Discrete Time Integrator Comparison for an Angle of Arrival Neural Network Detector on a Transmitter – Independent Receiver Network

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Abstract – A Recurrent First Order Artificial Neural Network may be used for target detection by Angle of Arrival on a Transmitter – Independent Receiver Network (multimode passive multistatic radar). In this paper a model of it is created and then simulation takes place in order to investigate the performance of various integration and accumulation methods. It is shown that construction simplicity that is essential to military systems is retained in this method in any case and the optimum integration method can be chosen.

Keywords – Multistatic Radar, Neural Networks, Angle of Arrival, Electronic Warfare, Digital Signal Processing.

1. Introduction.

Angle of Arrival method is a well known method of transmitter detection that consists of finding its angular (spherical) coordinates from at least two receivers and then solving a linear system to extract its coordinates (Gaussian or spherical) in a standard reference coordinate system.

This concept may be used also for target detection in a Transmitter – Independent Receiver Network (TIRN); this is multistatic radar that may detect targets illuminated by transmitters of opportunity.

In order to achieve robustness and redundancy needed for a military system plus multimode operation a four receiver model is considered. [1]

The Artificial Neural Network (ANN) considered here is one adapted from a general purpose design [2] used for linear problem solving.

It differs though consists of two linear variable gain layers, one linear fixed gain layer an integrator layer and a loop. It is supported by Target Matrix Generator preprocessing units. (Fig.1)

2. The AOA TIRN model description.

The linear system that is created is as it has been said in [1] an over determined linear system. Its geometrical representation is presented in Fig. 2.

This detection method can be described as follows:

Let \( f_p = 400\text{Hz} \) and \( r(x_i, y_i, z_i) \) be the locations of one of the receivers \((i \in \{1,2,3,4\})\) and the target respectively. Then the equations connecting the Cartesian coordinates with the spherical coordinates measured on each receiver by its monopulse antenna are given below:

\[
\begin{align*}
x - x_i &= R_i \cos \phi \sin \theta_i \\
y - y_i &= R_i \sin \phi \sin \theta_i \\
z - z_i &= R_i \cos \theta_i
\end{align*}
\]  

With the elimination of \( R_i \) (target receiver distance) it becomes:

\[
\begin{bmatrix}
\cos \theta_i & 0 & -\cos \phi_i \sin \theta_i \\
0 & \cos \phi_i & -\sin \phi_i \sin \theta_i
\end{bmatrix}
\begin{bmatrix}
x_i \\
y_i \\
z_i
\end{bmatrix} = \begin{bmatrix}
x \\
y \\
z
\end{bmatrix}
\]  

As it is clear, even two receivers would give an over determined 4X3 linear equation system.

Redundancy reasons and reasons of multimode operations [1], [3] is the main reason that at least a four receiver model is used, comprised by two pairs of receivers. This would give an 8X3 over determined equation system as shown below.

\[
Ax = b
\]

With:

\[
x = r_T = \begin{bmatrix}
x \\
y \\
z
\end{bmatrix}
\]  

\[(4a)\]
An on – line method of linear equation solving has been described thoroughly in [2] with a method that can be easily adapted for the AOA method. In order to solve (3), an energy function for an estimated AOA method is defined which in this case is:

$$E(x) = 0.5(Ax - b)^T (Ax - b)$$

(5)

The differential equation system below (where $t$ is in time units) describes gradient descent approximation for the minimization of the energy function $E(x)$ for an initial value $x(0)$ (initial value problem).

$$\frac{dx}{dt} = -\mu \nabla E(x), \quad \nabla E(x) = A^T (Ax - b)$$

(6)

The above system in its analytical form is equivalent to:

$$\frac{dx_j}{dt} = -\sum_{p=1}^{n} \mu_{jp} \left( \sum_{i=1}^{m} a_{ip} \left( \sum_{k=1}^{n} a_{ik} x_k - b_i \right) \right)$$

(7)

Choice of $\mu_{jp}$ must ensure the stability of the differential equation and an appropriate convergence speed to the desired solution. It has been proven that the system (6) or (7) is stable and has a solution that converges to a vector $x$ as $t$ tends to the infinite as it is:

$$\frac{dE}{dt} = \sum_{j=1}^{n} \frac{\partial E(x)}{\partial x_j} \frac{dx_j}{dt} = -\nabla E(x)^T M \cdot \nabla E(x) \leq 0$$

(8)

The above is always true if $M$ with elements $\mu_{jp}$ is a (predefined) positive definite matrix. Further analysis of (7) gives:

$$e_i(x) = \sum_{k=1}^{n} a_{ik} x_k - b_i, \quad i = 1, 2, ..., m$$

(9a)

$$\frac{\partial E(x)}{\partial x_p} = \sum_{i=1}^{m} a_{ip} e_i(x),$$

(9b)

$$p = 1, 2, ..., n, \quad x_j(0) = x_j^{(0)}$$

(9c)

The recurrent ANN, shown in fig.1, (Appendix) consists of integrators (as many as the dimension of the problem) and weighted input adders. The weights $\alpha_{ij}$ and $\mu_{ij}$ are the elements of the matrices $A$ and $M$, Second array elements they are constant. That makes this network easy to construct. In this particular case, in equations (6) to (9), $m = 8$ and $n = 3$. They are small values that make the construction easier.

There are three layers in that ANN connected in feed – forward mode. First layer named "sensor layer" because it senses the actual variables $x_j$ and computes errors $e_j(x)$ as they defined in (9a). Error signals $e_j(x)$ are inputs to the second “association” layer, which gives the gradient components of the system. Here the weights are approximately equal to weights of the first layer as (6) denotes. The third, "response layer" is consisted of response elements, which define the convergence rate.

In this case $\mu_{ij}$ (third layer elements) are considered constant for simplicity reasons. In order to simplify the network, $\mu_{ij}$ may be considered elements of a positively defined diagonal matrix, thus making the elimination of the adders in the third layer possible. This is the case simulated here in order to prove that even a simplified design can give the expected results.

Equation 9b then suggests that a feedback loop must be constructed with integrators. When discrete integration takes place (this is when digital circuits are used for signal processing), then the integrators defined by their transfer ($n z – transform$) functions. They are:

- For a trapezoidal integrator:
  $$H_I(z) = \frac{K \cdot T_s \cdot (z + 1)}{2 \cdot (z - 1)}$$

(10)

- For a Backward Euler integrator:
  $$H_I(z) = \frac{K \cdot T_s \cdot z}{z - 1}$$

(11)
• And finally a Forward Euler integrator:

$$H_3(z) = \frac{K \cdot T_s}{z - 1}$$  \hfill (12)

3. Simulation results

Test system was simulated using Matlab’s SIMULINK ®. This consists of four receivers at:

$$\begin{align*}
\begin{bmatrix}
x_1 \\
y_1 \\
z_1 \\
x_2 \\
y_2 \\
z_2 \\
x_3 \\
y_3 \\
z_3
\end{bmatrix}
&=egin{bmatrix}
0 \\
0 \\
1.414 \\
20.544 \\
0.743 \\
0.525 \\
-6.614 \\
-10.487 \\
0.545
\end{bmatrix}

\begin{bmatrix}
x_4 \\
y_4 \\
z_4
\end{bmatrix}
&=egin{bmatrix}
0 \\
20.443 \\
0.822
\end{bmatrix}
\end{align*}$$  \hfill (13)

This set of receivers is placed on a rough land surface as receiver altitudes denote. Coordinate axes $x'$,$y'$ denote position from West (negative) to East and from South to North respectively while axis $z'$ denotes altitude (height) placement.

PRF defines the sample time of the simulation and the trials have been made at $f_p = 400 \text{ Hz}$ . Thus the sample time is $T_s = 2.5 \mu \text{sec}$ . The target is a tri-sonic aircraft manoeuvring on hard turns. All the defined first order integrators are tried. Simulation takes place for 1000 seconds.

For the trapezoidal integrator a compromise must be done about the constant $K$ (integrator gain) Very large gains give most inaccurate results, but on the other hand very small gains give a very low convergence rate thus making initial acquisition of target more difficult.

For example while $K = 10^{-4}$ gives accurate results (Fig.3 Fig.4 Fig.5) and converges in 10 samples when $K = 10^{-6}$ then errors of the orders of 200 $m$

Forward Euler integration method must be avoided. Although integrators of this type were thoroughly tested with different integrator gains they all failed to acquire the target.

Backward Euler Integration has given the most accurate results. (Fig.7, Fig.8). The order of errors is in one millimeter while this method converges in the first 3 samples.

4. Conclusions

Although Artificial Neural Networks give accurate results, an engineer must be careful in every design In order to achieve nearly optimal results. In the case presented here only changing a small part of the design may give totally different results.

For the specified problem Backward Euler integration method gives perfect results (errors are only of academic importance) while Trapezoidal Integration may give acceptable results if integration constant is compatible with the PRF of the transmitting radar or the sampling period if it is a CW transmitter.

References:


Appendix. Figures and Diagrams.

**Fig. 1. Simulation model:**

**Fig. 2. Geometry of the AOA problem**
**Fig. 3.** Computed Cartesian coordinates in Km

**Fig. 4.** Errors in coordinates in mm (Trapezoidal Integrator, $K=10^{-4}$)

**Fig. 5.** Range error in mm (Trapezoidal Integrator, $K=10^{-4}$)

**Fig. 6.** Convergence. (Trapezoidal Integrator, $K=10^{-4}$)

**Fig. 7.** Errors in coordinates in mm. (Backward Euler Integrator, $K=1$)

**Fig. 8.** Range error in mm. (Backward Euler Integrator, $K=1$)