Proceedings of the 10th WSEAS International Conference on CIRCUITS, Vouliagmeni, Athens, Greece, July 10-12, 2006 (pp212-216)

Decay Interval of the Inductor Currents in DC-to-DC Fourth-order PWM Converters

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Abstract: - This paper presents a computational algorithm for the parameter D_2 (quantity that determines the decay interval D_2T_s) in DC-to-DC fourth-order PWM converters (Cuk, SEPIC, Zeta, low-ripple input-current boost converter etc.), with discontinuous conduction mode and parasitic resistances. The computation algorithm, based on the steady-state model obtained through the state-space averaging method and easy implemented with MATLAB, supplies the value of the parameter D_2 that is a necessary quantity for dynamic and static analysis, and design of control loop of these converters. A fourth-degree equation has been obtained for the low-ripple input-current PWM boost converter, SEPIC, Zeta and Cuk PWM converters with parasitics. Including the parasitic resistances increases the degree of the equation with unknown D_2 and makes the value of this parameter dependent on the duty ratio, load resistance, all converter parameters and coupling coefficient of inductors.

Key-Words: - Decay interval of inductor currents, DC-to-DC fourth-order PWM converters

1 Introduction

The discontinuous conduction mode (DCM) of operation in the DC-to-DC PWM converters is a very interesting option for many applications. Many designers prefer to operate the converter in DCM either to avoid the reverse recovery problem of the diode or to simplify the control [1]. Selecting DCM as an operating mode it is possible to eliminate current sensing circuits, current loops and all associated circuitry. So, DCM is looked as an interesting option even for multiphase converters [2].

The steady-state properties of the converter (dc voltage conversion ratio M, input and output resistance, average output voltage, and input and output current) and dynamic (audio-susceptibility and control-to-output voltage transfer function, and input and output impedance) with discontinuous conduction mode (DCM) depend on the parameter D_2. This parameter characterizes the decay interval D_2T_s that is the second time interval within an operating cycle for the converter switched with constant frequency and DCM.

The most important approaches to model PWM converters in DCM are well analyzed in [3]. The control-to-inductor current transfer function obtained using the system impulse response reveals that the high-frequency dynamics of DCM converters are directly related to the duty ratio D_1 and to the delayed response of the inductor current. The decay interval D_2T_s is exactly the length of the converter impulse response, i.e., the length of the delayed response in the inductor current. Concerning the fast dynamics and delay effect associated with the inductor current, various aspects of averaged modelling of PWM converters in DCM have been studied in [1]. New full-order averaged models of the PWM converters with DCM were derived in order to identify the origin of the fast dynamics associated with the inductor current.

The topologies as Cuk, SEPIC, Zeta and low-ripple input-current PWM boost are known as fourth-order PWM converters. They consist of two inductors, two capacitors, and a single switch realized by transistor and diode combination. In these converters, the two inductors conduct some constant current I_0, even for the third switched interval D_3T_s. In this third switched interval, the sum of the two-inductor currents that traverses the diode is zero [4], [5].

Using the inductor voltage and current waveforms and the volt-second-balance law on the inductors, for an assumed 100% efficiency, the unknown D_2 has been easily found for a non-isolated Cuk converter (no parasitic resistances) with DCM as the positive root of a second-degree equation by the form [5]:

$$D_2 = \sqrt{k_t} \quad (1)$$

where

$$k_t = \frac{2L_e f_s}{R} \quad (2)$$

with $L_e=L_1//L_2$ playing the role of an effective inductance value. As it can be seen from (1) and (2), the length of the decay interval D_2T_s is independent of the duty ratio D_1 (controlled quantity). Similarly, for a low-ripple input-current PWM boost converter [6], the parameter D_2 has been found as

$$D_2 = \frac{k_t\left(1+\sqrt{1+4D_1^2/k_t}\right)}{2D_1}. \quad (3)$$

All the averaged models compared in [1] predict the same dc as well as low-frequency responses. Consequently, a reduced-order averaged model such that obtained through the state-space averaging approach can be used to analyze the steady-state (dc) and low-frequency properties of the converter. This approach associated with a numerical simulator such as MATLAB allow to conduct studies concerning the effect of some parameters and non-idealities on the behaviour of the PWM converter in DCM. The previous analyses based on the analog simulators as SPICE, SABER etc. that take into account the parasitics follow to highlight the impact of these parasitics on the global performance or the fast- and slow-scale instabilities of a converter and less on the behavioural parameters such as parameter D_2 and critical load resistance [1], [3], [6], [7].

The paper proposes a computational algorithm that allow to study the impact of the parasitics and some circuit parameters on the length of the decay interval of inductor currents in fourth-order PWM converters such as Cuk, SEPIC, Zeta and low-ripple input-current boost converters. The studies presented here for the coupled-inductor case are obtained using the MatLab package. The paper is organized as follows: The next section presents the derivation and the stages of the proposed algorithm. Section 3 provides some results obtained by numerical simulations and comments. Section 4 offers conclusions.

2 Computational algorithm

A general fourth-degree equation with unknown D_2 has been derived for four topologies of fourth-order converters with coupled or non-coupled inductors and parasitic resistances such as Cuk, SEPIC, Zeta and low-ripple input-current PWM boost. The parameter D_2 is found as the positive root of this equation of fourth degree, which accomplishes the condition $D_2<1-D$.

In order to derive the fourth-degree equation with unknown D_2, the steady-state model of these converters, obtained through the state-space averaging method (SSA), is used. Some notations used:

i_1, i_2 - the currents through the inductors L_1 and L_2

v_1, v_2 - the voltage drops across the capacitors C_1 and C_2

$x=[i_1 \quad i_2 \quad v_1 \quad v_2]^T$ - the state variable vector

v_I - the line voltage (input quantity)

i_I, i_J, v_O -the absorbed and injected currents, output voltage (output quantities)

$y=[i_I \quad i_J \quad v_O]^T$ - the output vector.

In this approach, the steady-state model is obtained in the form

$$X=-A^{-1}BV_I \quad (4)$$

$$Y=FX \quad (5)$$

with the constraint

$$I_1+I_2=\frac{D_1(D_1+D_2)V_I}{R_{nom}}=\frac{D_1(D_1+D_2)V_I}{Rk_t} \quad (6)$$

where R_{nom} is a design parameter defined by $R_{nom}=2L_e f_s$ [3]. The state and output vectors for the steady state are $X=[I_1 \quad I_2 \quad V_1 \quad V_2]^T$ and $Y=[I_I \quad I_J \quad V_O]^T$. The (4x4) A, (4x1) B and (3x4) F matrices that describe the averaged behavior of the converter on an operating cycle are [1], [6]:

$$A=D_1A_1+D_2A_2+D_3A_3 \quad (7)$$

$$B=D_1B_1+D_2B_2+D_3B_3 \quad (8)$$

$$F=D_1F_1+D_2F_2+D_3F_3 \quad (9)$$

with $D_3=1-D-D_2$ from the assumed constant switching frequency f_s. The matrices A_i, B_i and F_i with $i=\overline{1,3}$ describe the three-switched networks of the converter with DCM. The elements of these matrices depend on all the parameters of the circuit (L_1, L_2, C_1, C_2, and R), the coupling coefficient k_c of the two inductors in the coupled inductor case and the duty ratio D_1. The elements of the matrices are functions on the parasitic resistances if these are included, namely: the loss resistance of the inductors (r_1 and r_2), the equivalent-series resistance of the capacitors (r_3 and r_4), and the conducting-state resistance of the transistor (r_5) and diode (r_6). The steady-state models for the four topologies of the fourth-order converters here considered differ from one another only by the number of the non null elements in the matrices A, B and F, and certainly by the make-up of the elements of the matrices A_i, B_i and F_i.

In order to cover all these four topologies of fourth-order PWM converters previously specified, the matrices A, B and F of the system described by (4) and (5) are chosen by the form:

$$A = \begin{bmatrix} a(1,1) & a(1,2) & a(1,3) & a(1,4) \\ a(2,1) & a(2,2) & a(2,3) & a(2,4) \\ a(3,1) & a(3,2) & 0 & 0 \\ a(4,1) & a(4,2) & 0 & a(4,4) \end{bmatrix}, \quad B = \begin{bmatrix} b(1) \\ b(2) \\ 0 \\ 0 \end{bmatrix}$$

$$F = \begin{bmatrix} c(1,1) & c(1,2) & 0 & 0 \\ c(2,1) & c(2,2) & 0 & 0 \\ c(3,1) & c(3,2) & 0 & c(3,4) \end{bmatrix}.$$

The two currents I_1 and I_2 expressed from (4) are functions of parameter D_2 and their sum can be set in the form

$$I_1 + I_2 = \frac{e_4 + e_5 D_2 + e_6 D_2^2}{e_0 + e_1 D_2 + e_2 D_2^2 + e_3 D_2^3}. \quad (10)$$

Equaling (10) with the constraint (6), a fourth-degree equation in unknown D_2 results:

$$m_4 D_2^4 + m_3 D_2^3 + m_2 D_2^2 + m_1 D_2 + m_0 = 0. \quad (11)$$

In order to compute the parameter D_2, the matrices A and B are set in the form:

$$A = A_e + D_2 A_{23}$$
$$B = B_e + D_2 B_{23}$$

where

$$A_e = D_1 A_1 + (1 - D_1) A_3 = [a_e(m,n)]$$
$$B_e = D_1 B_1 + (1 - D_1) B_3 = [b_e(m)]$$
$$A_{23} = A_2 - A_3 = [a_{23}(m,n)]$$
$$B_{23} = B_2 - B_3 = [b_{23}(m)].$$

The computation algorithm of the parameter D_2 has two main stages:

1.Determine the elements of (4x4) A_i, (4x1) B_i and (3x4) F_i matrices that describe the three switched-networks of the power stage in DCM:

$A_i = [a_i(m,n)]$, $B_i = [b_i(m)]$, $F_i = [c_i(l,n)]$ with $i = l = \overline{1,3}$ and $m = n = \overline{1,4}$.

2. Solve (11) and find the parameter D_2 as the solution that accomplishes the conditions: $D_2 \in \Re_+$ and $D_2 \langle 1 - D$.

The coefficients from (11) have the following expressions:

$m_4 = D_1 g_{16}$; $m_3 = D_1^2 g_{16} + D_1 g_{15}$;

$m_2 = D_1^2 g_{15} + D_1 g_{14} - R_{nom} g_{19}$;

$m_1 = D_1^2 g_{14} + D_1 g_{13} - R_{nom} g_{18}$;

$m_0 = D_1^2 g_{13} - R_{nom} g_{17}$.

The factors that appear in the coefficients in (11) are computed with the following formula:

$$g_0 = a_e(4,4) a_e(3,2) [a_e(1,3) a_e(2,1) - a_e(1,1) a_e(2,3)]$$

$$g_1 = a_e(4,4) \times \left\{ \begin{array}{l} a_e(3,2) \begin{bmatrix} a_e(2,1) a_{23}(1,3) + a_e(1,3) a_{23}(2,1) \\ - a_e(2,3) a_{23}(1,1) - a_e(1,1) a_{23}(2,3) \end{bmatrix} + \\ a_{23}(3,1) [a_e(1,3) a_e(2,1) - a_e(1,1) a_e(2,3)] \end{array} \right\}$$

$$g_2 = a_e(4,4) \times \left\{ \begin{array}{l} a_e(3,2) [a_{23}(2,1) a_{23}(1,3) - a_{23}(1,1) a_{23}(2,3)] + \\ a_{23}(3,1) \begin{bmatrix} a_{23}(1,3) a_e(2,1) + a_e(1,3) a_{23}(2,1) \\ - a_e(2,3) a_{23}(1,1) - a_e(1,1) a_{23}(2,3) \end{bmatrix} \end{array} \right\}$$

$$g_3 = a_e(4,4) a_{23}(3,1) \begin{bmatrix} a_{23}(1,3) a_{23}(2,1) - \\ a_{23}(1,1) a_{23}(2,3) \end{bmatrix}$$

$$g_4 = a_e(1,3) a_e(2,4) - a_e(1,4) a_e(2,3)$$

$$g_5 = a_e(2,4) a_{23}(1,3) + a_e(1,3) a_{23}(2,4) - a_e(2,3) a_{23}(1,4) - a_e(1,4) a_{23}(2,3)$$

$$g_6 = a_{23}(1,3) a_{23}(2,4) - a_{23}(1,4) a_{23}(2,3)$$

$$g_7 = a_e(3,1) a_e(4,2)$$

$$g_8 = a_e(4,2) a_{23}(3,1) + a_e(3,1) a_{23}(4,1) - a_e(3,2) a_{23}(1,1)$$

$$g_9 = a_e(4,4) a_e(3,1) \begin{bmatrix} a_e(1,2) a_e(2,3) - \\ a_e(1,3) a_e(2,2) \end{bmatrix}$$

$$g_{10} = a_e(4,4) \times \left\{ \begin{array}{l} a_e(3,1) \begin{bmatrix} a_e(2,3) a_{23}(1,2) + a_e(1,2) a_{23}(2,3) \\ - a_e(2,2) a_{23}(1,3) - a_e(1,3) a_{23}(2,2) \end{bmatrix} + \\ a_{23}(3,1) [a_e(1,2) a_e(2,3) - a_e(1,3) a_e(2,2)] \end{array} \right\}$$

$$g_{11} = a_e(4,4) \times \left\{ \begin{array}{l} a_e(3,1) [a_{23}(2,3) a_{23}(1,2) - a_{23}(1,3) a_{23}(2,2)] + \\ a_{23}(3,1) \begin{bmatrix} a_{23}(1,2) a_e(2,3) + a_{23}(2,3) a_e(1,2) \\ - a_e(2,2) a_{23}(1,3) - a_e(1,3) a_{23}(2,2) \end{bmatrix} \end{array} \right\}$$

$$g_{12} = a_e(4,4) a_{23}(3,1) \times [a_{23}(1,2) a_{23}(2,3) - a_{23}(1,3) a_{23}(2,2)]$$

$$g_{13} = g_0 + g_4 g_7 + g_9$$

$$g_{14} = g_1 + g_5 g_7 + g_4 g_8 + g_{10}$$

$$g_{15} = g_2 + g_6 g_7 + g_5 g_8 + g_{11}$$

$$g_{16} = g_3 + g_6 g_8 + g_{12}$$
$$g_{17} = a_e(4,4)[a_e(3,2) - a_e(3,1)] \times [b_e(1)a_e(2,3) - b_e(2)a_e(1,3)]$$
$$g_{18} = a_e(4,4)[a_e(3,2) - a_e(3,1)] \times \begin{bmatrix} b_{23}(1)a_e(2,3) + b_e(1)a_{23}(3,2) \\ - b_{23}(2)a_e(1,3) - b_e(2)a_{23}(1,3) \end{bmatrix}$$
$$g_{19} = a_e(4,4)[a_e(3,2) - a_e(3,1)] \times [b_{23}(1)a_{23}(2,3) - b_{23}(2)a_{23}(1,3)] .$$

The coefficients m_i with $i = \overline{1,4}$, from (11) depend on all the parameters of the circuit elements (L_1, L_2, C_1, C_2, R), the duty ratio D_1, the switching frequency f_s, the parasitic resistances (r_1, r_2, r_3, r_4, r_5, r_6) and the coupling coefficient k_c of the inductors. Consequently, D_2 will be dependent on all these parameters, unlike the no parasitic case when D_2 only depends on L_1, L_2, R, D_1, k_c and f_s.

MATLAB environment offers a very simple implementation of the computational algorithm proposed in this paper.

3 Simulation Results

The effect of the load resistance, duty ratio, coupling coefficient and capacities C_1 and C_2 over the parameter D_2 has been studied on a low-ripple input-current PWM boost converter with the specifications: L_1=5.1 μH, L_2=0.7 μH, C_1=18 μF, C_2=1000 μF and f_s=300 kHz [4]. The following parasitic resistances have been considered here: r_1=r_2=r_4=r_5=r_6=0.1 Ω and r_3=0.01 Ω. The diagram of this modified PWM boost converter showing the parasitics is given in Fig. 1.

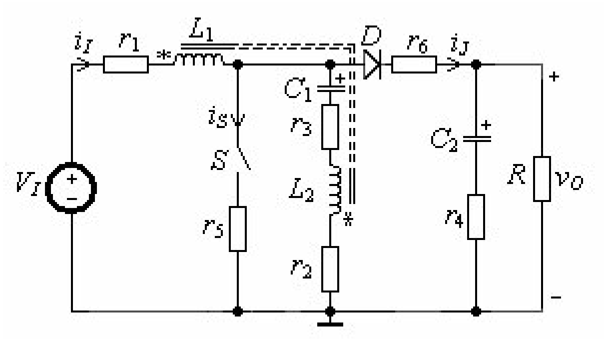

Fig. 1. The topology of the modified boost PWM converter with parasitics

Some simulation results obtained by means of the computational algorithm implemented with MATLAB package are shown in Fig. 2 and 3.

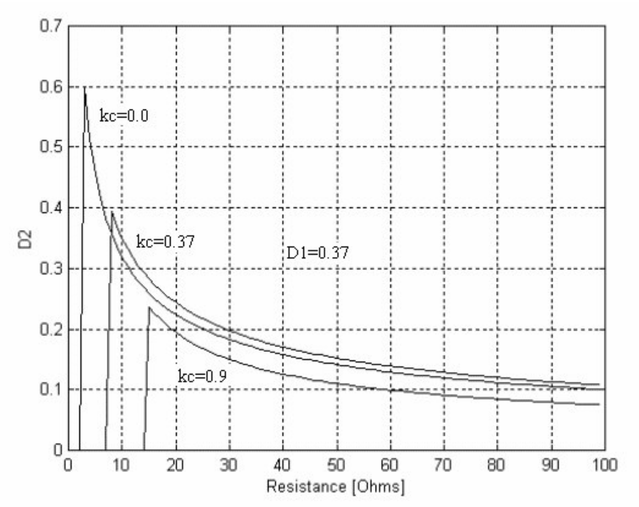

Fig. 2. D_2 plots versus the load resistance for several values of the coupling inductor coefficient

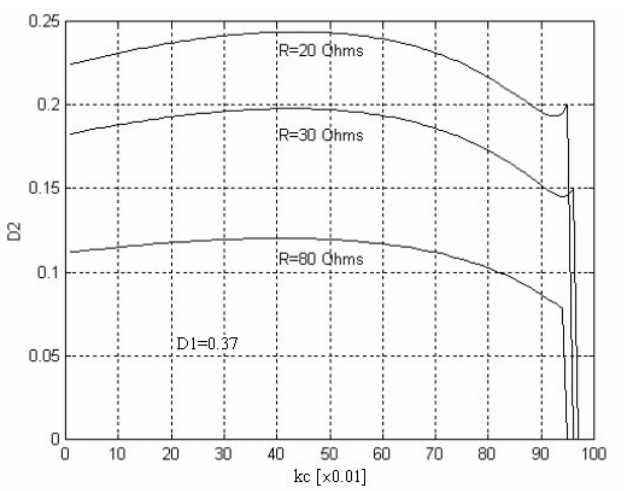

Fig. 3 D_2 plots versus the coupling inductor coefficient for several values of the load resistance

The plots of the parameter D_2 versus load resistance for constant switching frequency and duty ratio show that the decay interval length of the inductor currents is a continuous function on the coupling coefficient k_c from k_c=0 (separate inductors) to k_c=0.9, with a flat maximum between k_c=0.4 and k_c=0.5.

The effect of the duty ratio D_1 and load resistance R is to cause D_2 to decrease. The maximum value of D_2 decreases with the increasing of D_1 and R. None influence has the increasing and decreasing of the values C_1 or C_2 over the value of D_2.

After calculating the parameter D_2, the steady-state model of the converter with DCM (with coupled or separate inductors), in the form given by (4) and (5), and the dynamic (ac small signal) properties can be computed. The external characteristics of converter, $M = f\left(R / R_{nom}\right)$, and the boundary between the continuous and discontinuous conduction modes (as R_{crit} or k_{emcrit}) can be calculated and plotted too.

4 Conclusion

The paper proposes a computational algorithm for the parameter D_2, the necessary quantity for the steady-state and dynamic analysis, and the design of control loop of DC-to-DC fourth-order PWM converters with DCM and parasitic resistances (Cuk, SEPIC, Zeta, low-ripple input-current PWM boost converter etc.).

This computational algorithm supplies the value of the parameter D_2 taking into account the effect of all the parasitic resistances (loss resistance of inductors, equivalent-series resistance of capacitors, resistance of transistor and diode in conducting state etc.). The algorithm is based on the steady-state model obtained through the state-space averaging method. It is easy implemented with MATLAB package.

Unlike the no parasitic case when the value of parameter D_2 is found by solving a second-degree equation, this parameter is the positive root of fourth-degree equation with the unknown D_2. Moreover, the value of this parameter depends on the operating conditions (duty ratio and load resistance), all the parameters of the circuit elements and their parasitics, the switching frequency and the coupling coefficient of the inductors.

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