y_{1}$ is $Y_{1}=Y_{11}+Y_{12}$. Then the CE (7) tends to this form

$$
\begin{equation*}
Y_{11} Y_{4}+Y_{12} Y_{4}+Y_{2} Y_{3}+Y_{3} Y_{4}+Y_{2} Y_{4}=0 \tag{8}
\end{equation*}
$$

## 3. Simplest structure

From the general characteristic equation (4) we can readily derive real filter of the simplest structure CFA1Y4A, choosing there the particular admittances $Y_{\mathrm{i}}$, to obtain desired form (2) or (3) and postulating two capacitors for the $2^{\text {nd }}$ order canonical circuit. For the choosing: $Y_{1}=G_{1}, Y_{2}=s C_{1}, Y_{3}=G_{2}, Y_{4}=s C_{2}$
(Fig.2) the CE (4) has the following concrete form


Fig. 2. ARC biquad with the structure CFA1Y4A

$$
\begin{equation*}
s^{2} C_{1} C_{2}+s\left[C_{1}\left(G_{2}-G_{1}\right)+C_{2} G_{1}\right]+G_{1} G_{2}=0 \tag{9}
\end{equation*}
$$

Comparing the formulas (9) and (3) the following design equations are done

$$
\begin{equation*}
\omega_{0}=\sqrt{\frac{G_{1} G_{2}}{C_{1} C_{2}}}, \text { (10) } \quad Q=\frac{\sqrt{C_{1} C_{2} G_{1} G_{2}}}{C_{1}\left(G_{2}-G_{1}\right)+C_{2} G_{1}} \tag{11}
\end{equation*}
$$

From the autonomous circuits given in Fig 1a, we can build a frequency filter so that we disconnect some grounded branch ( $Y_{2}, Y_{3}$ and or $Y_{4}$ ) and we put there an independent input voltage source, as shown in Fig. 2. It is based on the known idea, that the CE remains the same if an ideal voltage source is connected into an arbitrary branch. Then we try to find a suitable output port examining all node voltages of this circuit. The Fig. 2 indicates all possibilities of driving (inputs) and loading (outputs) of this filter structure.
The first suitable variant CFA1Y4A11 has the input $V_{\mathrm{i} 1}$ ( $V_{\mathrm{i} 2}=V_{\mathrm{i} 3}=0$, these sources are shorted). There the output $V_{01}=V_{\text {o2 }}$ and the following transfer function with band pass (BP) character was derived

$$
\begin{equation*}
\frac{V_{o 1}}{V_{i 1}}=\frac{V_{o 2}}{V_{i 1}}=\frac{-s G_{1} C_{1}}{D(s)}, \tag{12}
\end{equation*}
$$

where the denominator is

$$
\begin{equation*}
D(s)=s^{2} C_{1} C_{2}+s\left[C_{1}\left(G_{2}-G_{1}\right)+C_{2} G_{1}\right]+G_{1} G_{2} \tag{13}
\end{equation*}
$$

Similarly for the output $V_{03}$

$$
\begin{equation*}
\frac{V_{o 3}}{V_{i 2}}=\frac{s C_{1} G_{2}}{D(s)}, \quad \text { (14) } \quad \frac{V_{o 3}}{V_{i 3}}=\frac{s^{2} C_{1} C_{2}}{D(s)} \tag{14}
\end{equation*}
$$

Note that from the independent loading point of view, the best is the output $V_{01}$, with zero internal impedance.

## 4. Structures with four admittances

The previous structure CFA1Y4A (Fig. 1a) can be interpreted as a simplest one with two resulting admittances only. We will discuss now the structures with four separated admittances. In this case the structure CFA1Y4D is taken (Fig. 1d) with the CE (7). For the first variant (CFA1Y4D-a) the admittances are chosen to obtain the biquad in Fig. 3. This circuit has suitable transfer functions

$$
\begin{gather*}
\frac{V_{o 3}}{V_{i 1}}=\frac{V_{o 2}}{V_{i 1}}=\frac{-s C_{1} G_{3}}{D(s)},  \tag{16}\\
\frac{V_{o 1}}{V_{i 1}}=\frac{-s^{2} C_{1} C_{2}}{D(s)},(17) \quad \frac{V_{o 1}}{V_{i 2}}=\frac{s C_{2} G_{2}}{D(s)} \tag{18}
\end{gather*}
$$

with the form of the BP (16), (18) and or HP one (17) respectively. There the denominator is

$$
\begin{equation*}
D(s)=s^{2} C_{1} C_{2}+s C_{2}\left(G_{2}+G_{3}\right)+G_{2} G_{3} . \tag{19}
\end{equation*}
$$

The other variant (CFA1Y4D-b) can be obtained interchanging the elements $\mathrm{R} \leftrightarrow \mathrm{C}$ in Fig. 3. Then also the low pass character has the transfer function $V_{01} / V_{\mathrm{i} 1}$.


Fig. 3. ARC biquad with the structure CFA1Y4D-a

## 5. Structures with five admittances

A modification of the biquad CFA1Y4D-a given above (Fig. 3) into the version with five admittances (CFA1Y5D-a) is quite straightforward, namely the other resistor $R_{1}$ is parallel connected to the capacitor $C_{1}$, as shown in Fig. 4. All possible inputs and outputs are depicted there (Fig. 4) as a result of the routine symbolical nodal analysis using computer tool SNAP. The transfer functions are LP, BP and HP types as follows
$\frac{V_{o 2}}{V_{i 2}}=\frac{V_{03}}{V_{i 2}}=\frac{-G_{1} G_{3}}{D(s)}$,
(20) $\frac{V_{o 2}}{V_{i 1}}=\frac{V_{o 3}}{V_{i 1}}=\frac{-s C_{1} G_{3}}{D(s)}$,
$\frac{V_{o 1}}{V_{i 2}}=\frac{s C_{2} G_{1}}{D(s)}$, (22) $\frac{V_{o 1}}{V_{i 3}}=\frac{s C_{2} G_{2}}{D(s)}$,
(23) $\frac{V_{o 1}}{V_{i 1}}=\frac{s^{2} C_{1} C_{2}}{D(s)}$.

There the resulting denominator is

$$
\begin{equation*}
D(s)=s^{2} C_{1} C_{2}+s C_{2}\left(G_{1}+G_{2}+G_{3}\right)+G_{2} G_{3} . \tag{25}
\end{equation*}
$$

and the design equations are


Fig. 4. ARC biquad with the structure CFA1Y5D-a

$$
\begin{gather*}
\omega_{0}=\frac{1}{\sqrt{C_{1} C_{2} R_{2} R_{3}}},  \tag{26}\\
Q=\sqrt{\frac{C_{1}}{C_{2}}}\left[\frac{\sqrt{R_{2} R_{3}}}{R_{1}}+\sqrt{\frac{R_{3}}{R_{2}}}+\sqrt{\frac{R_{2}}{R_{3}}}\right]^{-1} . \tag{27}
\end{gather*}
$$

A dual version (CFA1Y5D-b) is shown in Fig. 5. The routine symbolic analysis of this circuit yields the following transfer functions:

$$
\begin{array}{ll}
\frac{V_{o 2}}{V_{i 2}}=\frac{V_{o 3}}{V_{i 2}}=\frac{-s^{2} C_{1} C_{3}}{D(s)},(28) & \frac{V_{o 1}}{V_{i 1}}=\frac{s C_{1} G_{2}}{D(s)}, \\
\frac{V_{o 2}}{V_{i 2}}=\frac{V_{o 3}}{V_{i 2}}=\frac{-s C_{3} G_{1}}{D(s)},(30) & \frac{V_{o 1}}{V_{i 2}}=\frac{G_{1} G_{2}}{D(s)},
\end{array}
$$

whit the denominator

$$
\begin{equation*}
D(s)=s^{2} C_{2} C_{3}+s G_{2}\left(C_{1}+C_{2}+C_{3}\right)+G_{1} G_{2} . \tag{32}
\end{equation*}
$$



Fig. 5. ARC biquad with the structure CFA1Y5D-b
Then the natural frequency of this biquad is given by (26) and the quality factor is

$$
\begin{equation*}
Q=\sqrt{\frac{R_{1}}{R_{2}}}\left[\frac{C_{1}}{\sqrt{C_{2} C_{3}}}+\sqrt{\frac{C_{3}}{C_{2}}}+\sqrt{\frac{C_{2}}{C_{3}}}\right]^{-1} . \tag{33}
\end{equation*}
$$

Another biquadratic structure, with five admittances, is CFA1Y5C (Fig. 6), associated with the general circuit diagram in Fig. 1c, decomposing the admittance $Y_{4}$ and choosing in the CE (7) admittances:
$Y_{1}=G_{2}, Y_{2}=s C_{2}, Y_{3}=G_{3}, Y_{4}=s C_{1}+G_{1}$ (Fig. 6).
For this circuit (Fig. 6) the resulting suitable transfer functions are as follows:

$$
\begin{array}{lll}
\frac{V_{o 2}}{V_{i 1}}=\frac{V_{o 3}}{V_{i 1}}=\frac{G_{2} G_{3},}{D(s)} & \text { (34) } & \frac{V_{o 1}}{V_{i 2}}=\frac{-s C_{2} G_{3}}{D(s)}, \\
\frac{V_{o 1}}{V_{i 3}}=\frac{s C_{2} G_{1}}{D(s)}, & \text { (36) } & \frac{V_{o 1}}{V_{i 4}}=\frac{s^{2} C_{1} C_{2}}{D(s)}
\end{array}
$$

where the denominator is

$$
\begin{equation*}
D(s)=s^{2} C_{1} C_{2}+s\left[C_{1} G_{2}+C_{2}\left(G_{1}-G_{3}\right)\right]+G_{1} G_{3} . \tag{38}
\end{equation*}
$$

The quality factor in this case can be adjusted using the following equation


Fig. 6. ARC biquad with the structure CFA1Y5C

## 6. DC precise low pass filter

The structure CFA1Y5D-b (Fig. 5) can be ingeniously used as the low-drift DC precise second order low pass filter. This filter is suitable e.g. for D/A convertors and some measurement application. There the output node $V_{01}$ (internal node of the topology) and transfer function (31) is taken.

Nevertheless, for this application the circuit can be modified in the DC precise LP of the third order as shown in Fig. 7. Analysis yields the voltage transfer function in this polynominal form

$$
\begin{equation*}
\frac{V_{o 1}}{V_{i 2}}=\frac{1}{D(s)}, \tag{40}
\end{equation*}
$$

where the denominator is


Fig. 7. Third-order DC precise low pass filter

$$
\begin{align*}
& D(s)=s^{3} C_{1} C_{2} C_{3}+s^{2} C_{1} C_{2}\left(G_{1}+G_{2}\right) G_{0}+  \tag{41}\\
& +s\left(C_{1}+C_{2}\right) G_{1} G_{2}+G_{0} G_{1} G_{2}
\end{align*}
$$

Note that this circuit (Fig. 7) can be generalized in higher n-order DC precise LP filter, putting in the port Z more RC feedback cells.

## 7. Simulation

To verify the functionality of the proposed filter structures, the PSpice simulation has been carried out, using an adequate model of the ideal CFA. Results are confirming the theoretical assumptions and the given symbolical analysis. Additional studying of the parasitic influences and modelling of the real components and parasitic effects will be done.

## 8. Conclusions

The systematic uniform synthesis based on the general autonomous network with suitable characteristic equation is really good way how to obtain the ARC filters or oscillators with modern functional blocks, as has been shown here for the transimpedance operational amplifiers.

The given filters based on the single CFA can operate at higher frequency as active filters with standard (voltage) operational amplifiers. They can be also implemented in IC technologies. Only one CFA means smaller chip area, lower power consumption and lower noise.

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