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Antimonotonicity in Chua's Canonical Circuit with a Smooth Nonlinearity

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Abstract: We have studied the dynamics of Chua's canonical circuit, when the v-i characteristic of the nonlinear resistor of the circuit is a smooth cubic function. Unlike the monotone bifurcation behavior of the members of Chua's circuit family with a piecewise linear resistor, reverse period doublings, as a parameter of the circuit is varied in a monotone way, have been observed in the circuit we have studied. Dynamics of the circuit is very sensitive to initial conditions, as chaotic attractors coexist with period-1 limit cycles.

Key-Words: Nonlinear circuit, chaos, antimonotonicity, cubic nonlinearity, bubbles, crisis, coexisting attractors.

1 Introduction

Electric circuits have emerged as a simple yet powerful experimental and analytical tool in studying chaotic behavior in nonlinear dynamics. Most chaotic and bifurcation effects cited in the literature have been observed in electric circuits e.g. the period-doubling route to chaos [1-4], intermittency route to chaos [5-8], quasiperiodicity route to chaos [9-11], crisis [12-14], antimonotonicity [15,16]. Chua's circuit is a paradigm for chaos [17]. Among the members of Chua's circuit family, the autonomous canonical Chua's circuit introduced by Chua and Lin [18] is of considerable importance. This is because it is capable of realizing the behavior of every member of the Chua's circuit family [18, 8, 14]. It consists of two active elements, one linear negative conductor, and one nonlinear resistor with odd-symmetric piecewise linear υ-i characteristic.

For a piecewise-linear nonlinearity, very extensive literature exists on theoretical, numerical and experimental aspects of the dynamics of the members of Chua's circuit family. The reasons for the previous choice of a piecewise-linear nonlinearity were the following:

- The corresponding circuits can be easily built with off-the-shelf components.
- Explicit Poincaré map can be derived which allows a *rigorous* mathematical proof, that Chua's circuit is chaotic in the sense of Shil'nikov's theorem.

Later, some bifurcation phenomena were obtained for Chua's circuit with a smooth nonlinearity, in particular, cubic nonlinearity [19,20]. The choice of a cubic nonlinearity has several advantages over a piecewise-linear one. It does not require absolute-valued functions and it is smooth, which is desirable from a mathematical perspective. Moreover, all phenomena found in the piecewise linear version also exist in the cubic version, e.g. Hopf bifurcation phenomenon requires C^3 functions and in the piecewise-linear case the corresponding phenomenon is not really Hopf but Hopf-like: in particular, the amplitude of oscillation jumps suddenly from 0 to a *finite* amplitude.

Cascades of period-doubling bifurcations have long been recognized to be one of the most common routes to chaos, as exemplified e.g. by the one-dimensional (1–D) logistic map $x_{n+1} = \lambda x_n (1 - x_n)$. As the parameter λ in such a map is increased, it is known that periodic orbits are only created but never destroyed. Unlike the monotone bifurcation behavior of the logistic map, however it has been shown that, in many common nonlinear dynamical systems, periodic orbits can be both created as well as destroyed, via reverse bifurcation sequences as a parameter is varied. Dawson et al., [21], named this type of creation and annihilation of periodic orbits *antimonotonicity*.

Reversals of period-doubling cascades have been observed in various nonlinear physical systems both numerically and experimentally. In one of the first studies of this phenomenon [22], the occurrence of such reverse sequences was connected to the dynamics of a cubic 1–D map. As examples of numerical simulations, we cite the van der Pol equation [23], Duffing's oscillator [24], a RC-ladder chaos generator [25], and an autonomous 4^{th}-order nonlinear electric circuit [26]. Experimental manifestations of antimonotonicity have also been observed on the driven R,L,p-n junction nonlinear circuit [2,27,28], and on Chua's circuit, with an asymmetric v-i characteristic [29]. In this paper, we have studied the dynamics of Chua's canonical circuit [18] with an odd symmetric cubic nonlinearity and we have focused on the phenomenon of antimonotonicity, which has never observed in the members of Chua's circuit family with a piecewise linear symmetric i-v characteristic.

2 The Canonical Chua's Circuit

Chua's canonical circuit is a nonlinear autonomous 3^{rd}-order electric circuit (Fig.1). The nonlinear element is a nonlinear resistor, while G_n is a linear negative conductance. In this paper, the v-i characteristic of the nonlinear resistor is a smooth cubic function, Fig.2, of the form

$$i_N = -k_1 v_{C1} + k_3 v_{C1}^3 \tag{1}$$

where $k_1, k_3 > 0$, instead of the piecewise linear type-N characteristic used in the previous studies [8,14,18]. The laboratory realization of this nonlinear resistor can be found in [19].

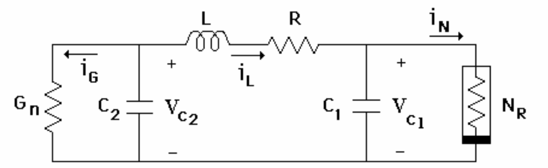

Fig.1. Chua's canonical circuit

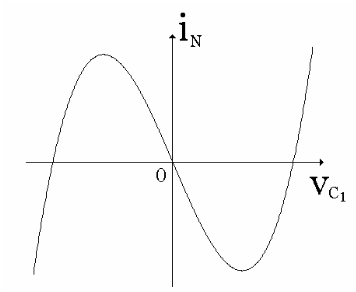

Fig.2. The cubic v-i characteristic of the nonlinear resistor.

The state equations of the circuit are the following:

$$\frac{dv_{C1}}{dt} = \frac{1}{C_1}\left(i_L + k_1 v_{C1} - k_3 v_{C1}^3\right) \tag{2}$$

$$\frac{dv_{C2}}{dt} = -\frac{1}{C_2}\left(i_L + G_n v_{C2}\right) \tag{3}$$

$$\frac{di_L}{dt} = \frac{1}{L}\left(-v_{C1} + v_{C2} - Ri_L\right) \tag{4}$$

3 Dynamics of the Circuit

We have chosen the following values for the circuit parameters: L = 100 mH, R = 330 Ω, and G_n = –0.40 mS, while k_1 = 0.3 mS and k_3 = 0.1 mA/V^3. Giving constant values to capacitance C_1, we have plotted the bifurcation diagrams v_{C1} vs. C_2. The comparative study of the bifurcation diagrams gives the qualitative changes of the dynamics of the system, as C_1 takes different discrete values.

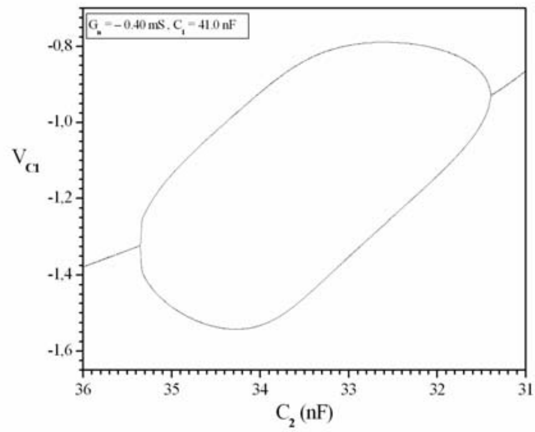

Fig.3. The bifurcation diagram, v_{C1} vs. C_2, for C_1 = 41.0 nF.

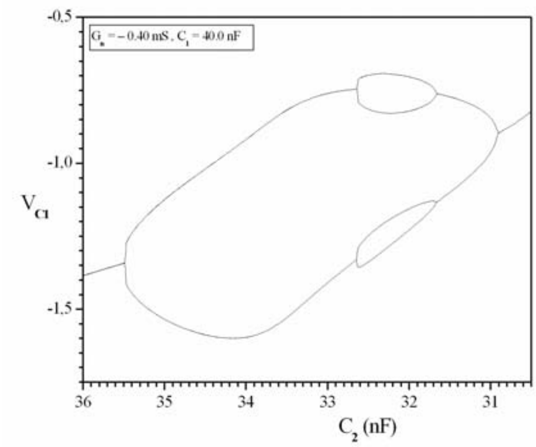

Fig.4. The bifurcation diagram, v_{C1} vs. C_2, for C_1 = 40.0 nF.

The bifurcation diagram, v_{C1} vs. C_2, for C_1 = 41.0 nF is shown in Fig.3. As C_2 is decreased, the system always remains in a periodic state following the scheme: period–1 → period–2 → period–1 (or p–1 → p–2 → p–1). Bier and Bountis, [22], named this scheme "primary bubble". The bifurcation diagram v_{C1} vs. C_2, for $C_1 = 40.0$ nF is shown in Fig.4. The system remains again in a periodic state, but a period–4 state is now formed.

As C_2 is decreased, chaotic states appear, as we can observe in Fig.5, where the bifurcation diagram v_{C1} vs. C_2, for $C_1 = 39.5$ nF is shown. The bubble is now *chaotic*. Chaotic states become enlarged, as C_2 is decreased (Figs.6-9). For all the bubbles, the initial and the final dynamic state is a period–1 state, so the bubbles are "period–1 bubbles".

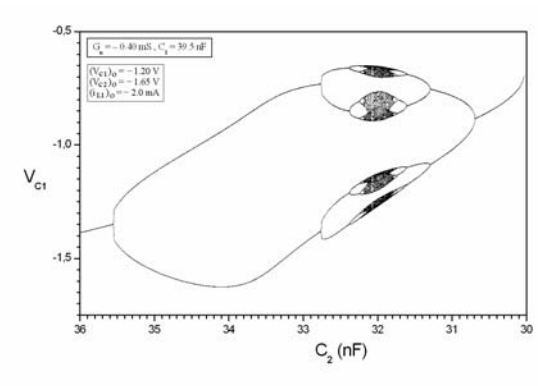

Fig.5. The bifurcation diagram v_{C1} vs. C_2, for $C_1 = 39.5$ nF.

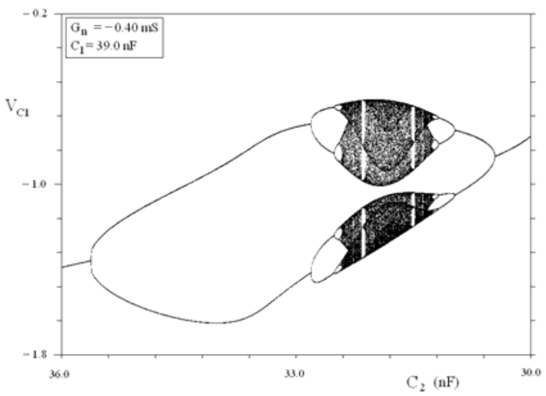

Fig.6. The bifurcation diagram v_{C1} vs. C_2, for $C_1 = 39.0$ nF.

Reverse period doublings are destroyed, when $C_1 = 36.0$ nF (Fig.10). A sudden transition, from a chaotic to a periodic state is observed at $C_2 = 31.0$ nF, a phenomenon called *crisis*,[30]. In Fig.11, the chaotic spiral attractor ($C_2 = 31.0$ nF) is shown, while in Fig.12 the periodic attractor ($C_2 = 30.8$ nF) is shown.

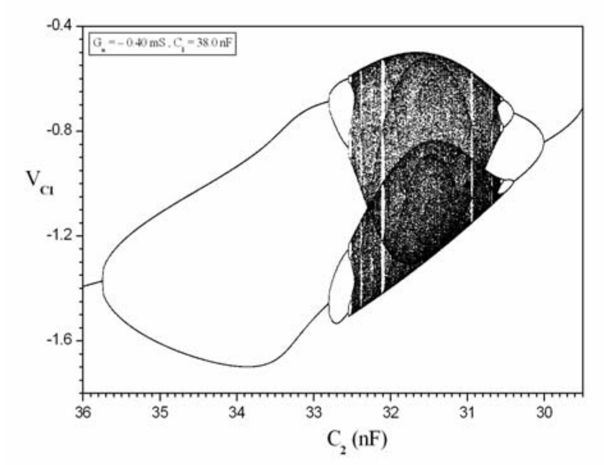

Fig.7. The bifurcation diagram v_{C1} vs. C_2, for $C_1 = 38.0$ nF.

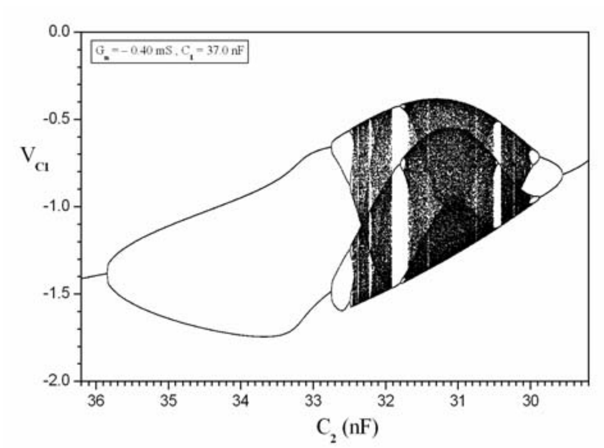

Fig.8. The bifurcation diagram v_{C1} vs. C_2, for $C_1 = 37.0$ nF.

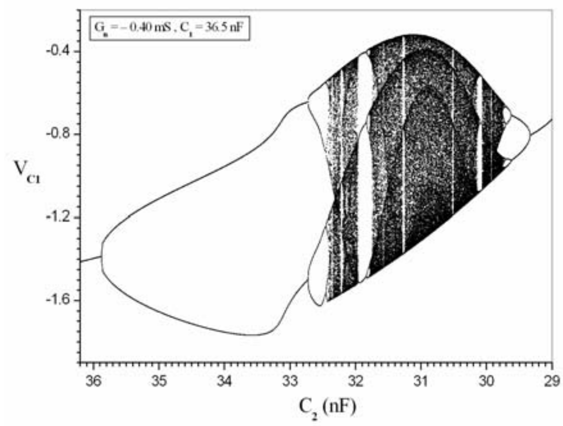

Fig.9. The bifurcation diagram v_{C1} vs. C_2, for $C_1 = 36.5$ nF.

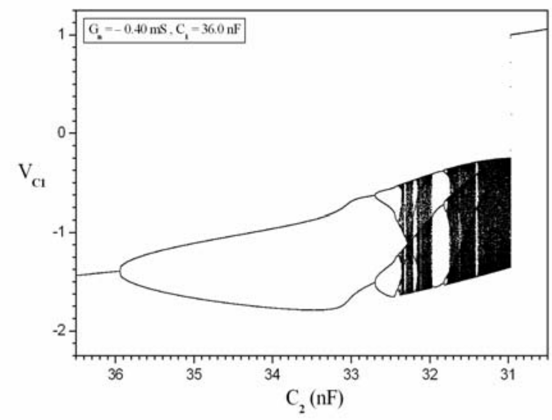

Fig.10. The bifurcation diagram v_{C1} vs. C_2, for $C_1 = 36.0$ nF.

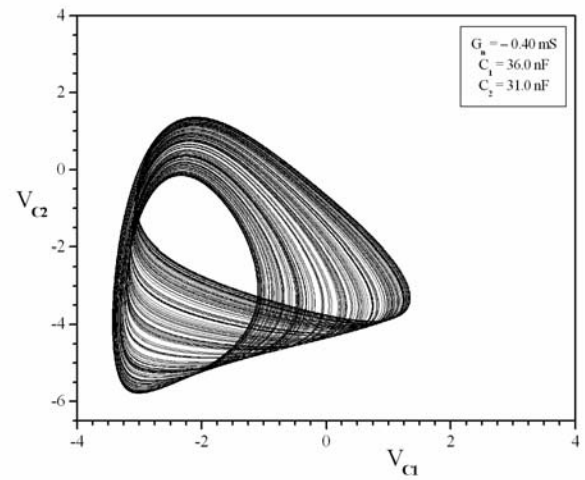

Fig.11. Phase portrait for $C_1 = 36.0$ nF and $C_2 = 31.0$ nF. Chaotic spiral attractor.

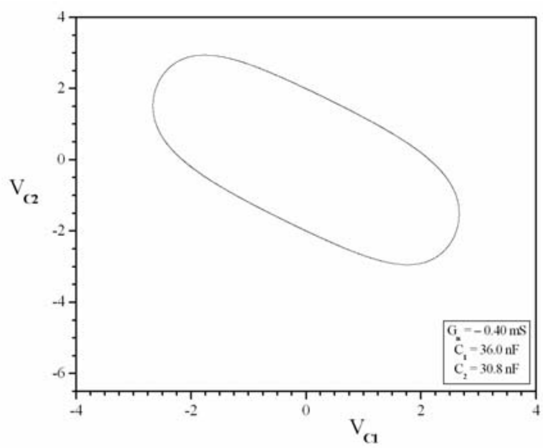

Fig.12. Phase portrait for $C_1 = 36.0$ nF and $C_2 = 30.8$ nF. Limit cycle.

4 Discussion and Conclusions

Bier and Bountis, [22], demonstrated that reverse period doubling sequences are expected to occur, when a minimum number of conditions is fulfilled. Their main result was, that a reverse period doubling sequence is likely to occur in any nonlinear system, where there is a symmetry transformation, under which the state equation remains invariant.

Indeed our system of differential equations (2-4) under the transformation

$$v_{C1} \to -v_{C1}\ ,\ v_{C2} \to -v_{C2}\ ,\ i_L \to -i_L \tag{5}$$

remains invariant. In addition, it has also been demonstrated in the literature, [22,31], that reverse period doubling commonly arises in nonlinear dynamical systems involving the variation of two parameters. It is important, however, that the period doubling “trees” develop symmetrically *towards* each other along some line in parameter space. This would allow them to terminate, by joining their “branches” to form “bubbles”, thus exhibiting the phenomenon of antimonotonicity.

We have to notice, that antimonotonicity is present, when the well-known “double-scroll” Chua’s attractor is *absent*. This is an explanation, that antimonotonicity has not been observed in Chua’s circuit with a piecewise linear resistor, although the former criteria are fulfilled.

The creation of bubbles is also very sensitive to initial conditions. The spiral attractors coexist with limit cycles like the one in Fig.12, so the circuit can be driven to two quite different states, depending on the initial condition.

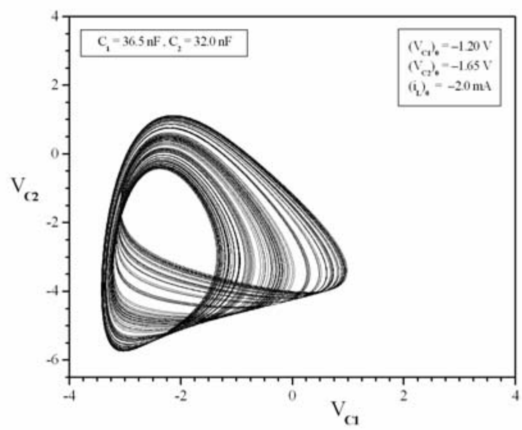

Fig.13. Chaotic spiral attractor for $C_1 = 36.5$ nF, $C_2 = 32.0$ nF and initial conditions $(v_{C1})_0 = -1.20$V, $(v_{C2})_0 = -1.65$V, and $(i_L)_0 = -2.0$mA

As an example, in Fig.13, the chaotic spiral attractor is shown for $C_1 = 36.5$ nF and $C_2 = 32.0$ nF and initial conditions $(v_{C1})_0 = -1.20$V, $(v_{C2})_0 = -1.65$V, and $(i_L)_0 = -2.0$mA, while in Fig.14, a period-1 limit cycle is shown for $C_1 = 36.5$ nF and $C_2 = 32.0$ nF and initial conditions $(v_{C1})_0 =$

+0.20V, $(v_{C2})_0 = -1.65$V, and $(i_L)_0 = -2.0$mA. The power spectra of the two attractors are shown in Figs. 15 and 16, and they have not anything common. The high peak in Fig.15 corresponds to a frequency of 2700 Hz, while the frequency of the periodic limit cycle is 3500 Hz, so this trajectory is not embedded in the chaotic attractor.

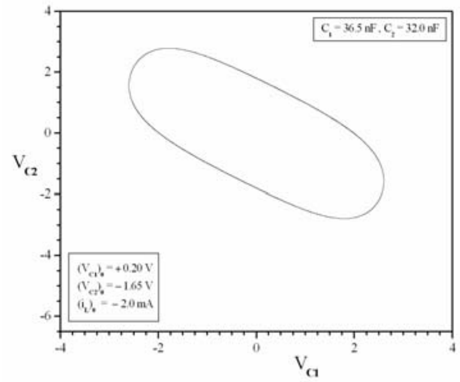

Fig.14. Period-1 limit cycle for $C_1 = 36.5$ nF, $C_2 = 32.0$ nF and initial conditions $(v_{C1})_0 = +0.20$V, $(v_{C2})_0 = -1.65$V, and $(i_L)_0 = -2.0$mA.

Coexisting attractors play an important role in dynamics of identical coupled nonlinear systems, especially in the case of synchronization.

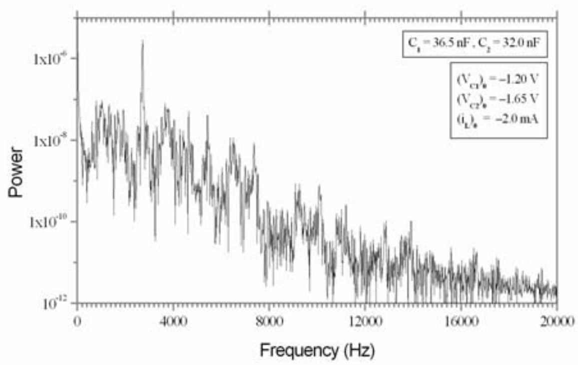

Fig.15. Power spectrum of the chaotic spiral attractor of Fig.13.

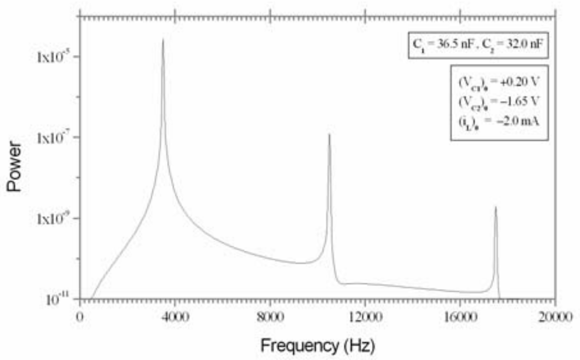

Fig.16. Power spectrum of the periodic attractor of Fig.14.

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