# Impedance-Transforming Lumped Element Two-Branch $90^{0}$ Couplers in Case of Type C 

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#### Abstract

An impedance-transforming lumped-element (LE) two-branch $90^{\circ}$ coupler is discussed in this paper. The LE coupler discussed here can be derived from a branch-line coupler consisting of one-quarter or three-quarters wavelength transmission-lines (that is, case of type C) and classified into five groups. Normally, the LE coupler needs 8 circuit elements. However, the existence of reduced LE couplers consisting of 6 circuit elements is shown in this paper during the process for deriving the five circuit groups. The circuit element values are given by terminal admittance $\mathrm{y}_{02}$ and power division ratio $\mathrm{k}^{2}$.


Key-Words: - Impedance-transforming, Lumped-element, Two-branch, $90^{\circ}$ coupler, 6 elements

## 1 Introduction

A branch-line (BL) $90^{\circ}$ coupler is a kind of co-directional coupler that divides incoming signals into two output ports. The phase difference of the output signals is odd multiples of $90^{\circ}$ [1], and the coupler is therefore important in the front end of transmitter/receiver systems [2], [3]. The fundamental circuit is composed of four quarter wavelength transmission-lines (TLs). The TL components occupy a large area on a printed circuit board and create dimensional problems in microwave integrated circuits, especially at frequencies below 10 GHz .

In recent years, lumped-elements (LEs) have become attractive in systems in which size-reduction techniques are important, since techniques for fabricating inductors and capacitors have improved even in the ultra-high frequency (UHF) band. Therefore, a capacitive coupled two-branch LE coupler was proposed by Gupta [4]. To achieve a wider bandwidth, broad-band matching design techniques were proposed by Vogel, Ohta, Chiang and Sakagami [1], [5]-[8].

Recently, an impedance-transforming LE two-branch 3-dB $90^{\circ}$ coupler has been discussed [9]. Therefore, more general discussion, including the case of unequal power division, will be presented under the condition of Gupta's circuit model [10].

The circuit shown in Fig. 1 consists of four TLs of which normalized characteristic admittances are $\mathrm{y}_{\mathrm{b}}, \mathrm{y}_{\mathrm{T}}$ and $y_{b 2}$. The normalized characteristic admittances at four terminals are 1 and $y_{02}$. The two lines between ports $\# 1$ and \#4 and between ports \#2 and \#3 are called branch lines. The other two lines are called through lines.

### 2.1 Four realizations

Four cases are considered in terms of wavelength $\lambda_{0}$ of center frequency $\mathrm{f}_{0}$.

Type A: The four TLs are all $\lambda_{0} / 4$ in length.
Type B: The four TLs are all $3 \lambda_{0} / 4$ in length.
Type C: The two branch lines are $\lambda_{0} / 4$ and the through lines are $3 \lambda_{0} / 4$ in length.

Type D : The two branch lines are $3 \lambda_{0} / 4$ and the through lines are $\lambda_{0} / 4$ in length.

Type A is well known and is called a branch-line coupler [2].

### 2.2 Design values

In terms of $S$ parameters, the ideal branch-line coupler must satisfy the following conditions at the center $f_{0}$ :

$$
\begin{equation*}
\mathrm{S}_{11}=\mathrm{S}_{41}=0, \text { and }\left|\mathrm{S}_{21}\right|:\left|\mathrm{S}_{31}\right|=1: \mathrm{k}, \tag{1}
\end{equation*}
$$

where k is the coupling factor.
The branch-line coupler admittances shown in Fig. 1 are given by [10]:

$$
\begin{equation*}
\mathrm{y}_{\mathrm{b} 1}=\mathrm{k}, \quad \mathrm{y}_{\mathrm{b} 2}=\mathrm{ky}_{02}, \text { and } \mathrm{y}_{\mathrm{T}}=\left\{\left(1+\mathrm{k}^{2}\right) \mathrm{y}_{02}\right\}^{0.5} . \tag{2}
\end{equation*}
$$



Fig. 1. Two-branch TL $90^{\circ}$ coupler.

## 3 LE Realizations in the Case of Type C

When two-branch TL $90^{\circ}$ couplers of types A and B are transformed into LE couplers, the circuit structure will be the same as that reported previously [5]. Therefore, we will discuss couplers of type C.

### 3.1 Equivalent transformation

A TL of line length $\lambda_{0} / 4\left(\right.$ or $\left.3 \lambda_{0} / 4\right)$ is shown in Fig. 2, where $z_{0}$ and $y_{0}$ are the normalized characteristic impedance and admittance, respectively. The single line can be transformed into $\pi$ equivalents as shown in Fig. 3 (a) and (b) [11]. The T equivalents are omitted here.


Fig. 2. A TL of $\lambda_{0} / 4$ (or $3 \lambda_{0} / 4$ ).

(a) In case of $\lambda_{0} / 4$.
(b) In case of $3 \lambda_{0} / 4$.

Fig. 3. $\pi$ equivalents, where $l$ and $c$ represent normalized inductance and capacitance.

### 3.2 Case of type C

When the relationship between Figs. 2 and 3 is applied to the circuit in the case of type C , the circuit shown in Fig. 4 can be obtained.

The series components are given by

$$
\begin{equation*}
\mathrm{c}_{\mathrm{T}}=\mathrm{y}_{\mathrm{T}}, l_{\mathrm{b} 1}=\mathrm{z}_{\mathrm{b} 1}, \text { and } l_{\mathrm{b} 2}=\mathrm{z}_{\mathrm{b} 2} . \tag{3}
\end{equation*}
$$

The shunt components are shown in Fig. 5 as $P_{1}$ - and $\mathrm{P}_{2}$-circuits, and the following equations hold:

$$
\begin{array}{ll}
\mathrm{c} l_{\mathrm{p} 1}=\mathrm{y}_{\mathrm{b} 1} \mathrm{Z}_{\mathrm{T}}=\left\{\mathrm{k}^{2} \mathrm{z}_{02} /\left(1+\mathrm{k}^{2}\right)\right\}^{0.5} & \text { for } \mathrm{P}_{1} \text {-circuit }, \\
\mathrm{c} l_{\mathrm{p} 2}=\mathrm{y}_{\mathrm{b} 2} \mathrm{Z}_{\mathrm{T}}=\left\{\mathrm{k}^{2} \mathrm{y}_{02} /\left(1+\mathrm{k}^{2}\right)\right\}^{0.5} & \text { for } \mathrm{P}_{2} \text {-circuit. } \tag{5}
\end{array}
$$

In general, the shunt circuit can be classified into three cases.

Case $1: c />1$, case of a capacitive circuit
Case 2 : $\mathrm{cl}=1$, case in which c and $l$ can be removed
Case $3: c l<1$, case of an inductive circuit.


Fig. 4. LE Circuit of type C.


Fig. 5. Shunt components in Fig. 4.

### 3.2.1 Case of $\mathrm{c}=1$

From the above three cases, cases 1, 2 and 3, a reduced element circuit, an LE coupler of 6 elements, can be expected.

From (4) under the condition of $\mathrm{cl}=1$,

$$
\begin{equation*}
\mathrm{Z}_{02}=1+\mathrm{k}^{-2} . \tag{6}
\end{equation*}
$$

From (5) under the condition of $\mathrm{c}=1$,

$$
\begin{equation*}
\mathrm{y}_{02}=1+\mathrm{k}^{-2} . \tag{7}
\end{equation*}
$$

It is found from (6) and (7) that the LE coupler of 6 elements can be realized only in the case of impedance transformation under the condition of finite power division, and the $\mathrm{P}_{1}$ - and $\mathrm{P}_{2}$-circuits can not be removed at the same time.
Now, let us show that the circuit shown in Fig. 4 can be classified into 5 groups.

### 3.2.2 Five circuit structures

From (4) and (5) and cases 1, 2, and 3, the following circuit structures and element values can be obtained:
(A) When $\mathrm{y}_{02}>1+\mathrm{k}^{-2},\left.\mathrm{c}\right|_{\mathrm{p} 1}<1$ and $\left.\mathrm{c} l\right|_{\mathrm{p} 2}>1$ hold. Fig. 6(a) shows the circuit in this case.

$$
\begin{equation*}
l_{a}=1 /\left(\mathrm{y}_{\mathrm{T}}-\mathrm{y}_{\mathrm{b} 1}\right) \text { and } c_{b}=\mathrm{y}_{\mathrm{b} 2}-\mathrm{y}_{\mathrm{T}} . \tag{8}
\end{equation*}
$$

(B) When $\mathrm{y}_{02}=1+\mathrm{k}^{-2},\left.\mathrm{c}\right|_{\mathrm{p} 1}<1$ and $\left.\mathrm{c} l\right|_{\mathrm{p} 2}=1$ hold. The resultant circuit is shown in Fig. 6(b).

$$
\begin{equation*}
l_{a}=1 /\left(\mathrm{y}_{\mathrm{T}}-\mathrm{y}_{\mathrm{b} 1}\right)=\mathrm{k} . \tag{9}
\end{equation*}
$$

(C) When $1 /\left(1+\mathrm{k}^{-2}\right)<\mathrm{y}_{02}<1+\mathrm{k}^{-2},\left.\mathrm{cl}\right|_{\mathrm{p} 1}<1$ and $\left.\mathrm{c}\right|_{\mathrm{p} 2}<1$ hold. The resultant circuit is shown in Fig. 6(c).

$$
\begin{equation*}
l_{a}=1 /\left(\mathrm{y}_{\mathrm{T}}-\mathrm{y}_{\mathrm{b} 1}\right) \text { and } l_{b}=1 /\left(\mathrm{y}_{\mathrm{T}}-\mathrm{y}_{\mathrm{b} 2}\right) . \tag{10}
\end{equation*}
$$

(D) When $\mathrm{y}_{02}=1 /\left(1+\mathrm{k}^{-2}\right),\left.\mathrm{c}\right|_{\mathrm{p} 1}=1$ and $\left.\mathrm{c}\right|_{\mathrm{p} 2}<1$ hold. The resultant circuit is shown in Fig. 6(d).

$$
\begin{equation*}
l_{b}=1 /\left(\mathrm{y}_{\mathrm{T}}-\mathrm{y}_{\mathrm{b} 2}\right)=\mathrm{k}+\mathrm{k}^{-1} . \tag{11}
\end{equation*}
$$

(E) When $\mathrm{y}_{02}<1 /\left(1+\mathrm{k}^{-2}\right),\left.\mathrm{c}\right|_{\mathrm{p} 1}>1$ and $\left.\mathrm{c}\right|_{\mathrm{p} 2}<1$ hold. The resultant circuit is shown in Fig. 6(e).

$$
\begin{equation*}
c_{a}=\mathrm{y}_{\mathrm{b} 1}-\mathrm{y}_{\mathrm{T}} \text { and } l_{b}=1 /\left(\mathrm{y}_{\mathrm{T}}-\mathrm{y}_{\mathrm{b} 2}\right) . \tag{12}
\end{equation*}
$$

### 3.2.3 Examples for the case of 2:1 power division

In the case of $2: 1$ power division, from (1) and (2), $\mathrm{k}=\mathrm{y}_{\mathrm{b} 1}=1 / 2^{0.5}, \mathrm{y}_{\mathrm{b} 2}=\mathrm{ky}_{02}$, and $\mathrm{y}_{\mathrm{T}}=\left(1.5 \mathrm{y}_{02}\right)^{0.5}$. Therefore, the series components shown in Fig. 4 or Fig. 6(a)-(e) are determined using (3) as follows:

$$
\begin{equation*}
\mathrm{c}_{\mathrm{T}}=\mathrm{y}_{\mathrm{T}}=\left(1.5 \mathrm{y}_{02}\right)^{0.5}, l_{\mathrm{b} 1}=\mathrm{z}_{\mathrm{b} 1}=2^{0.5}, \text { and } l_{\mathrm{b} 2}=\mathrm{z}_{\mathrm{b} 2}=2^{0.5} / \mathrm{y}_{02} \tag{13}
\end{equation*}
$$

The shunt components are determined by (8)-(12) according to $\mathrm{y}_{02}$.

In the following, calculated examples of $2: 1$ power division are presented for a $50-\Omega$ system.
(A) Case of $50-$ to $10-\Omega \operatorname{LE} 90^{\circ}$ coupler

Since $\mathrm{k}=1 / 2^{0.5}$ and $\mathrm{y}_{02}=5$, the circuit is given by Fig. 6(a). The element values are derived using (8) and (13):


Fig. 6. Five circuit structures for arbitrary $y_{02}$ and $k$.

$$
\begin{aligned}
& l_{\mathrm{a}}=2^{0.5} /\left(15^{0.5}-1\right), \mathrm{c}_{\mathrm{a}}=\left(5-15^{0.5}\right) / 2^{0.5}, \text { and } \\
& \mathrm{c}_{\mathrm{T}}=7.5^{0.5}, l_{\mathrm{b} 1}==^{0.5}, l_{\mathrm{b} 2}=2^{0.5} / 5 .
\end{aligned}
$$

(B) Case of 50- to 50/3- $\Omega$ LE $90^{\circ}$ coupler Since $\mathrm{k}=1 / 2^{0.5}$ and $\mathrm{y}_{02}=3$, the circuit is given by Fig. 6(b). The element values are

$$
l_{\mathrm{a}}=1 / 2^{0.5} \text {, and } \mathrm{c}_{\mathrm{T}}=4.5^{0.5}, l_{\mathrm{b} 1}=2^{0.5} \text {, and } l_{\mathrm{b} 2}=2^{0.5} / 3 .
$$

The frequency characteristics for the above two cases are shown in Fig. 7(a) and (b). The perfect input match, isolation, power transfer to port $\# 2,1.76 \mathrm{~dB}$, and power transfer to port $\# 3,4.77 \mathrm{~dB}$, are satisfied at the center frequency. Other examples are omitted.


Fig. 7. Frequency characteristics for $\mathrm{y}_{02}=5$ and 3 .

## 4 Conclusion

Impedance-transforming two-branch lumped element $90^{\circ}$ couplers have been investigated by introducing two-branch $90^{\circ}$ couplers consisting of one-quarter and three-quarters wavelength transmission-lines and Gupta's terminal conditions.

In this paper, the circuit structure is classified into five groups, and reduced lumped element $90^{\circ}$ couplers have been presented, as shown in Fig. 6 (b) and (d). It is proved that the reduced lumped element $90^{\circ}$ couplers are realizable only in the case of impedance transformation, as seen from (6) and (7). Although the circuit models shown in Fig. 6 (a), (c) and (e) were pointed out in [1], general and simple expressions for
the lumped element values, which can be determined by the coupling factor k and terminal load $\mathrm{y}_{02}$, have been presented for the first time. The circuit model for type D will be discussed in the near future.

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