

A Review of Chaotic circuits, Simulation and Implementation

Cherif Aissi¹ and Demetrios Kazakos²

¹College of Engineering, University of Louisiana at Lafayette
Lafayette, LA 70504, USA

²Department of Electrical & Computer Engineering, University of Idaho
Moscow, Idaho 83843-1023

Abstract: - Understanding how to design circuits that display a chaotic behavior is of great importance both to students and to practicing engineers. The benefit is to learn not only how to avoid chaos, which can be a nuisance in a circuit, but also to develop applications that actually exploit this phenomenon. In this paper, we present two autonomous and two nonautonomous circuits that behave chaotically. Electronics Workbench is used to simulate circuits and show the presence of chaos. Complete circuits implementation and oscilloscope plots are all given.

Key-Words: -Chua's circuit, chaos, attractor, bifurcation, oscillator.

1. Introduction

For decades, the engineering undergraduate education in the area of circuit and design has been mainly focused in linear models. The reason is that linear system theory has been thoroughly developed and mathematical tools are available to analyze such systems. This philosophy has led many scientists and experimentalists to disregard many observed phenomena because linear system theory can not explain them. In the last decade, there is a strong interest in exploring systems that display unusual complicated waveforms, commonly known as strange attractors. These attractors have been increasingly observed in several nonlinear deterministic systems.

Therefore, it is important for today's students to be exposed to these complex chaos phenomena. From the educational aspect, students need to learn not only how to control and avoid chaos but also how to design chaotic circuits and develop applications, which explore these phenomena. Realizing the educational value to introduce undergraduate students to the

phenomena of chaos, Lonngren [1] describes an interesting electronics experiment to illustrate the existence of chaos. The described laboratory experiment with the accompanying theory is a good start for the student to grasp and understand chaos. As a continuation of this goal and to enhance the student understanding of chaos, Hamill [2] presented a collection of ten chaotic circuits simulated using PSpice. These circuits, some are quite simple, illustrate how chaos can be generated.

This paper can be viewed as a logical follow up on the progress made to understand chaos. It is a continuation to strengthen the student understanding not only on how chaos is simulated but also how to implement it. It is worthwhile to mention that some of the presented circuits, such as the Colpitts oscillator, are taught to students in electronics courses. Yet, there is no mention that chaotic behavior may occur. We have chosen to stay away from circuits that require a degree of mathematical sophistication beyond the undergraduate level.

In this paper, several chaotic circuits are presented for the student and the practicing engineer to study and experiment with. We have selected to use Electronics Workbench (EW) to simulate circuits since it provides an interface as close as to the real implementation environment. In addition, complete circuits implementation and oscilloscope graphical plots are all presented.

Section II presents examples of autonomous circuits. Section III is devoted to nonautonomous circuits. It is our hope that this paper will raise many questions in the mind of students and practicing engineer.

2. Examples of autonomous chaotic circuits:

Chua's circuit: The first example we present is the Chua's circuit (a third-order autonomous, dissipative electrical circuit). It has been investigated thoroughly at the experimental, numerical and analytical levels [3-9]. This circuit, known for its rich repertoire of nonlinear dynamical phenomena, has become a universal paradigm for chaos. Fig. 1(a) shows the Chua's circuit which includes two capacitors, a resistor, an inductor and a nonlinear resistor NR. Applying KCL and KVL, the Chua's circuit is described by three differential equations:

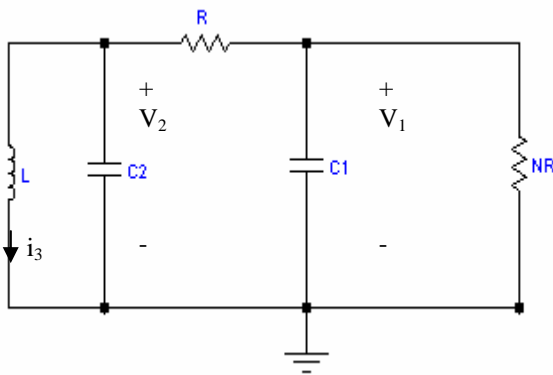


Fig 1. (a) The unfolded Chua's circuit

$$\begin{aligned} C_1 \frac{dv_1}{dt} &= \frac{1}{R}(v_1 - v_2) - f(v_1) \\ C_2 \frac{dv_2}{dt} &= \frac{1}{R}(v_1 - v_2) + i_3 \\ L \frac{di_3}{dt} &= -v_2 \end{aligned} \quad (1)$$

Where the nonlinear Chua's function, shown in Fig. 1(b), is described by

$$f(v_R) = m_o v_R + \frac{1}{2}(m_1 - m_o) \left\{ |v_R + B_p| - |v_R - B_p| \right\} \quad (2)$$

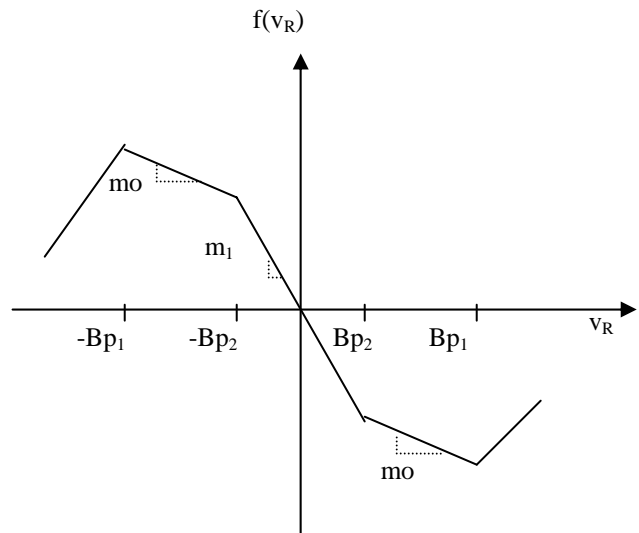


Fig.1(b) Chua's nonlinear function

The realization of the Chua's circuit is shown in Fig 1.(c). The constant m_o , m_1 , and B_p can be easily computed.

$$\left(\begin{aligned} m_1 &= -\frac{R_2}{R_1 R_3} - \frac{R_5}{R_4 R_6}, m_o = -\frac{R_2}{R_1 R_3} + \frac{1}{R_4}, \\ B_{p1} &= \frac{R_3}{R_2 + R_3} E_{sat}, B_{p2} = \frac{R_6}{R_5 + R_6} \end{aligned} \right)$$

Where, E_{sat} is the saturation voltage of the operational amplifier. The complete implementation of the Chua's circuit is shown in Fig. 1(c).

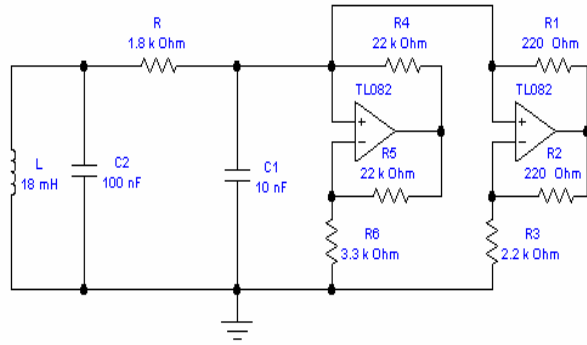


Fig 1.(c) The realization of the Chua's circuit [1]

The results of Electronics Workbench simulation show the phase portrait of the probed signals in fig. 1(d).

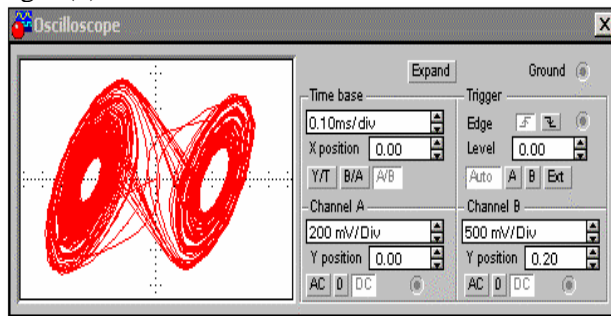


Fig. 1(d). Phase portrait V_{c2} versus V_{c1} .

The Chua's circuit was implemented using the TL082 operational amplifier. The double scroll is shown in fig. 1(e) and the voltage $v_{c1}(t)$ as well as its spectrum are shown in fig.1(f). The reader is encouraged to experiment with this circuit by connecting a variable resistor in series with the inductor L.

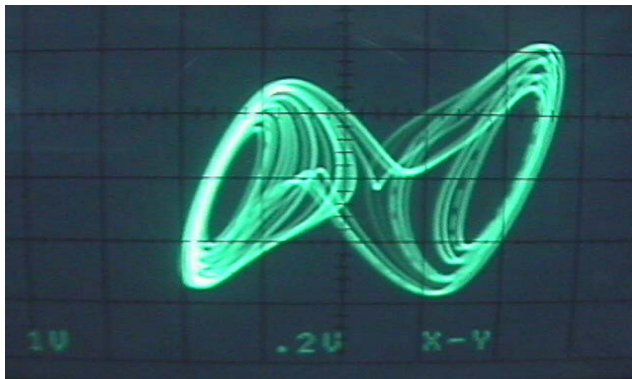


Fig.1(e). double scroll V_{c2} versus V_{c1} for $R=1.53K\Omega$

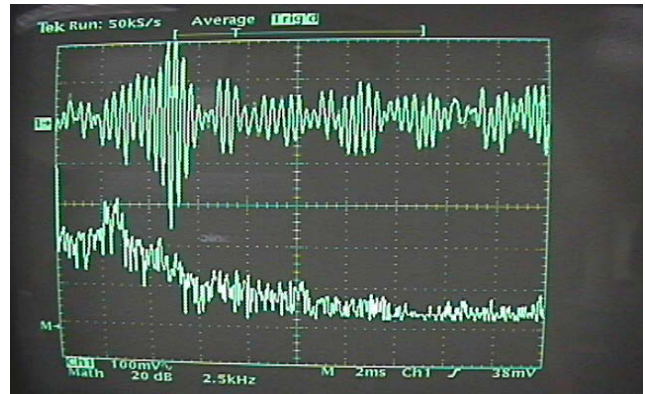


Fig.1(f). Top: $V_{c1}(t)$. Spectrum of $V_{c1}(t)$.

Colpitts oscillator: The second example is the Colpitts oscillator [10,11] shown in Fig. 2(a).

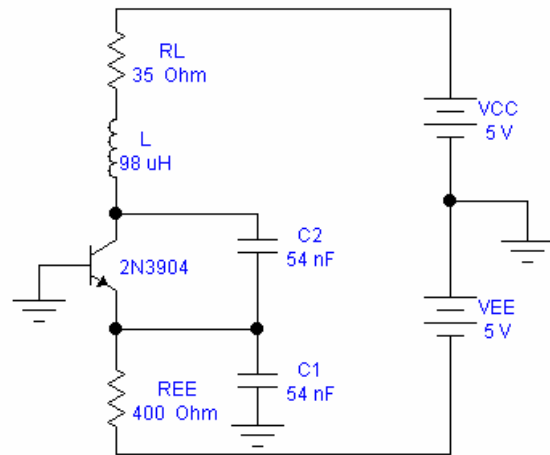


Fig. 2(a). Colpitts oscillator.

It is described by a system of three state equations:

$$\begin{aligned} C_1 \frac{dV_{CE}}{dt} &= i_L - i_C \\ C_2 \frac{dV_{BE}}{dt} &= -\frac{V_{EE} + V_{BE}}{R_{EE}} - i_L - i_B \quad (3) \\ L \frac{di_L}{dt} &= V_{CC} - V_{CE} + V_{BE} - i_L R_L \end{aligned}$$

The transistor is modeled as follows

$$i_B = \begin{cases} 0 & \text{if } V_{BE} \leq V_{TH} \\ \frac{V_{BE} - V_{TH}}{R_{ON}} & \text{if } V_{BE} > V_{TH} \end{cases}$$

$$i_C = \beta_F i_B$$

Where, V_{TH} is the threshold voltage ($\approx 0.75V$), R_{ON} is the small signal on-resistance of the base-emitter junction, and β_F is the forward current gain of the device. In most undergraduate electronics books, this circuit is shown to oscillate but yet it can be driven to chaos. Fig. 2(b) shows the chaotic phase portrait. By changing the value of resistance R_L we obtain different trajectories.

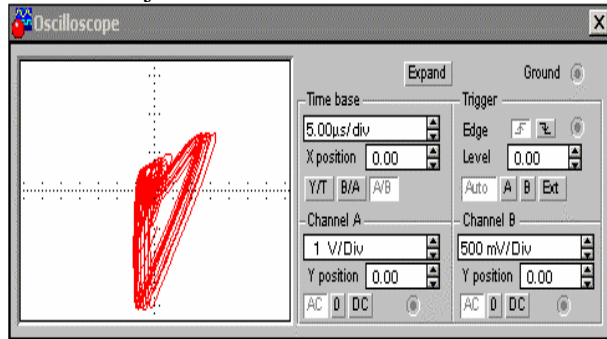


Fig. 2(b). Phase portrait of V_C versus V_E

The Colpitts oscillator was implemented and its phase portrait scroll is shown in Fig. 2(c). The voltage $V_C(t)$ and its spectrum are shown in Fig. 2(d)

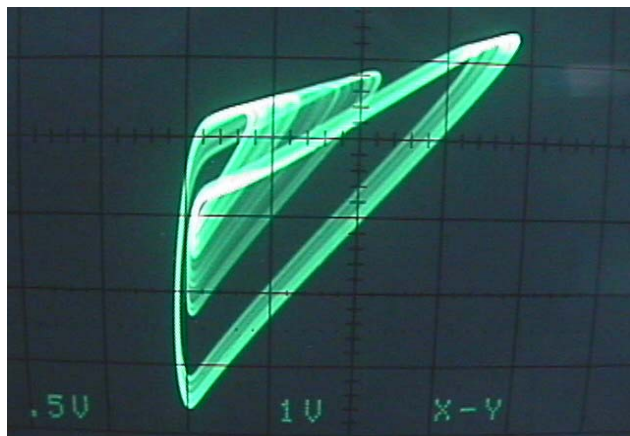


Fig.2(c). Phase portrait of V_C versus V_E for $R_{EE}=466\Omega$

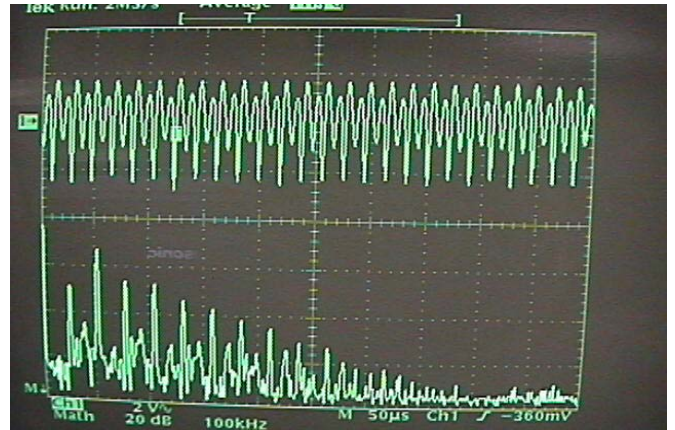


Fig.2(d). Top: $V_C(t)$. Bottom: spectrum of $V_C(t)$

3. Examples of nonautonomous chaotic circuits

RL-Diode circuit: The first example of nonautonomous chaotic circuit is the driven RL-diode circuit [12,13] shown in Fig 3(a).

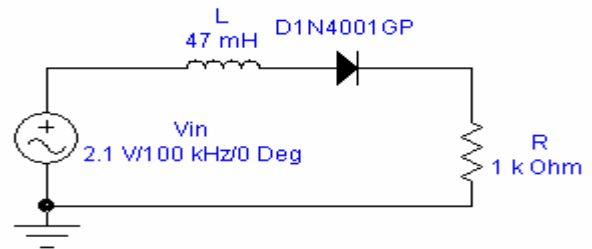


Fig. 3(a). RL-Diode chaotic circuit

It consists of a series connection of an ac-voltage source, a linear resistor, a linear inductor and a diode which is the only nonlinear circuit element. The state equations describing this circuit is:

$$V_{in} = Ri + L \frac{di}{dt} + V_D$$

$$i = I_S \left(e^{\frac{V_D}{nV_T}} - 1 \right)$$

Where V_D is the voltage across the diode, I_S is the diode saturation current, n is a constant which has a value between 1 and 2 depending on the material and the physical structure of the diode, and V_T is the thermal voltage. The i - v characteristic of a diode is shown in Fig. 3(b).

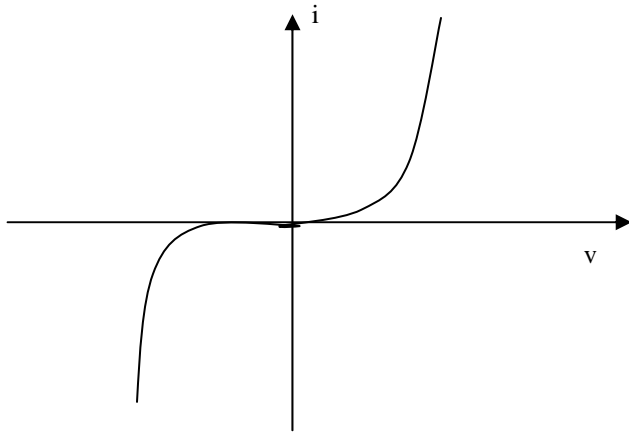


Fig. 3(b). I-V diode characteristic.

An important feature of this circuit is that the current i (or the voltage across the resistor R) can be chaotic although the input voltage V_{in} is nonchaotic. The results of the Electronic Workbench simulation, are shown in Fig. 3(c) for $R=1k\Omega$.

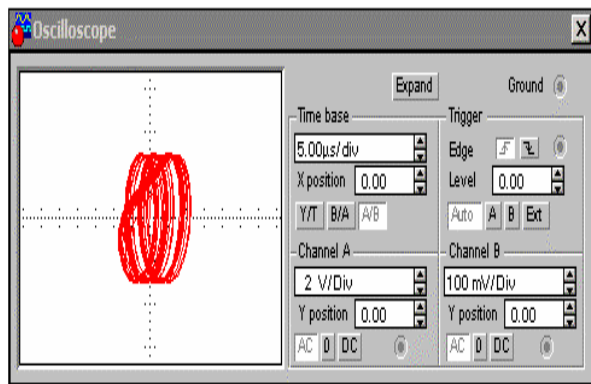


Fig. 3(c). Phase portrait of V_{in} versus V_R

The RL-diode was implemented and its phase portrait is shown in fig. 3(d). The voltage across the resistance R and its spectrum are shown in fig. 3(e). The reader is encouraged to experiment with different values of R to obtain different phase portraits.

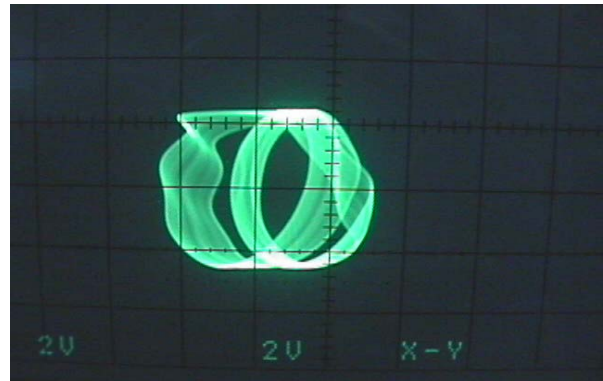


Fig.3(d) Plot of V_{in} versus V_R for $f = 130KHz$, V_{in} peak-peak=6.5V and $R=26K\Omega$.

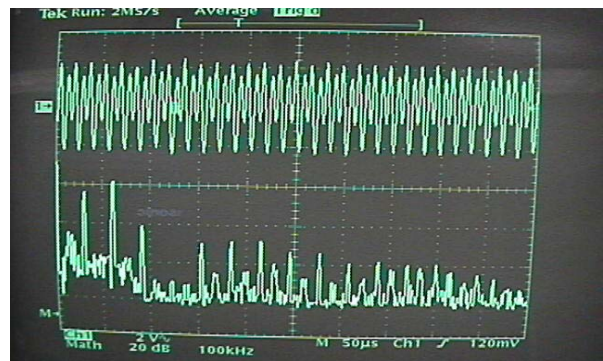


Fig.3(f). Top: $V_R(t)$. Bottom: spectrum of $V_R(t)$.

Nonautonomous Chua's family circuit: The second example is a simple sinusoidal driven circuit shown in Fig.4(a) [14].

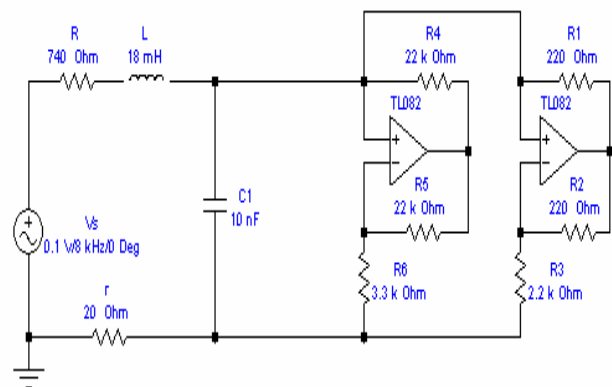


Fig. 4(a). Dissipative autonomous chaotic circuit

It consists of an external periodic source, a linear resistor, an inductor, a capacitor and the

nonlinear Chua's function NR. By applying Kirchoff's laws, the resultant state equations are:

$$C \frac{dv_1}{dt} = i_L - f(v_1)$$

$$L \frac{di_L}{dt} = -Ri_L - v_1 + a \sin(\omega t)$$

Where the nonlinear Chua's function, shown previously in Fig. 1(b), is described by equation (2). In addition to the implementation of the Chua's function, a small resistor, connected in series with the external source, is added to the circuit in order to probe the current i_L . The results of Electronics Workbench simulation show the phase portrait of the probed signal in Fig.4(b).

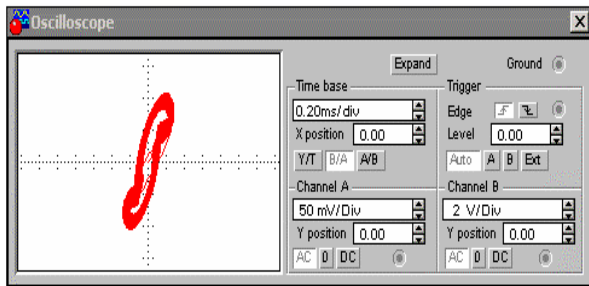


Fig. 4(b). Phase portrait of the probed signals V_{C1} versus V_r

This circuit was implemented and the phase portrait is shown in fig. 4(c). The voltage $v_{C1}(t)$ and its spectrum are shown in fig. 4(d). The reader is encouraged to experiment with this circuit by changing the amplitude of the external source as well as the frequency.

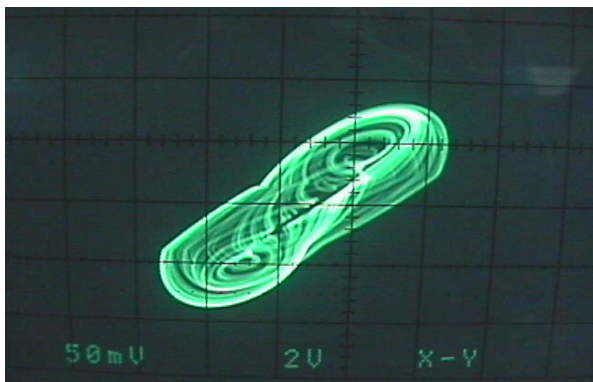


Fig.4(c): V_{C1} versus V_r for $R=740\Omega$, $f = 15\text{KHz}$ and $V_{in}\text{-peak-peak}=2\text{V}$

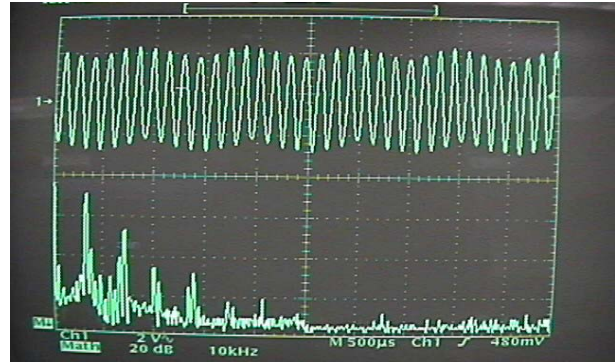


Fig.4(d):Top: $V_{C1}(t)$. Bottom: spectrum of $V_{C1}(t)$

Conclusion

Introducing the phenomena of chaos to students and practicing engineers is very important not only to investigate its existence even in simple nonlinear circuits but also to explore it and build some sophisticated applications. The circuits presented here are selected because of their simplicity from the mathematical point of view. Electronics Workbench simulation provides a virtual electronic lab environment to experiment with chaos. This will enhance the learning process by being able to make all circuit changes before purchasing any component to implement actual circuits. Complete circuits implementations are presented to show the existence of chaos behavior.

It is our hope that this paper will entice the reader to experiment with this nonlinear phenomena. Currently we are investigating the applications of chaos.

References

- [1] K. E. Lonngren, "Notes to accompany a student laboratory experiment on chaos," IEEE Transactions on Education, Vol. 34, No.1, February 1991.
- [2] D. C. Hamill, "Learning about chaotic circuits with Spices," IEEE Transactions on Education, Vol. 36, No.1, pp. 123-128, February 1993.

- [3] R. Madan, Chua's Circuit: A Paradigm for Chaos. Singapore: World Scientific, 1993.
- [4] M.P. Kennedy, "Three Steps to chaos-Part I: Evolution," IEEE Transaction on Circuits and Systems, Vol.40, no.10, pp.640-656, October 1993.
- [5] M.P. Kennedy, "Three Steps to chaos-Part II: A Chua's Circuit Primer," IEEE Transaction on Circuits and Systems, Vol.40, no.10, pp.657-674, October 1993.
- [6] L.O. Chua and G.N. Lin, "Canonical realization of Chua's circuit family," IEEE Transaction on Circuits and Systems, Vol.37, no.7, pp.885-902., 1990.
- [7] L.O. Chua, et Al, "A Universal Circuit for Studying and Generating Chaos -Part I : Routes to Chaos," IEEE Transaction on Circuits and Systems, Vol.40, no10, October 1993, pp.732-744.
- [8] L.O. Chua, et Al, "A Universal Circuit for Studying and Generating Chaos -Part II : Strange Attractors," IEEE Transaction on Circuits and Systems, Vol.40, no10, October 1993, pp.745-762.
- [9] R. Brown, "Generalization of the Chua's Equations," International Journal of Bifurcation and Chaos, Vol.2, no.4, pp.889-909, 1992.
- [10] M.P. Kennedy, "Chaos in the Colpitts Oscillator," IEEE Transaction on Circuits and Systems, Vol.41, pp.771-774., 1994.
- [11] M.P. Kennedy, "On the relationship Between the Chaotic Colpitts Oscillator and Chua's Oscillator," IEEE Transaction on Circuits and Systems-I, Vol.42, pp.376-379., June 1995.
- [12] T. Matsumoto, L. Chua and S. Tanaka, "Simplest Chaotic Nonautonomous Circuit," Phys. Rev A, Vol. 30, pp. 1155-1157, August 1984.
- [13] A. Azzouz and M. Hasler, "Orbits of the R-L-Diode Circuit," IEEE Transaction on Circuits and Systems, Vol.37, pp.1330-1339, November 1990.
- [14] K. Murali, M. Lakshmanan and L.O. Chua, "The Simplest Dissipative Nonautonomous Chaotic Circuit," IEEE Transaction on Circuits and Systems I, Vol.41, no.6, June 1994, pp. 462-463.