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Discrete Decentralized Stabilization of Large-Scale Systems With Prescribed Degree of Convergence

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Abstract: - In this paper the decentralized stabilization problem of discrete linear time invariant large scale interconnected systems without any assumption of system structure is considered. The design is based on stability result that employs the notion of block diagonal dominance in matrices and improves upon the exiting results for this problem. Our main result here is the sufficient condition for discrete decentralized stabilization. A new and simple algorithm is proposed. Simulation results on a numerical example are given to verify the proposed design.

Key-Words: - Large scale system, Decentralized stabilisation, Diagonal dominance, Continuous Lyapunov equation, Discrete Lyapunov equation, Decentralized control.

1 Introduction

Decentralized control has received renewed interests during the past years, motivated by its importance in applications to large complex engineering systems. Since 1960 many authors have considered this problem. The challenge is to find classes of decentralized controllers from which specific designs can be selected achieving a reasonable compromise in terms of simplicity of design, implementation and performance.

In earlier work, the approach that has received a high degree of popularity is to start with appropriately constructed Lyapunov function for the isolated subsystems and to impose suitable restrictions on the interconnections between the subsystems such that the overall system can be proved stable using a Lyapunov which is the sum of the subsystem Lyapunov functions. This approach have been applied in diverse practical problems such as spacecraft control [5],[6], control power systems[7] and control of industrials manipulators. More recently Solheim[8] gave a graphical approach based on the Gerschgorin theorem and Geromel and Bernussou [9] proposed a parametric optimisation scheme for the choice of subsystem Lyapunov functions. M. Sundareshan and R.M.Elbanna [1],[2] presented a systematic constructive procedure based on a stability result that employs the notion of block-diagonal dominances in matrices. But the implementation of these controllers is very complicated because the resolution of equation of algorithm imposes constraining conditions on the interconnections matrices and lead to restricted classes of the interconnected system Moreover the obtained gains are very high. M.Kacim and N.Elalami [3] proposed a balanced decentralized control always based on Gerschgorin theorem and upon techniques known in the theory of the balanced realisation of systems. Despite the simplicity of this approach, the procedures as the above mentioned designs are generally of trial and error nature and if the interconnections do not satisfy the required conditions, one is obliged to start with an alternate selection of the subsystem Lyapunov functions and repeat the process. To reduce this uncertainty in the Lyapunov function selection, F.Elmarjany and N.Elalami [4] studied the decentralized stabilization via eigenvalues assignment and developed the sufficient condition under which exponential stabilization with a prescribed convergence rate is achieved. The obtained gains of controllers are smaller than those found in other designs. The objective of this paper is to extend the previous works to discrete time systems and develop a new and simple approach for the design of discrete decentralized controllers of large scale interconnected linear systems.

2 Problem Formulation

Consider a large-scale continue system s described as an interconnection of s subsystems $s_1, s_2, \ldots, s_s$, by:

$$\dot{x}_i(t) = A_i x_i(t) + B_i u_i(t) + \sum_{\substack{j=1 \\ j \neq i}}^{s} H_{ij} x_j(t)$$

$$y_i = C_i x_i(t) \quad i = 1 \ldots\ldots s \tag{1}$$

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$x_i \in \Re^{ni}$ is the state of the subsystem s, $u_i \in \Re^{mi}$ is its input vector and $y_i \in \Re^{pi}$ is its output vector.

$\sum_{\substack{j=1 \\ j\neq i}}^{s} H_{ij}$ is the term due to interconnection of the other subsystems. $A_i \in \Re^{ni\times ni}$; $B_i \in \Re^{ni\times mi}$, and $H_{ij} \in \Re^{ni\times nj}$ are matrices of appropriate dimensions.

It assumed that all pair (A_i, B_i) are controllable and (A_i, C_i) are completely observable.

The problem is to design a local controller

$$u_i(t) = -K_i x_i(t) \qquad (2)$$

Stabilises the large scale interconnected system.

Applying the i controller (2) to the plant (1) give:

$$\dot{x}_i(t) = F_i x_i(t) + \sum_{\substack{j=1 \\ j\neq i}}^{s} H_{ij} x_j(t) \qquad i = 1...s$$

Where $F_i = A_i - B_i K_i$ (3)

Let $\lambda_{\min}(X)\, et\, \lambda_{\max}(X)$ respectively denote the minimum and the maximum of reel matrix X, the notation λ_i (X) and σ_i (X) denote the ith eigenvalue and singular value of the matrix X, (λ_i (X)) are arranged in descending order when they are reel, i.e $\lambda_1(X) \geq \lambda_2(X) \geq .... \geq \lambda_n(X)$, also for any $P \in \Re^{n\times m}$ $\|P\| = \lambda_{\max}^{\frac{1}{2}}(P^T P) = \sigma_1(P)$

Definition 2.1 Let A $\in R^{n\times n}$ be partitioned in the form:

$$A = \begin{bmatrix} A_{11} & A_{12} & \Lambda & A_{1n} \\ A_{21} & A_{22} & \Lambda & A_{2n} \\ M & & & \\ A_{n1} & A_{n2} & \Lambda & A_{nn} \end{bmatrix} \qquad (4)$$

Where $A_{ii} \in \Re^{ni\times ni}$ et $A_{ij} \in \Re^{ni\times nj}$ i, j=1, 2…n.

If A_{ii} are non singular and

$$\left\|A_{ii}^{-1}\right\|^{-1} \geq \sum_{\substack{j=1 \\ j\neq i}}^{s} \left\|A_{ij}\right\|, \forall \text{ i=1,2…s} \qquad (5)$$

Then A is said to be block diagonal dominant relative to the partitioning in (4) . If strict inequality holds in (5), then A is strictly block diagonal dominant.

Lemma 2.2

Let the matrix A partitioned as in (4) satisfy the conditions:

(i) $A = A^T$

(ii) A_{ii}=1,2….s. are positive definite

(iii) A is strictly block diagonal dominant

Then, all eigenvalues of A are real and positive

Theorem 2.3

Let spec $(F_i) \subset$ LHP $\forall$ i=1, 2,….s and let P_i the symmetric matrix solution of the Lyapunov equation

$$F_i^T P_i - P_i F_i + Q_i = 0 \qquad (6)$$

For an arbitrarily selected symmetric matrix $Q_i \in \Re^{ni\times ni}$, then (3) is asymptotically stable if:

$$\lambda_{\min}(Q_i) \phi \lambda_{\max}(P_i) \sum_{j=1(j\neq i)}^{s} \left\|H_{ij}\right\| + \sum_{j=1(j\neq i)}^{s} \lambda_{\max}(P_j) \left\|H_{ji}\right\| \qquad (7)$$

Proof: see [1] and [2].

The objective of this paper is to extend this work to discrete systems and to give the sufficient conditions of existence of discrete decentralized controllers.

Let us consider a discrete large-scale system described as interconnections of s subsystems by:

$$z_i(k+1) = F_i z_i(k) + \sum_{j=1(j\neq i)}^{s} G_{ij} z_j(k)$$

$\forall$ i=1……s (8)

F_i is asymptotically stable matrix

$F_i \in \Re^{ni\times nj}$, $G_{ij} \in \Re^{ni\times nj}$

Theorem 2.4

Let F_i an asymptotically stable matrix $\forall$ i=1, 2… s and let P_i the symmetric matrix solution of the discrete algebraic Lyapunov matrix equation:

$$F_i^T P_i F_i - P_i + Q_i = 0 \qquad (9)$$

For an arbitrarily selected symmetric matrix $Q_i \in \Re^{ni\times ni}$, then (8) is asymptotically stable if:

$$\lambda_{\min}(Q_i) \phi \lambda_{\max}(P_i) \left\|F_i\right\| \sum_{j=1(j\neq i)}^{s} \left\|G_{ij}\right\| + \sum_{j=1(j\neq i)}^{s} \lambda_{\max}(P_j) \left\|G_{ij}\right\| \left\|F_j\right\| + \sum_{k=1(k\neq i)}^{s} (\lambda_{\max}(P_k) \left\|G_{ki}\right\| \; (\sum_{j=1(j\neq i)}^{s} \left\|G_{kj}\right\|)) + \sum_{j=1(j\neq i)}^{s} \lambda_{\max}(P_j) \left\|G_{ji}\right\|^2 \qquad (10)$$

To prove this result, we will use the standard results for eigenvalues of symmetric matrices [5], [6].

For $M = M^T \in R^{n*n}$

$$\lambda_{\min}(M) I \leq M \leq \lambda_{\max}(M) I \qquad (11)$$

For $X = X^T$, $Y = Y^T \in R^{n*n}$

$$\lambda_i(X) + \lambda_n(Y) \leq \lambda_i(X+Y) \leq \lambda_i(X) + \lambda_1(Y) \qquad (12)$$

For $X=X^T$, $Y\in R^{n*n}$

$$\sigma_n^2(Y)\lambda_i(X) \le \lambda_i(Y^T XY) \le \sigma_1^2(Y)\lambda_i(X) \qquad (13)$$

Proof

selecting $V(z(k)) = z^T(k)\, P\, z(k)$, $P=diag(P_1,P_2,\Lambda\ P_s)$ as a discrete Lyapunov function and evaluating its variation along the trajectories of (8)

Δ V(z)=V(z(k+1))-V(z(k))

$=z^T(k)\,(F^T P G+G^T P F+G^T P G- Q)z(k)$

$= -z^T(k)\, W\, z(k)$

Where

$Q=diag(Q_1,Q_2,\Lambda\ \Lambda\ Q_s)$,

$F=diag[F_1,F_2,\Lambda\ ,F_s],\ G=[G_{ij}],\ i,j=1,2,....s$

$$W=(Q-F^T PG-G^T PF-G^T PG) \qquad (14)$$

satisfies $W^T=W$

The diagonal elements:

$$W_{ii}=Q_i-\sum_{j=1(j\neq i)}^{s} G_{ji}^T P_j G_{ji}\ ,\ i=1,2.....,s. \qquad (15)$$

$W_{ii}^T=W_{ii}$, applying (11) to (15) gives:

$$\lambda_{\min}(Q_i)I-\sum_{\substack{j=1\\ j\neq i}}\lambda_{\max}(G_{ji}^T P_j G_{ji})I \le W_{ii} \le$$

$$\lambda_{\max}(Q_i)I-\sum_{\substack{j=1\\ j\neq i}}\lambda_{\min}(G_{ji}^T P_j G_{ji})I$$

If $\lambda_{\min}(Q_i)\phi \sum_{j=1(j\neq i)}^{s}\lambda_{\max}(G_{ji}^T P_j\, G_{ji})$,

W_{ii} is positive definite

and $\left\|W_{ii}^{-1}\right\|^{-1}=\lambda_{\min}(Q_i-\sum_{j=1(j\neq i)}^{s} G_{ji}^T P_j G_{ji})$

Applying (12) and (13) we have:

$$\lambda_i(Q_i)-\lambda_1(\sum_{\substack{j=\\ j\neq i1}}^{s} G_{ji}^T P_j G_{ji}) \le \lambda_i(Q_i-\sum_{\substack{j=1\\ j\neq i}}^{s} G_{ji}^T P_j G_{ji}$$

$$\le \lambda_i(Q_i)-\lambda_{\min}(\sum_{\substack{j=1\\ j\neq i}}^{s} G_{ji}^T P_j G_{ji})$$

$$\lambda_{\min}(Q_i)-\sum_{\substack{j=1\\ j\neq i}}^{s}\sigma_1^2(G_{ji})\lambda_{\max}(P_j) \le$$

$$\lambda_{\min}(Q_i)-\sum_{\substack{j=1\\ j\neq i}}^{s}\lambda_{\max}(G_{ij}^T P_j G_{ji}) \le \left\|W_{ii}^{-1}\right\|$$

$$W_{ij}=-((F^T PG)_{ij}+(G^T PF)_{ij}+(G^T PG)_{ij})$$

$$= -F_i^T P_i\, G_{ij} - G_{ji}^T P_j\, F_j - \sum_{\substack{k=1\\ k\neq i}}^{s} G_{ki}^T P_k\, G_{kj}$$

From lemma 2.2 , W is positive definite if it is strictly block-diagonal dominant, i.e.

$$\left\|W_{ii}^{-1}\right\|^{-1}\ \phi\ \sum_{\substack{j=1\\ j\neq i}}^{s}\left\|W_{ij}\right\|$$

$\forall$ i=1……..s

It is simple to observer that for $i\neq j$:

$$\left\|w_{ij}\right\| \le \lambda_{\max}(P_i)\left\|F_i\right\|\left\|G_{ij}\right\| + \lambda_{\max}(P_j)\left\|G_{ji}\right\|\left\|F_j\right\| +$$

$$\sum_{\substack{k=1\\ k\neq i}}\lambda_{\max}(P_k)\left\|G_{ki}\right\|\left\|G_{kj}\right\|$$

and hence (10) implies

$W_{ii}\ \phi\ 0$ and $\left\|W_{ii}^{-1}\right\|^{-1}\phi\sum_{\substack{j=1\\ j\neq i}}^{s}\left\|W_{ij}\right\|$

The implementation of this control by using theorem 2.4 is very complicated because the resolution of equation of algorithm imposes constraining conditions on the interconnections matrices and leads to restricted classes of the interconnected system. Moreover the obtained gains are very high. In this paper we propose a new algorithm for decentralised stabilisation based on the theorem Gerschgorin such that the overall system can be stabilized with a prescribed convergence rate.

3 An Algorithm for discrete decentralized stabilization

Let consider the discrete system:

$$x_i(k+1) = A_i x_i(k) + B_i u_i(k) + \sum_{\substack{j=1\\ j\neq i}}^{s} G_{ij} x_j(k) \qquad (16)$$

The problem is to design a local dynamic controller:

$u_i(k) = -\, k_i\, x_i(k)$ (17)

Stabilizes the system (16).from(16) and (17) we have:

$$x_i(k+1) = f_i x_i(k) + \sum_{\substack{j=1\\ j\neq i}}^{s} G_{ij}$$

Where $f_i = A_i - B_i\ k_i$

Spec (f_i)= $\left\{\lambda_1^i \quad \lambda_2^i \quad \Lambda \quad \lambda_n^i\right\}$

Let us consider the following transformation:

$X_i(k) = \alpha^{-k} X(k), \qquad U_i(k) = \alpha^{-k} u_i(k)$

α is a positive scalar ($\alpha \ge 1$)

$$X_i(k+1) = \alpha^{-(k+1)} X_i(k) = \alpha^{-1} A X_i + \alpha^{-1} B U_i(k) + \sum_{\substack{j=1 \\ j \neq 1}}^{s} \alpha^{-1} h_{ij} X_j(k) \qquad (18)$$

$$U_i(k) = -K_i X_i(k)$$

The goal is to select the feedback gain K_i and the required scalar α which stabilize (18)

$F_i = \alpha^{-1} (A_i - B_i K_i)$

$Spec(F_i) = \{\beta_1^i \quad \beta_2^i \quad \Lambda \quad \beta_n^i\}$

From theorem2.4, the system (18) is asymptotically stable if :

$$\lambda_{min}(Q_i) \phi \alpha^{-1}\lambda_{max}(P_i)\|F_i\| \sum_{j=1(j\neq i)}^{s} \|G_{ij}\| + \alpha^{-1} \sum_{j=1(j\neq i)}^{s} \lambda_{max}(P_j)\|G_{ij}\|\|F_j\| + \alpha^{-2} \sum_{k=1(k\neq i)}^{s} (\lambda_{max}(P_k)\|G_{ki}\| \; (\sum_{j=1(j\neq i)}^{s} \|G_{kj}\|)) + \alpha^{-2} \sum_{j=1(j\neq i)}^{s} \lambda_{max}(P_j)\|G_{ji}\|^2 \qquad (19)$$

We shall now give the procedure for the construction of the controllers gains k_i:

Step1: α =1, select K_i such that spec (A_i-B_i K_i) is inside the unit circle. This selection can be made by a standard pole placement design.

Step2: choose an arbitrary matrix positive Q_i,

Step 3 : Solve the Lyapunov matrix equation.

If condition (19) is checked then :

Calculate k_i such as

spec (A_i-B_i k_i)= spec (α^{-1} (A_i-B_i K_i))

If not, $\alpha = \alpha$ +1 and go to step 3.

3 Illustrative example

We consider the following example which was treated by Sundershan and Elbanna[1].

$$\dot{x}_1 = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ -2 & -1 & -1 \end{bmatrix} x_1 + \begin{bmatrix} 2 & 0 \\ 3 & 4 \\ 2 & 1 \end{bmatrix} x_2 + \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix} u_1$$

$$\dot{x}_2 = \begin{bmatrix} 0 & 1 \\ -3 & -2 \end{bmatrix} x_2 + \begin{bmatrix} 4 & 0 & 0 \\ 5 & 6 & 0 \end{bmatrix} x_1 + \begin{bmatrix} 0 \\ 1 \end{bmatrix} u_2$$

The discrete-time model is obtained from its continuous –time model by discretizing it using MATLAB c2d with the sampling period T=0.1

$$x_1(k+1) = \begin{bmatrix} 0.9997 & 0.0998 & 0.0048 \\ -0.0097 & 0.9948 & 0.0950 \\ -0.1900 & -0.1047 & 0.8998 \end{bmatrix} x_1(k) + \begin{bmatrix} 0.2153 & 0.0201 \\ 0.3085 & 0.4042 \\ 0.1552 & 0.0744 \end{bmatrix} x_2(k) + \begin{bmatrix} 0.0002 \\ 0.0048 \\ 0.0950 \end{bmatrix} u_1(k)$$

$$x_2(k+1) = \begin{bmatrix} 0.9860 & 0.0902 \\ -0.2705 & 0.8056 \end{bmatrix} x_2(k) + \begin{bmatrix} 0.4215 & 0.0280 & 0 \\ 0.3949 & 0.5411 & 0 \end{bmatrix} x_1(k) + \begin{bmatrix} 0.0047 \\ 0.0902 \end{bmatrix} u_2(k)$$

Selecting K_1= [12,6590 14,4390 5,3373] and K_2 = [7,0324 3,6090] to place the eigenvalues of ($A_1 - B_1 K_1$) and ($A_2 - B_2 K_2$) at (0,8084+0,0977i 0,8084-0,0977i 0,6987) and (0,7166+0,1020i 0,7166-0,1020i)

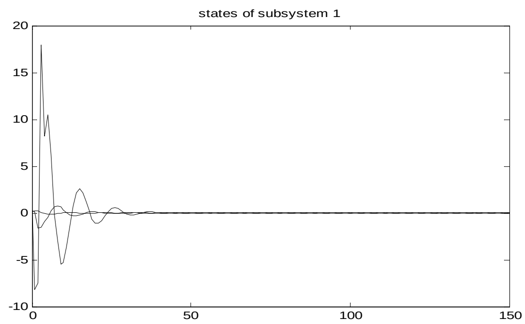
Following the steps of the algorithm, we obtain α =4;

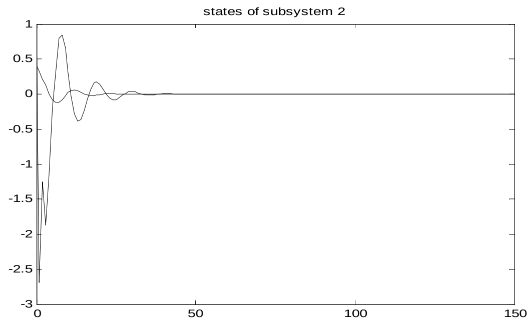
The required controller gains k_1 and k_2 to be used are then:

k_1= [551.1286 148.6881 15. 8668] and

k_2= [71.5979 12.1847].

All the states of the subsystems are plotted in figures. From the simulation results it can be seen that each subsystems are asymptotically stable.





4 Conclusion

This paper has introduced a new approach for designing a decentralized control for discrete large scale systems. The present results are developed in the context of decentralized stabilization; they have a wider application in that they can be extended to the design of decentralized observation algorithms and to the design of decentralized model reference adaptive identification schemes.

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