Proceedings of the 10th WSEAS International Conference on CIRCUITS, Vouliagmeni, Athens, Greece, July 10-12, 2006 (pp191-196)

Dynamics of Two Resistively Coupled Electric Circuits of 4th Order

M. S. PAPADOPOULOU, I.M. KYPRIANIDIS, and I. N. STOUBOULOS
Physics Department
Aristotle University of Thessaloniki
Thessaloniki, 54124
GREECE

Abstract: - We have studied the case of chaotic synchronization of two identical nonlinear autonomous electric circuits, which are mutually (bidirectionally) coupled via a linear resistor R_C. The two circuits can be either υ_{C1}-coupled or υ_{C2}-coupled. Both, experimental and simulation results, have shown that chaotic synchronization is possible only in the case of υ_{C2}-coupling.

Key-Words: - Nonlinear circuit, Chaos, Attractor, Mutual coupling, Reverse period-doubling.

1 Introduction

Since the discovery by Pecora and Carroll [1], that chaotic systems can be synchronized, the topic of synchronization of coupled chaotic circuits and systems has been studied intensely [2] and some interesting applications, as in broadband communications systems, have come out of this research [3-5]. Generally, there are two methods of chaos synchronization available in the literature. In the first method, due to Pecora and Carroll [1], a stable subsystem of a chaotic system could be synchronized with a separate chaotic system, under certain suitable conditions. The second method to achieve chaos synchronization between two identical nonlinear systems is due to the effect of resistive coupling without requiring to construct any stable subsystem [6-8]. According to Carroll and Pecora [9], periodically forced synchronized chaotic circuits are much more noise-resistant than autonomous synchronized chaotic circuits.

In this paper we have studied the case of two identical fourth-order autonomous nonlinear electric circuits (Fig.1) with two active elements, one linear negative conductance and one nonlinear resistor with a symmetrical piecewise linear v - i characteristic (Fig.2). Using the capacitances C_1 and C_2 as the control parameters, we have observed a reverse period-doubling sequence, as well as a crisis phenomenon, when the spiral attractor suddenly widens to a double-scroll attractor [10].

2 The Nonlinear Circuit

The circuit, we have studied, is shown in Fig.1 and has two active elements, a nonlinear resistor R_N and a linear negative conductance G, which were implemented using the circuits shown in Figs.3 and 4 respectively. The real v-i characteristic of the conductance G is shown in Fig.5.

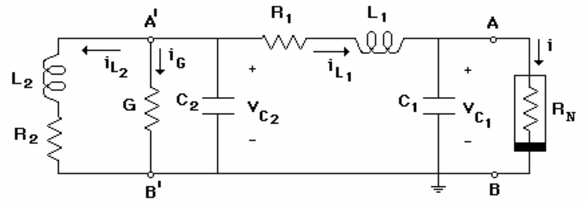

Fig.1. The fourth-order autonomous nonlinear electric circuit.

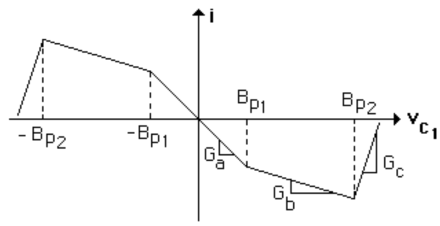

Fig.2. The piecewise-linear and symmetrical v-i characteristic of the nonlinear resistor R_N.

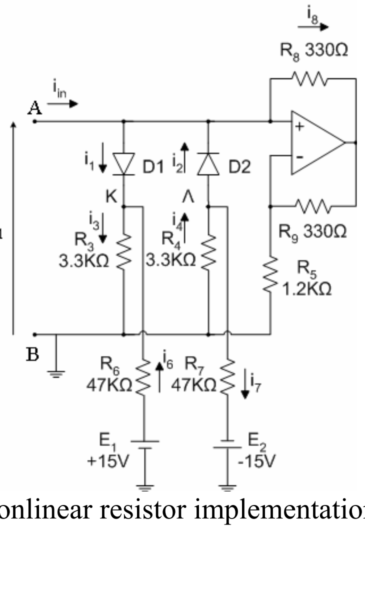

Fig.3. The nonlinear resistor implementation

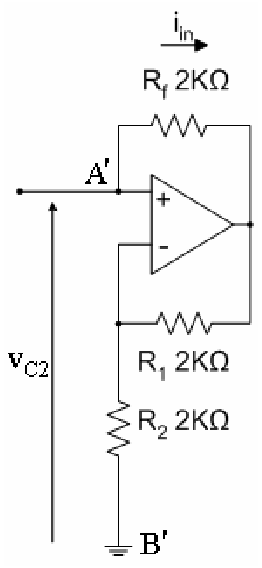

Fig.4. The negative conductance implementation

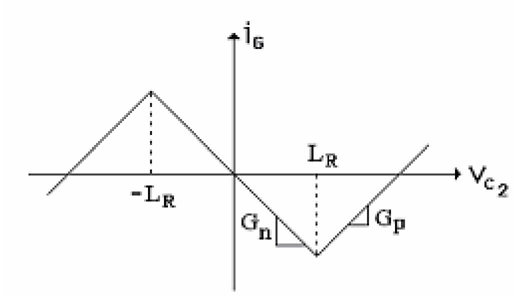

Fig.5. The real v-i characteristic of the negative conductance G.

The state equations of the circuit are:

$$\frac{d\upsilon_{C1}}{dt}=\frac{1}{C_1}\left(i_{L1}-i\right) \tag{1}$$

$$\frac{d\upsilon_{C2}}{dt}=-\frac{1}{C_2}\left(G\cdot\upsilon_{C2}+i_{L1}+i_{L2}\right) \tag{2}$$

$$\frac{di_{L1}}{dt}=\frac{1}{L_1}\left(\upsilon_{C2}-\upsilon_{C1}-R_1 i_{L1}\right) \tag{3}$$

$$\frac{di_{L2}}{dt}=\frac{1}{L_2}\left(\upsilon_{C2}-R_2 i_{L2}\right) \tag{4}$$

where

$$\begin{aligned} i=g(\upsilon_{C1})=&G_C\upsilon_{C1}+0.5(G_a-G_b)(|\upsilon_{C1}+B_{p1}|-|\upsilon_{C1}-B_{p1}|)\\ &+0.5(G_b-G_c)(|\upsilon_{C1}+B_{p2}|-|\upsilon_{C1}-B_{p2}|)\end{aligned} \tag{5}$$

The values of the circuit parameters are: L_1=10.2mH, L_2=21.5mH, R_1=2.02KΩ, R_2=108Ω, G_n=-0.5mS, G_p=-0.5mS, L_R=7.5V, G_a=-0.833mS, G_b=-0.514mS, G_c=2.9mS, B_{p1}=1.46V and B_{p2}=10.1V.

The Bifurcation diagram of the dynamics of the circuit is shown in Fig.6. We can observe forward and reverse period-doubling cascades, i.e. antimonotonocity [10].

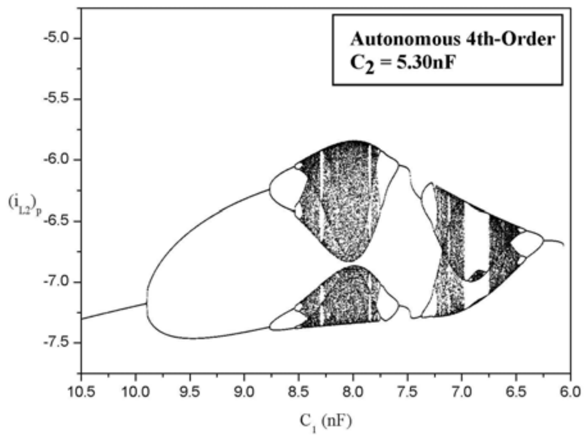

Fig.6. The bifurcation diagram $(i_{L2})_p$ vs. C_1 for C_2=5.30nF.

3 Mutual Resistive Coupling

If two identical circuits are resistively coupled, complex dynamics can be observed, as well as chaotic synchronization, as the coupling parameter R_C is varied. The two circuits can be either υ_{C1}-coupled or υ_{C2}-coupled, as we can see in Figs.7 and 8 respectively.

3.1 υ_{C1}-coupling

The system of the two identical 4^{th} order nonlinear circuits υ_{C1}-coupled via the linear resistor R_C is shown in Fig.7. The state equations of the system are:

$$\frac{d\upsilon_{C11}}{dt}=\frac{1}{C_1}\left\{-\frac{\upsilon_{C11}-\upsilon_{C12}}{R_C}+i_{L11}-g\left(\upsilon_{C11}\right)\right\} \tag{6}$$

$$\frac{d\upsilon_{C21}}{dt}=\frac{1}{C_2}\left\{-G\upsilon_{C21}-i_{L11}-i_{L21}\right\} \tag{7}$$

$$\frac{di_{L11}}{dt}=\frac{1}{L_1}\left\{-\upsilon_{C11}+\upsilon_{C21}-R_1 i_{L11}\right\} \tag{8}$$

$$\frac{di_{L21}}{dt}=\frac{1}{L_2}\left\{\upsilon_{C21}-R_2 i_{L21}\right\} \tag{9}$$

$$\frac{d\upsilon_{C12}}{dt}=\frac{1}{C_1}\left\{\frac{\upsilon_{C11}-\upsilon_{C12}}{R_C}+i_{L12}-g\left(\upsilon_{C12}\right)\right\} \tag{10}$$

$$\frac{d\upsilon_{C22}}{dt}=\frac{1}{C_2}\left\{-G\upsilon_{C22}-i_{L12}-i_{L22}\right\} \tag{11}$$

$$\frac{di_{L12}}{dt} = \frac{1}{L_1}\left\{-\upsilon_{C12}+\upsilon_{C22}-R_1 i_{L12}\right\} \quad (12)$$

$$\frac{di_{L22}}{dt} = \frac{1}{L_2}\left\{\upsilon_{C22}-R_2 i_{L22}\right\} \quad (13)$$

where

$$\begin{aligned} g(\upsilon_{C1j}) = {} & G_c\upsilon_{C1j}+0.5(G_a-G_b)(|\upsilon_{C1j}+B_{p1}|-|\upsilon_{C1j}-B_{p1}|) \\ & +0.5(G_b-G_c)(|\upsilon_{C1j}+B_{p2}|-|\upsilon_{C1j}-B_{p2}|) \end{aligned} \quad (14)$$

and j=1, 2.

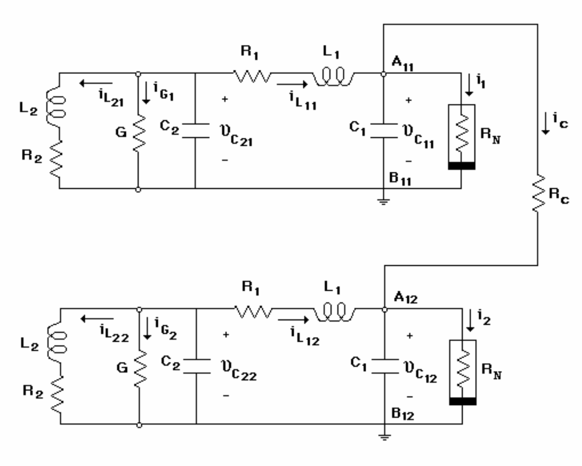

Fig.7. The υ_{C1}-coupled system of the two identical 4th order nonlinear circuits.

3.2 υ_{C2}-coupling

The system of the two identical 4th order nonlinear circuits υ_{C2}-coupled via the linear resistor R_C is shown in Fig.8. The state equations of the system are:

$$\frac{d\upsilon_{C11}}{dt} = \frac{1}{C_1}\left\{i_{L11}-g\left(\upsilon_{C11}\right)\right\} \quad (15)$$

$$\frac{d\upsilon_{C21}}{dt} = \frac{1}{C_2}\left\{-\frac{\upsilon_{C21}-\upsilon_{C22}}{R_C}-G\upsilon_{C21}-i_{L11}-i_{L21}\right\} \quad (16)$$

$$\frac{di_{L11}}{dt} = \frac{1}{L_1}\left\{-\upsilon_{C11}+\upsilon_{C21}-R_1 i_{L11}\right\} \quad (17)$$

$$\frac{di_{L21}}{dt} = \frac{1}{L_2}\left\{\upsilon_{C21}-R_2 i_{L21}\right\} \quad (18)$$

$$\frac{d\upsilon_{C12}}{dt} = \frac{1}{C_1}\left\{i_{L12}-g\left(\upsilon_{C12}\right)\right\} \quad (19)$$

$$\frac{d\upsilon_{C22}}{dt} = \frac{1}{C_2}\left\{\frac{\upsilon_{C21}-\upsilon_{C22}}{R_C}-G\upsilon_{C22}-i_{L12}-i_{L22}\right\} \quad (20)$$

$$\frac{di_{L12}}{dt} = \frac{1}{L_1}\left\{-\upsilon_{C12}+\upsilon_{C22}-R_1 i_{L12}\right\} \quad (21)$$

$$\frac{di_{L22}}{dt} = \frac{1}{L_2}\left\{\upsilon_{C22}-R_2 i_{L22}\right\} \quad (22)$$

where $g(\upsilon_{C1j})$ is given by Eq. (14)

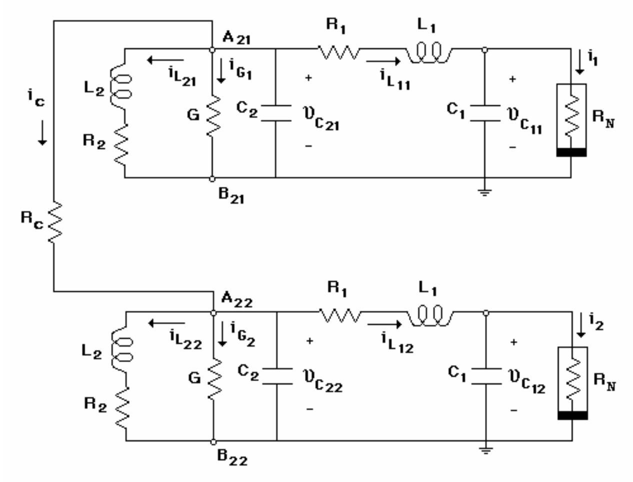

Fig.8. The υ_{C2}-coupled system of the two identical 4th order nonlinear circuits.

4 Dynamics of the Coupled System

We have studied the evolution of the two identical resistively coupled autonomous and nonlinear 4th order electric circuits from nonsynchronized to synchronized oscillations, when they exhibit chaotic behavior. By ″synchronization of chaos″ we mean, that the chaotic oscillations of individual circuits are identical to each other.

Using the bifurcation diagram of Fig.6, we have chosen the values of C_1 where chaotic behavior is observed and have studied the possibility of chaotic synchronization by plotting the diagrams of υ_{C22}-υ_{C21} vs. R_1/R_C, the coupling factor.

Chaotic synchronization was never observed in the case of υ_{C1}-coupling.

4.1 Chaotic attractors

The chaotic attractors of the two circuits, when they are uncoupled, are shown in the following figures.

In Figs. 9 and 10 we can see the theoretical phase portraits for C_2=5.3nF and C_1=8.0nF and C_1=8.4nF respectively, while in Figs. 11-14 the experimental chaotic attractors are shown, for the two identical circuits.

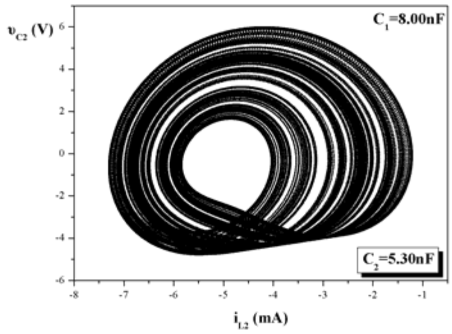

Fig.9. Chaotic attractor for C_2=5.30nF and C_1=8.00nF.

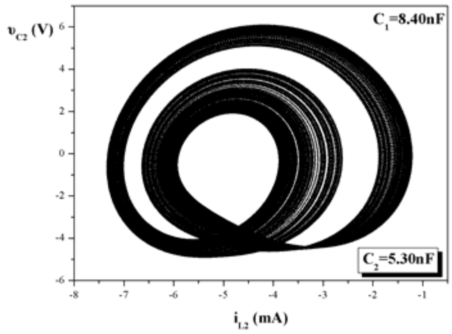

Fig.10. Chaotic attractor for C_2=5.30nF and C_1=8.40nF.

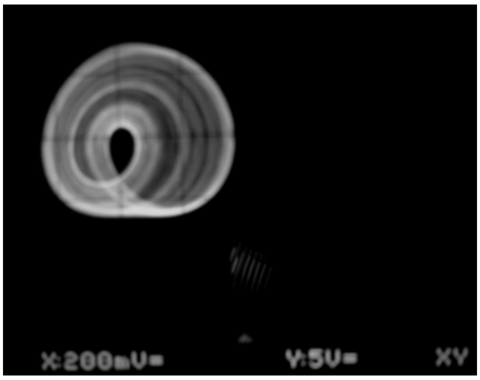

Fig.11. Chaotic attractor for C_2=5.30nF and C_1=8.00nF for the first circuit (i_{L2} vs. υ_{C2}).

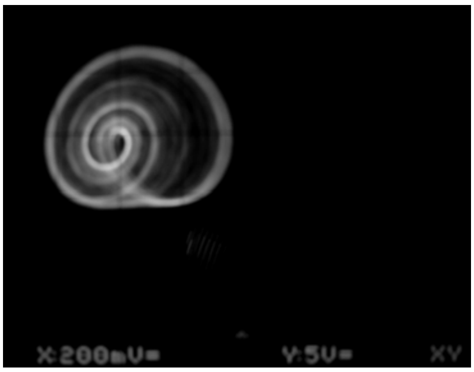

Fig.12. Chaotic attractor for C_2=5.30nF and C_1=8.40nF for the first circuit (i_{L2} vs. υ_{C2}).

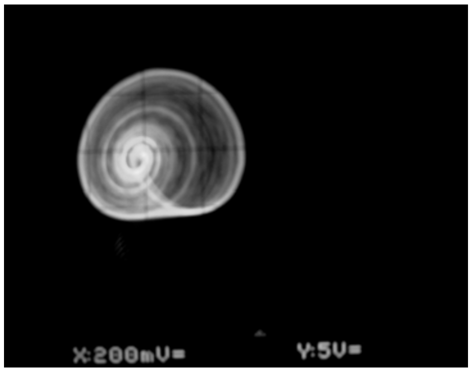

Fig.13. Chaotic attractor for C_2=5.30nF and C_1=8.00nF for the second circuit (i_{L2} vs. υ_{C2}).

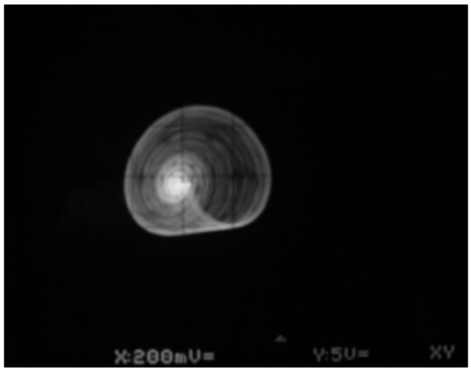

Fig.14. Chaotic attractor for C_2=5.30nF and C_1=8.40nF for the second circuit (i_{L2} vs. υ_{C2}).

4.2 The case of υ_{C2}-coupling

We have studied the possibility of chaotic synchronization when the initial conditions of the coupled circuits belong to the same basin of attraction.

The diagrams of υ_{C22}-υ_{C21} vs. R_1/R_C are shown in Figs. 15 and 16 correspond to the case, when the initial conditions of the coupled circuits belong in the same basin of attraction.

In Figs. 17-19 we can see the experimental results from the coupled system for C_2=5.30nF, C_1=8.00nF and various values of the coupling resistor R_C.

In Figs. 20-22 we can see the experimental results for C_2=5.30nF and C_1=8.40nF, as the coupling resistance R_C is varied.

We observe how the system passes from nonsynchronized states to synchronized ones, depending on the coupling resistance R_C.

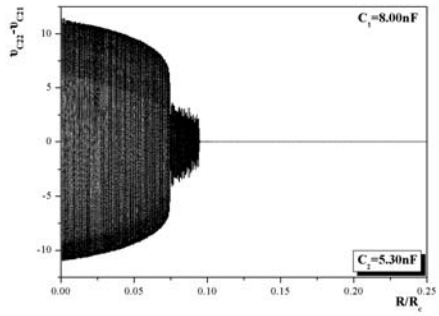

Fig.15. The diagram of υ_{C22}-υ_{C21} vs. R_1/R_C, for C_2=5.30nF and C_1=8.00nF.

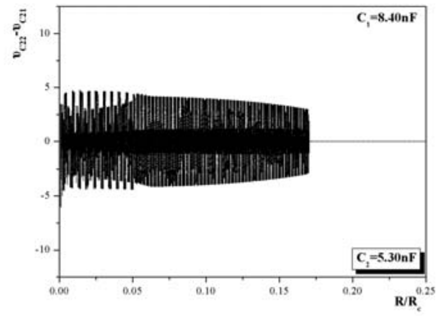

Fig.16. The diagram of υ_{C22}-υ_{C21} vs. R_1/R_C, for C_2=5.30nF and C_1=8.40nF.

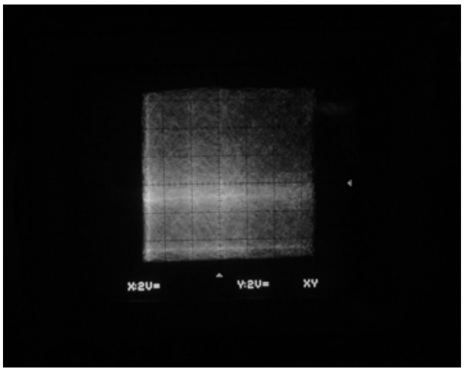

Fig.17. υ_{C2}-coupling for C_2=5.30nF, C_1=8.00nF and R_C= ∞ (υ_{C22} vs. υ_{C21}).

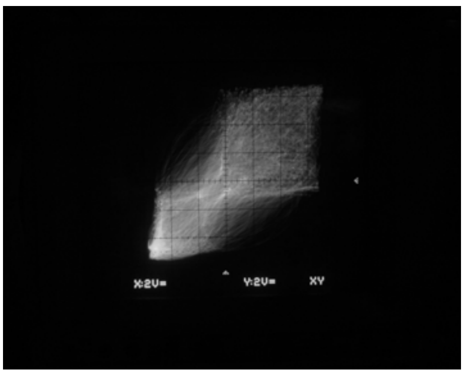

Fig.18. υ_{C2}-coupling for C_2=5.30nF, C_1=8.00nF and R_C=49KΩ (υ_{C22} vs. υ_{C21}).

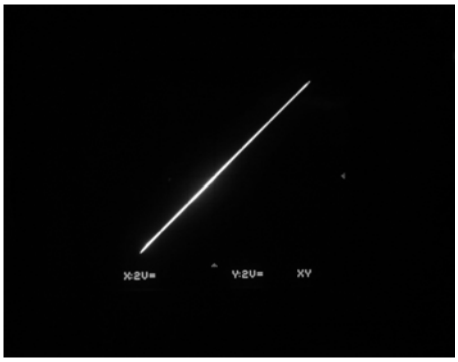

Fig.19. υ_{C2}-coupling for C_2=5.30nF, C_1=8.00nF and R_C=1.3KΩ (υ_{C22} vs. υ_{C21}). Chaotic synchronization.

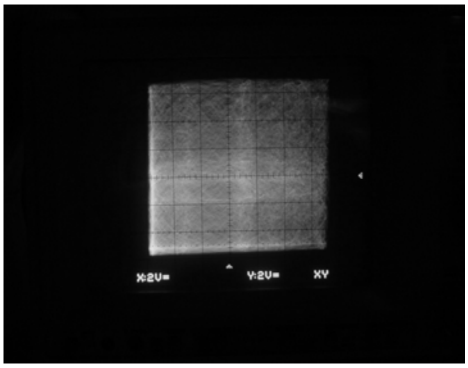

Fig.20. υ_{C2}-coupling for C_2=5.30nF, C_1=8.40nF and R_C= ∞KΩ (υ_{C22} vs. υ_{C21}).

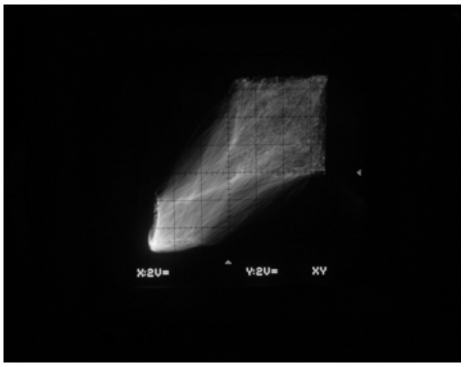

Fig.21. υ_{C2}-coupling for C_2=5.30nF, C_1=8.40nF and R_C=40KΩ (υ_{C22} vs. υ_{C21}).

Fig.22. υ_{C2}-coupling for C_2=5.30nF, C_1=8.40nF and R_C=1.5KΩ (υ_{C22} vs. υ_{C21}). Chaotic synchronization.

5 Conclusion

In this paper we have shown the process of chaotic synchronization using computer simulation and experiment.

We have studied the dynamics of the individual nonlinear autonomous electric circuit and determined the parameter's region, where the circuit exhibits chaotic behavior. Furthermore, we observed the chaotic synchronization of two identical circuits, which are mutually coupled via a linear resistor R_C, when the initials conditions of the coupled circuits belong to the same basin of attraction. Both, experimental and simulation results have shown that chaotic synchronization is possible in the case of υ_{C2}-coupling and not possible in the case of υ_{C1}-coupling. In the first case, the coupling is established at the node of the linear part of the autonomous circuit, while in the second case the coupling is established at the node of the nonlinear part. The transferred energy from the coupling branch is not enough to synchronize the coupled system in the second case, as it has been verified by simulation and experiment. This is also enhanced in the case when conductance G_n becomes nonlinear. The increased complexity of the system does not allow synchronization, even in the case of υ_{C2}-coupling configuration [11].

Acknowledgements:
This work has been supported by the research program "EPEAEK II, PYTHAGORAS II", code number 80831, of the Greek Ministry of Education and E.U.

References:

[1] Pecora, L. M. and Carroll, T. L., Synchronization in chaotic systems, *Physical Review Letters*, Vol. 64, No.8, 1990, pp. 821-824.
[2] Wu, C. W., *Synchronization in Coupled Chaotic Circuits and Systems*, World Scientific, 2002.
[3] Cuomo, K. M. and Oppenheim, A. V., Circuit implementation of synchronized chaos with applications to communications, *Physical Review Letters*, Vol.71, No.1, 1993, pp. 65-68.
[4] Kocarev, L. and Parlitz, U., General approach for chaotic synchronization with applications to communications, *Physical Review Letters*, Vol.74, No.25, 1995, pp. 5028-5031.
[5] Kolumban, G., Kennedy, M. P. and Chua, L. O., The role of synchronization in digital communications using chaos - part I: Fundamentals of digital communications, *IEEE Transactions Circuits Systems-I*, Vol.44, 1997, pp. 927-936.
[6] Murali, K. and Lakshmanan, M., Drive-response scenario of chaos synchronization in identical nonlinear systems, *Physical Review E*, Vol.49, No.6, 1994, pp. 4882-4885.
[7] Kapitaniak, T., Chua, L. O. and G.-Q. Zhong, Experimental synchronization of chaos using continuous control, *International Journal of Bifurcation and Chaos*, Vol.4, No.2, 1994, pp. 483-488.
[8] Murali, K., Lakshmanan, M. and Chua, L. O., Controlling and synchronization of chaos in the simplest dissipative nonautonomous circuit, *International Journal of Bifurcation and Chaos*, Vol.5, No.2, 1995, pp. 563-571.
[9] Caroll, T. L. and Pecora L. M., Synchronizing nonautonomous chaotic circuits, *IEEE Transactions Circuits Systems-II*, Vol.40, 1993, pp. 646-650.
[10] Kyprianidis, I. M., Stouboulos, I. N., Haralabidis, P. and Bountis T., Antimonotonicity and chaotic dynamics in a fourth-order autonomous nonlinear electric circuit, *International Journal of Bifurcation and Chaos*, Vol.10, No.8, 2000, pp. 1903-1915.
[11] Kyprianidis I. M. and Stouboulos I. N., Synchronization of two resistively coupled nonautonomous and hyperchaotic oscillators, *Chaos Solit. Fract.*, Vol.17, 2003, pp. 317-325.